On the Relation between Private Information and Non-Fundamental Volatility*

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Abstract

It is a well-established property that more precise private information leads to lower non-fundamental volatility in a coordination economy with dispersed information. In this note, we identify conditions under which such an argument holds or does not hold. In particular, we show that the opposite relationship holds when (1) there is a strong positive correlation between private information of different agents and (2) public information is endogenously generated.

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1 Introduction

In many economic environments, economic agents receive incomplete information on variables of interests, in the form of private and public signals à-la Morris and Shin (2002), and consider both the value of such variables (i.e. economic fundamentals) and actions of other agents when making decisions. Other agents' actions are particularly important in the environment where there exists strategic complementarity (eg. Bertrant competition) or substitutability (eg. Cournot competition) in actions. Because of such strategic considerations, actions of economic agents are affected by not only fundamentals but also non-fundamental factors such as noise in signals and others' actions. This raises non-fundamental volatility in actions at the aggregate level, which has been pointed out as an important source of the fluctuations in the previous literature (Angeletos and Pavan (2007); and Hébert and La'O (2021)).

Importantly, non-fundamental volatility becomes lower as private information that each agent receives becomes more precise (which we call an "individual signal channel"), regardless of the underlying strategic motives. This is because agents equipped with better private information put more weights on their own private signals and hence conditional on the fundamental value, θ, aggregate action will be less dependent on public information (Angeletos and Pavan (2004)). As pointed out by Angeletos and Werning (2006) and Rondina and Shim (2015), however, public information is endogenously generated in the presence of the financial market and such a property would alter the equilibrium properties held in the economy with exogenous public information. While consequences of the endogenous public information on the equilibrium have been widely studied in the previous literature, less attention has been paid to the question if the equilibrium property of non-fundamental volatility mentioned above still holds in the economy in which financial price serves as an endogenous public signal. This paper aims to fill this gap in the literature.

In particular, we generalize a model introduced in Rondina and Shim (2015) in two dimensions: First, we introduce a coordination economy that can exhibit both complementarity and substitutability in actions. The outcome from the coordination economy determines the return of an asset traded in the financial market where the price of the asset provides public information. Second, a connectedness of private information between different types of agents is generalized. In our model economy, there are two types of agents: a player in the coordination game who receives private information with precision $\alpha_x > 0$ and a trader in the financial market who receives private information with precision $\alpha_y > 0$. In order to identify the role of information structure in determining non-fundamental volatility, we assume that α_y is non-negatively related to α_x ($\alpha_y = f(\alpha_x)$ with $f'(\cdot) \geq 0$).[1] This nests both Angeletos and Werning (2006) ($f(\alpha_x) = \alpha_x$) and Rondina and Shim (2015) ($f(\alpha_x) = c$) as special cases, and hence we refer to our model economy a "linked-precision" economy.[2]

Equipped with this model, we identify conditions under which non-fundamental volatility is an increasing, decreasing, or constant function of the precision of a private signal for the player. We find that the extent to which private signals of different types of agents are related is crucial: As is well-documented in Rondina and Shim (2015), public information becomes more (resp. less) precise when private information of the player (resp. trader) improves while that of the trader (resp. player) is held constant. We refer to this particular effect arising from the changes in public information a "public signal channel." When public information improves with better private information, it raises dependence of the private action on public information so that it can potentially counteract the individual signal channel. If there is no connection between the two private signals (Rondina and Shim (2015)), for example, this channel amplifies the individual signal channel as public signal becomes *less* precise due to heightened strategic uncertainty. If there is a positive relationship between the two private signals, public signal can improve even when the private information of the player improves because that of the trader also becomes more precise ($f'(\cdot) > 0$). Hence, the public signal channel would counteract the individual signal channel when the change in the precision of the trader's private signal substantially dominates that of player's. In this case, public signal would become sufficiently *more* precise to increase non-fundamental volatility of the aggregate action. We further show that a linear relationship ($\alpha_y = \alpha_x$) assumed in Angeletos and Werning (2006) is a knife-edge case in which both channels are of the same magnitude so that non-fundamental volatility and precision of the player's private signal are seemingly unrelated at the equilibrium.

Our finding contributes to the literature in twofold. First, a careful modelling of the information structure in this class of models may be more important than previously thought, as it delivers very different equilibrium properties. For example, different from a usual conjecture (for example, see Atkeson (2000)), endogeneity of public information does not always lead to different implications from those of models with an exogenous public signal. Hence, it should be carefully examined with alternative information structures. Second, key findings of this paper have implications on the literature on in-

[1]We come back to the non-negativity assumption in Section 3.2.

[2]We thank an anonymous referee for suggesting a proper name for our model economy.

formation design (see Bergemann and Morris (2019) for a nice summary on the literature), which has been recently burgeoned: individual welfare as well as social welfare are functions of non-fundamental volatility. So whether non-fundamental volatility increases in the precision of private signals or not has impacts on the optimal design of information.

2 The Model

Since the model environment is identical to Rondina and Shim (2015), except that the payoff structure of the coordination game is generalized, we only introduce the key parts of the model. Derivation of key equilibrium conditions can be found in Rondina and Shim (2015).

2.1 Financial Market The financial market is modeled as in a stylized CARA-Gaussian model. There is a continuum of risk-averse traders $j \in [0, 1]$ whose utility function is given by

$$u_j = V(w_j) = -e^{\gamma w_j} \tag{2.1}$$

subject to the budget constraint $w_j = w_0 + (f - p)k_j$. k_j denotes a trader j's demand for the risky asset, p is the price of the risky asset, f is a dividend or return, and w_0 is the initial wealth. The dividend is defined as $f = A(\theta, p)$ where $A(\theta, p)$ is an aggregate action that is determined at the coordination game where the aggregate action is assumed to increase the return.[3] Traders predict the dividend f conditional on the equilibrium price p and a private signal $y_j = \theta + \frac{1}{\sqrt{\alpha_y}}\epsilon_j$, where $\epsilon_j \sim \mathcal{N}(0, 1)$ and ϵ_j is i.i.d. across traders.

The exogenous aggregate supply of the asset is given by $\mathbb{K}^s(\xi) = \frac{1}{\sqrt{\alpha_\xi}}\xi$ with $\xi \sim \mathcal{N}(0, 1)$ to avoid a fully-revealing equilibrium. The equilibrium strategy of an individual trader is then determined as follows.

$$k(y, p) = \frac{\mathbb{E}(f|y, p) - p}{\gamma \mathbb{V}(f|y, p)} = \frac{\alpha_y}{\gamma\lambda}(y - p) \tag{2.2}$$

and hence $\mathbb{K}^d = \frac{\alpha_y}{\gamma\lambda}(\theta - p)$.

[3] This payoff structure is to keep tractability of the model.

2.2 Coordination Game

There is a continuum of players $i \in [0,1]$. A player's simple quadratic utility function is given by

$$u(a_i, A, \theta) = -\frac{1}{2}(a_i - (1-r)\theta - rA)^2 \tag{2.3}$$

where $A = \int_0^1 a_i di$ and $r \in (-1,1)$ measures complementarity ($r > 0$) and substitutability ($r < 0$) in the coordination game. θ denotes the fundamental value of this economy and is assumed to be drawn from an improper uniform distribution on $\mathbb{R}$. Each player receives a private signal x_i with precision α_x and a public signal p with precision α_p, which are defined as $x_i = \theta + \frac{1}{\sqrt{\alpha_x}}\epsilon_i$ and $p = \theta + \frac{1}{\sqrt{\alpha_p}}\xi$ where $\epsilon_i, \xi \sim \mathcal{N}(0,1)$ and θ, ϵ_i, and ξ are independent and uncorrelated.

Following Angeletos and Pavan (2007) and Rondina and Shim (2015), the equilibrium strategy of a player i that observes a private signal x_i and a public signal p is determined as follows.

$$a_i(x_i, p) = \lambda x_i + (1-\lambda)p \tag{2.4}$$

where $\lambda \equiv \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}$.

The aggregate action across players is defined as

$$A(\theta, p) = \lambda\theta + (1-\lambda)p \tag{2.5}$$

3 Non-Fundamental Volatility: the Role of Information Structure and Financial Prices

In this section, we first provide key equilibrium conditions and properties and then analyze the extent to which alternative information structures change the previous result on the relationship between the non-fundamental volatility and precision of private information.

3.1 Equilibrium

Using the market clearing condition, $\mathbb{K}^s(\xi) = \mathbb{K}^d$, we can obtain the following fixed point condition for α_p, the precision of public information.

$$\alpha_p = \alpha_\xi \left(\frac{\alpha_y}{\gamma\lambda}\right)^2 = \alpha_\xi \left(\frac{\alpha_y}{\gamma}\right)^2 \left(\frac{\alpha_p}{(1-r)\alpha_x} + 1\right)^2 \tag{3.1}$$

As is discussed in Rondina and Shim (2015), there might exist two solutions to the above equation and only one of them, the low-precision solution, is stable.[4] Thus, we only focus on a stable positive solution of α_p^* satisfying equation (3.1), whose exact form is given as follows.

$$\alpha_p^* = \frac{(1-(\alpha_\xi(\frac{\alpha_y}{\gamma})^2\frac{2}{(1-r)\alpha_x})) - \sqrt{((\alpha_\xi(\frac{\alpha_y}{\gamma})^2\frac{2}{(1-r)\alpha_x})-1)^2 - 4(\alpha_\xi(\frac{\alpha_y}{\gamma(1-r)\alpha_x})^2\alpha_\xi(\frac{\alpha_y}{\gamma})^2)}}{2\alpha_\xi(\frac{\alpha_y}{\gamma(1-r)\alpha_x})^2} \tag{3.2}$$

$2\alpha_y\sqrt{\alpha_\xi}/\gamma < \sqrt{(1-r)\alpha_x}$ is assumed to hold to ensure the existence of the solution.

We first present the following two lemmas that will be useful to understand our main findings.

Lemma 1. *Suppose that the solution to the above equation (3.2) exists. Then* $(1-r)\frac{\alpha_x}{\alpha_p^*} - 1 > 0$ *always hold.*

Proof. Define $Q \equiv \frac{4\left(\frac{\alpha_y}{\gamma}\right)^2\alpha_\xi}{(1-r)\alpha_x}$. The condition for the stable solution $((1-r)\alpha_x > 4(\alpha_y/\gamma)^2\alpha_\xi)$ implies $Q \in (0,1)$ and hence $\sqrt{1-Q} > 1-Q$. This implies $Q > 1-\sqrt{1-Q}$. Using the definition of Q, we can obtain $(1-r)\frac{\alpha_x}{\alpha_p^*} - 1 > 0$. □

Lemma 2 (Precision of Public Information)**.** *Precision of public information,* α_p^**,* ***increases*** *when* α_x *decreases or* α_y *increases. Formally,*

$$\frac{\partial\alpha_p^*}{\partial\alpha_x} < 0 \quad \textit{and} \quad \frac{\partial\alpha_p^*}{\partial\alpha_y} > 0 \tag{3.3}$$

Proof. Refer to Rondina and Shim (2015). □

For a detailed interpretation of the second lemma, we refer to Rondina and Shim (2015) and present only the key intuitions here. A negative effect of α_x on the precision of public information arises from heightened strategic uncertainty, which lowers a demand for assets as the improvement of player's private information is not associated with that of trader's private information. This results in less information aggregation at the financial market. A positive effect of α_y on the precision of public signal, on the other hands, comes from more information aggregation from a greater asset demand by traders since (aggregate) uncertainty about the state of the economy becomes smaller for them.

Following the literature, we measure non-fundamental volatility as the variance of aggregate action conditional on fundamentals:

[4] See also Vives (2008); Ganguli and Yang (2009); and Manzano and Vives (2011) for more related discussions.

$$
\begin{aligned}
Var(A(\theta,p)|\theta) &= \frac{(1-\lambda)^2}{\alpha_p} = \frac{\left(\frac{\frac{\alpha_p}{1-r}}{\alpha_x+\frac{\alpha_p}{1-r}}\right)^2}{\alpha_p} \\
&= \frac{1}{\left((1-r)\frac{\alpha_x}{\sqrt{\alpha_p}}+\sqrt{\alpha_p}\right)^2}
\end{aligned}
\tag{3.4}
$$

3.2 Non-Fundamental Volatility in Each Economy In order to examine the effect of the changes in the precision of private information on the non-fundamental volatility, we first consider a benchmark economy with an exogenous public signal, and then generalize the information structure to isolate the contribution of each change on the result.

Exogenous Economy. We first consider a benchmark economy, which we refer to an exogenous economy, by assuming a fixed α_p (c.f., Morris and Shin (2002) is one particular example of such an economy). In this economy, there is a negative relationship between the precision of private information and non-fundamental volatility:

Proposition 1 (Exogenous Economy)**.** *Suppose that α_x increases. Then non-fundamental volatility, $Var(A(\theta,p)|\theta)$, strictly* ***decreases****. Formally,*

$$
\frac{\partial Var(A(\theta,p)|\theta)}{\partial \alpha_x} < 0 \tag{3.5}
$$

Proof. If we differentiate the equation (3.4) with respect to α_x, we obtain

$$
\frac{\partial Var(A(\theta,p)|\theta)}{\partial \alpha_x} = -\left[\underbrace{\frac{2(1-r)}{\sqrt{\alpha_p}\left((1-r)\frac{\alpha_x}{\sqrt{\alpha_p}}+\sqrt{\alpha_p}\right)^3}}_{\text{IS}>0}\right] < 0 \tag{3.6}
$$

□

As a result, non-fundamental volatility always declines in the exogenous economy regardless of the coordination motive (r). Since the public signal is exogenous, changes in α_x affect the action of each agent only through private information (and changes in weights). We call this "individual (private) signal (IS) channel." Note that $\frac{\partial \lambda}{\partial \alpha_x} > 0$: When the precision of private signal improves, each player depends more on its own private signal when making a decision. Thus, the aggregate action becomes less dependent on the public signal, and hence non-fundamental volatility strictly decreases as α_x increases.

Linked-Precision Economy. We now consider an alternative economy in which (1) α_p is determined by the equilibrium condition (3.2) and (2) precision of private information for the player and that for the trader are potentially connected.[5] In particular, we assume that $\alpha_y = f(\alpha_x)$ where $f(\cdot)$ can take any arbitrary functional forms but satisfies $f'(\cdot) \geq 0$.[6] This captures the idea that improvement of private information might potentially have a positive spill-over effect on the (private) information of other type of agents. For instance, $f(\alpha_x) = \alpha_x > 0$ nests Angeletos and Werning (2006) as a special case and $f(\alpha_x) = c > 0$ nests Rondina and Shim (2015) as another special case. In the sense that we allow precisions of private signals to be linked, we refer to this economy as a "linked-precision economy."

In this economy, an interesting result arises as is summarized in the following proposition.

Proposition 2 (Linked-Precision Economy)**.** *Consider the economy in which $\alpha_y = f(\alpha_x)$ holds and precision of the public signal is determined by equation (3.1). If α_x increases, the followings hold.*

1. *Non-fundamental volatility, $Var(A(\theta, p)|\theta)$, decreases when $f(\alpha_x) > \alpha_x f'(\alpha_x)$.*

2. *Non-fundamental volatility, $Var(A(\theta, p)|\theta)$, does not change when $f(\alpha_x) = \alpha_x f'(\alpha_x)$.*

3. *Non-fundamental volatility, $Var(A(\theta, p)|\theta)$, increases when $f(\alpha_x) < \alpha_x f'(\alpha_x)$.*

Proof. See Appendix.

□

Hence, in opposite to the conventional wisdom that better private information lowers non-fundamental volatility at the aggregate level, the relationship between the two crucially depends on the extent to which private information of different agents are connected. In order to obtain the intuition behind the above proposition, we differentiate the equation (3.4) with respect to α_x:

[5] We appreciate two anonymous referees for giving us insightful comments to generalize the findings reported in the previous draft.

[6] If $f'(\cdot) < 0$ (private information of an agent is negatively related to that of the other agent), non-fundamental volatility is always negatively related to the precision of private information, which is a trivial result that directly follows from Proposition 2.

$$\frac{\partial Var(A(\theta,p)|\theta)}{\partial \alpha_x} = \underbrace{-\frac{2}{\sqrt{\alpha_p^*}\left((1-r)\frac{\alpha_x}{\sqrt{\alpha_p^*}} + \sqrt{\alpha_p^*}\right)^3}}_{<0} \times \left[\underbrace{(1-r)}_{\text{IS}} \underbrace{-\frac{1}{2}\frac{\partial \alpha_p^*}{\partial \alpha_x}\left((1-r)\frac{\alpha_x}{\alpha_p^*} - 1\right)}_{\text{PS}}\right] \tag{3.7}$$

Note that in addition to the 'IS' channel, there is an additional channel, Public Signal (PS) channel, through which the changes in α_x affect non-fundamental volatility. Since $(1-r)\alpha_x > \alpha_p^*$ holds (Lemma 1), it suffices to check the changes in public information with respect to the changes in precision of the player, α_x. Different from the case with the exogenous economy, better private information affects non-fundamental volatility also through the changes in the informativeness of public signal ($\frac{\partial \alpha_p^*}{\partial \alpha_x}$). However, the direction is indeterminate: if the PS channel takes a positive value, it reinforces the IS channel (the first part of proposition). If the PS channel is negative of the small magnitude, it cannot counteract the IS channel and the first part of Proposition 2 still holds. If both the IS channel and PS channel are of the same magnitude but with the opposite sign, the non-fundamental volatility is independent from the changes in the precision of private information (the second part of proposition). On the contrary, if PS channel is negative and of large magnitude, it can overturn Proposition 1 (the third part of proposition 2).

To further understand our finding, consider first the economy introduced in Rondina and Shim (2015), which satisfies the first part of the above Proposition as they assume that private information of different agents are independent from each other ($f'(\alpha_x) = 0$). If α_x becomes higher, the precision of the public information becomes lower without any ambiguity (Lemma 2). Hence, $\frac{\partial \alpha_p^*}{\partial \alpha_x} < 0$ makes the PS term to be always positive, which reinforces the IS channel. This is summarized by the following corollary:

Corollary 1 (Rondina-Shim Economy)**.** *Suppose that α_x increases and $\alpha_y = c > 0$. Then non-fundamental volatility, $Var(A(\theta,p)|\theta)$, strictly* ***decreases****. Moreover, non-fundamental volatility changes* *more than the exogenous economy. Formally,*

$$\frac{\partial Var(A(\theta,p)|\theta)}{\partial \alpha_x} < \left.\frac{\partial Var(A(\theta,p)|\theta)}{\partial \alpha_x}\right|_{exogenous\ economy} < 0 \tag{3.8}$$

Proof. Omitted because this is a direct implication of Proposition 2. □

Hence, there exists a "multiplier" effect from the financial market. That is, the effect of better private information on the aggregate non-fundamental volatility is exacerbated in the presence of the financial market in the Rondina-Shim economy.[7]

How about the economy introduced in Angeletos and Werning (2006)? In their model, $f(\alpha_x) = \alpha_x$ and hence it satisfies the second part of Proposition 2. Notice that α_y, the precision of the trader's private signal, also improves when α_x becomes higher. While α_x and α_y affect the precision of the public signal in the opposite directions, the effect of α_y dominates that of α_x as is shown in Rondina and Shim (2015), and hence $\frac{\partial \alpha_p^*}{\partial \alpha_x} > 0$ holds. However, such an improvement is exactly enough to cancel out the effect of the IS channel on the non-fundamental volatility, and hence we obtain the knife-edge case that is summarized in the following corollary.[8]

Corollary 2 (Angeletos-Werning Economy)**.** *Suppose that α_x increases and $\alpha_y = c\alpha_x$ with $c > 0$. Then non-fundamental volatility, $Var(A(\theta,p)|\theta)$, does not change.*

Proof. Omitted as this is a direct implication of Proposition 2. □

Intuitions to understand the last part of Proposition 2 are built upon discussions on Rondina-Shim economy and Angeletos-Werning economy: non-fundamental volatility can become higher even when the precision of the private information becomes greater if the positive linkage between precisions of the private information of different agents is non-negligible. Recall that the condition to obtain such a result is $f(\alpha_x) < \alpha_x f'(\alpha_x)$, implying that α_y improves much faster than α_x. For instance, in the case of $f(\alpha_x) = \alpha_x^2$, private information of a trader becomes better as that of a player improves, but at a much faster pace. This way, public information would become much more precise so that the PS channel counteracts the IS channel. Figure 1 shows that this is really a plausible equilibrium outcome: the left panel shows that non-fundamental volatility is decreasing in the precision of private information

[7] We thank a referee to point out this property.

[8] In this Corollary, we consider a slightly generalized version of Angeletos and Werning (2006) by assuming that the slope of $f(\alpha_x)$ can be different from one.

of a player. The solid black line traces the non-fundamental volatility in the Rondina-Shim economy, and it clearly shows that there exists a multiplier effect. The right panel, on the contrary, shows that non-fundamental volatility can increase at the equilibrium when $\alpha_y = \alpha_x^2$.

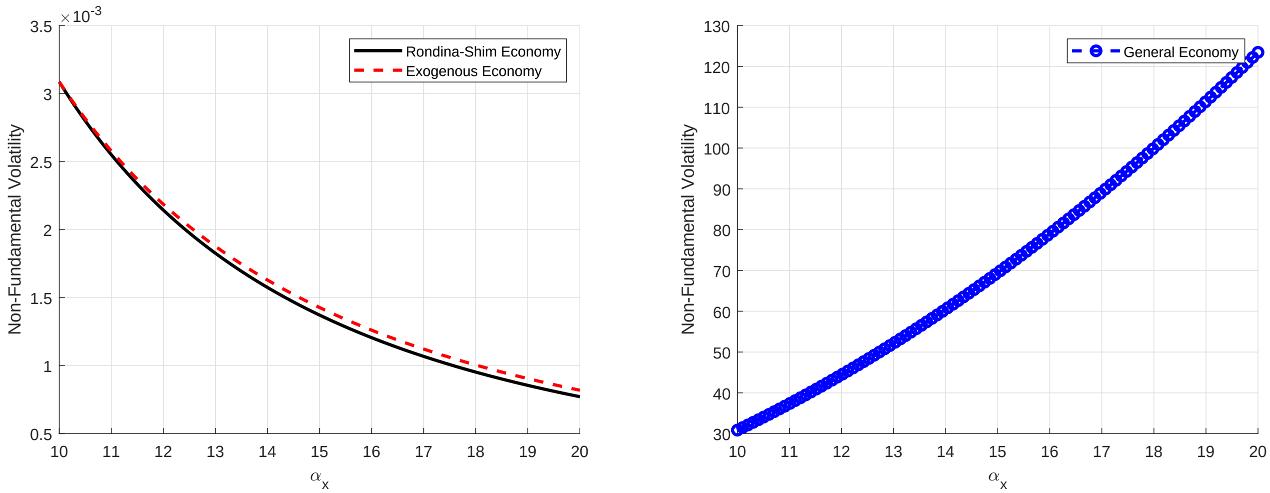

Figure 1a: NFV decreases in α_x Figure 1b: NFV increases in α_x

Figure 1: NFV as a function of α_x

Note: Horizontal axes represent precision of private information. Rondina-Shim Economy assumes $\alpha_y = c$, Exogenous Economy assumes α_p is fixed (at the same value with Rondina-Shim Economy), and General Economy assumes $\alpha_y = \alpha_x^2$.

4 Concluding Remark

In this paper, we analyze the effect of the changes in the precision of private information on the non-fundamental volatility under various assumptions on the information structure. We find that the equilibrium relationship between the non-fundamental volatility and precision of private information for the players is a byproduct of the information structure and endogeneity of the public information. While we do not explicitly show, allowing endogenous information acquisition as in Hébert and La'O (2021) hardly changes our key findings as the results do not particularly depend on the information acquisition cost.

An interesting and fruitful application of our finding would be to study the effect of changes in private information on the social welfare. Social welfare in this class of models can be decomposed into terms related to the non-fundamental volatility and dispersions in actions (Angeletos and Pavan (2004)), which implies that different equilibrium properties found in this paper would further deliver different welfare implications. We leave this as a future work.

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A Appendix. Proof for Proposition 2

We first define $g(x) \equiv \frac{f^2(\alpha_x)}{\alpha_x}$. Then precision of the public signal is expressed as follows:

$$\alpha_p^* = \frac{1 - \frac{2\alpha_\xi g(x)}{\gamma^2(1-r)} - \sqrt{1 - \frac{4\alpha_\xi g(x)}{\gamma^2(1-r)}}}{\frac{2\alpha_\xi g(x)}{\gamma^2(1-r)^2\alpha_x}} \tag{A.1}$$

$$= (1-r)\alpha_x \cdot \left[\frac{\gamma^2(1-r)}{2\alpha_\xi g(x)} - 1 - \frac{\gamma^2(1-r)}{2\alpha_\xi g(x)}\sqrt{1 - \frac{4\alpha_\xi g(x)}{\gamma^2(1-r)}}\right] \tag{A.2}$$

Define $H \equiv \frac{2\alpha_\xi g(x)}{\gamma^2(1-r)}$. Then

$$\frac{\partial \alpha_p^*}{\partial \alpha_x} = (1-r) \cdot \left(\frac{1}{H} - \frac{1}{H}\sqrt{1-2H} - 1\right) + (1-r)\alpha_x \frac{g'(x)}{g(x)} \cdot \left(-\frac{1}{H} + \frac{1}{H}\sqrt{1-2H} + \frac{1}{\sqrt{1-2H}}\right) \tag{A.3}$$

Recall that it suffices to determine the sign of $(1-r) - \frac{1}{2}\frac{\partial \alpha_p^*}{\partial \alpha_x}\left((1-r)\frac{\alpha_x}{\alpha_p^*} - 1\right)$ to analyze the effect of precision of the private signal on the non-fundamental volatility. Define $A \equiv \frac{1}{H}\left(1 - \sqrt{1-2H}\right)$ and note that $\left((1-r)\frac{\alpha_x}{\alpha_p^*} - 1\right) = \frac{2-A}{A-1}$.

If suffices to consider the following expression:

$$-\frac{1}{2}\left[(A-1) + \left(\frac{2\alpha_x f'(x)}{f(x)} - 1\right) \cdot \left(\frac{1}{\sqrt{1-2H}} - A\right)\right]\frac{2-A}{A-1} \gtreqless -1 \tag{A.4}$$

By rearranging the terms, we can obtain

$$A\left(1 - \frac{\alpha_x f'(x)}{f(x)}\right) \gtreqless 0 \tag{A.5}$$

As $A > 0$, we can get the following conditions stated in Proposition 2.