Information Inequality and the Role of Public Information*

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Abstract

Transparent disclosures of public information might be one natural policy to reduce information inequality among individuals. We conduct a welfare analysis of such policy by introducing ex-ante heterogeneity in individuals' private information in a class of economies with dispersed information and strategic complementarity or substitutability. We find that better private information for the already privately better-informed individuals increases welfare, even though it may be accompanied by greater information inequality. We also show that the better the private information for the already better-informed individuals, the more likely that increasing precision of public information reduces welfare. Our findings suggest caution in making information policies that aim to narrow informational gap with better public information.

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I. Introduction

Economic agents get their private information from many different sources. Some agents have access to sources that have very precise or accurate information while the others do not, creating information inequality—the non-homogeneity of private information quality—across agents. One natural way to reduce such information inequality is through disclosure of more transparent public information from government agencies or central banks. Is this policy always socially desirable? Can agents benefit even when information inequality widens? The goal of this paper is to answer these questions.

Starting with Morris and Shin (2002), a large literature has studied the social value of public information (see, e.g., Amador and Weill 2010; Angeletos and Pavan 2004, 2007; Goldstein and Yang 2019; James and Lawler 2008, 2011; Ui 2014). Our paper complements this line of research by considering the welfare effects of public disclosures in a class of environments á la Morris and Shin (2002). The point of departure of our paper is that, while Morris and Shin (2002) and many others consider ex-post information heterogeneity, we allow for ex-ante information heterogeneity by assuming that each agent's private information precision can be either high or low. The discrepancy between the two types of precisions captures the magnitude of inequality in private information.

Our paper is closely related to James and Lawler (2012) who also discuss the implications of changes in public and private information precisions for equilibrium behavior and social welfare in the presence of information inequality. In their model, agents' actions are strategic complements and private information precision differs between two groups each comprising half of the population. We consider a more flexible environment with either complementarity or substitutability in actions and varying fraction of the two groups, and derive additional results pertinent to those extensions.[1] A more important distinction is that James and Lawler (2012) model two types of private information precision by assuming that a second type's precision is lower than the homogeneous precision in

[1]Consider situations in which a group of financial analysts desires to accurately estimate a company's earnings over the coming years. In some cases, analysts might prefer to make unusually bold forecasts hoping to stand out from the crowd (strategic substitutability); in some other cases, they might prefer to herd (strategic complementarity). See Kim and Shim (2019) for more details on the two cases.

Morris and Shin (2002), whereas we assume it to be higher. So while their corresponding discussions are mirror images of some of our results, their interpretations on the welfare implications of heterogeneous private information quality for the Morris and Shin framework are imprecise. We consolidate the results and provide more complete and appropriate interpretations.

We show that if public information is more precise, then both types of agents find it optimal to rely more on public information, and that social welfare can decrease over some ranges of parameter values. That is, even in the presence of information inequality, greater provision of public information is not always desirable. We find that when there are more privately better-informed agents in the population, both types of agents respond more (resp. less) strongly to private information when actions are strategic complements (resp. substitutes). We also find that when the better-informed agent's private information gets even better, they adjust upward their reliance on private information regardless of complementarity or substitutability in actions, whereas the agents with worse private information adjust upward their reliance on private information only when actions are complements.

More interestingly, we find that either more privately better-informed agents in the population or more precise private information for those agents unambiguously increases social welfare, even if the latter accompanies greater inequality in private information. We also show that the better the private information for the already informationally-advantaged agents, the more likely that public information reduces welfare. The results of our paper expand those of Morris and Shin (2002) to a wider class of economies that have information inequality. James and Lawler (2012) ascertain that the set of parameter configurations under which public information reduces welfare is smaller when there is information inequality, whereas our results imply the opposite. The novelty of our paper is that we enrich the relevant discussions of James and Lawler (2012) by providing more clear and coherent interpretations of those results.

Our analysis suggests that when considering the welfare implications of public disclosure by government agencies or central banks, it is important to think about the specific structure of information possessed by agents and the level of information disparities among agents. Otherwise, disclosure of more transparent public information may deliver undesirable consequences for welfare.

The remainder of the paper is organized as follows. Section II presents the model, and solves for the equilibrium. Section III provides our main findings, which consider the comparative statics of equilibrium behavior as well as the welfare effects. Section IV concludes the paper. All proofs are contained in Appendix A, and additional technical details can be found in Appendix B.

II. The Model

Our model extends the beauty contest model of Morris and Shin (2002) to allow for either strategic complementarity or substitutability and for ex-ante heterogeneity of private information precision, resembling the model of James and Lawler (2012) in the latter sense.

In the economy there is a continuum of agents, indexed by i and uniformly distributed over the interval $[0, 1]$. We represent the state of the economy with an exogenous random variable $\theta \in \mathbb{R}$ drawn from a normal distribution with mean μ and variance α_θ^{-1}, which constitutes the agents' common prior about θ. The agents do not observe the realization of the true θ but instead observe noisy private signals and a noisy public signal.

The public signal is summarized by $y = \theta + (\alpha_y)^{-1/2}\varepsilon$, where α_y denotes the precision of y and the common noise ε follows $N(0, 1)$, independent of θ. The common posterior of θ given public signal alone is then normal with mean $p \equiv \mathbb{E}[\theta|y]$ and variance α_p^{-1}, where $\alpha_p \equiv \alpha_\theta + \alpha_y$. Hereafter, we refer to α_p as the (total) precision of public information, which combines the precision of the common prior and the public signal y; and we identify public information with p rather than y.[2]

Each agent is characterized by the precision of his private signal, which can be high

[2]This formulation directly follows Angeletos and Pavan (2007) and Colombo, Femminis and Pavan (2014). The latter paper provide an extensive analysis of information acquisition problem, which can be connected to our extended model allowing for one group of agents to choose the precision of their private information. This extension is beyond the scope of the present paper and is left for future work.

(type H) or low (type L).[3] The private signal of agent i of type $t \in \{H, L\}$ is summarized by $x_i^t = \theta + (\alpha_x^t)^{-1/2}\varepsilon_i$ where α_x^t denotes the precision of the private signal and the idiosyncratic noise ε_i follows $N(0,1)$, independent of one another, as well as of θ and ε.

After observing the signals (x_i^t, y), agent i of type t chooses an action $a_i^t \in \mathbb{R}$.[4] The payoff function for agent i of type t is given by

$$u_i^t(a_i^t, a_{-i}, \theta) = -(1-r)(a_i^t - \theta)^2 - r(L_i^t - \bar{L}),$$

where a_{-i} is the action profile of other agents, $L_i^t \equiv \int_0^1 (a_j - a_i^t)^2 d_j$ and $\bar{L} \equiv \int_0^1 L_j d_j$; the parameter $r \in (-1, 1)$ gives the weight on the second-guessing motive, as well as measuring the resulting degree of coordination—strategic complementarity ($r > 0$) or substitutability ($r < 0$) in agents' actions.[5]

We assume that $\alpha_x^L \equiv \alpha_x$ and $\alpha_x^H \equiv \mu\alpha_x$, where $\mu > 1$. The parameter μ captures the magnitude of the ex-ante difference between the precisions of the two types' private signals; which we call *information inequality.* Then a higher μ means increased inequality in private information. We let χ denote the fraction of agents of type H.

Condition 1. $-r(1-\chi)(\mu - 1)\frac{\alpha_x}{\alpha_x + \alpha_p} < 1.$

Proposition 1. *A linear equilibrium exists and is the unique equilibrium. Under Condition 1, the equilibrium action of agent i of type $t \in \{H, L\}$ is $a_i^t(x_i^t, y) = \lambda^t x_i^t + (1 - \lambda^t)p$ where*

$$\lambda^H = \frac{(1-r)\frac{\mu\alpha_x}{\mu\alpha_x + \alpha_p}}{1 - r\left(\chi\frac{\mu\alpha_x}{\mu\alpha_x + \alpha_p} + (1-\chi)\frac{\alpha_x}{\alpha_x + \alpha_p}\right)} \quad \text{and} \quad \lambda^L = \frac{(1-r)\frac{\alpha_x}{\alpha_x + \alpha_p}}{1 - r\left(\chi\frac{\mu\alpha_x}{\mu\alpha_x + \alpha_p} + (1-\chi)\frac{\alpha_x}{\alpha_x + \alpha_p}\right)}. \tag{1}$$

The coefficients λ^H and λ^L measure the sensitivities of the equilibrium actions to private information relative to public information for the agents of type H and L respectively. Condition 1 ensures that $\lambda^H < 1$ in the unique equilibrium, the derivation of which is in Appendix B.1. This condition always holds if $r > 0$ but may fail if $r < 0$.

[3]For ease of exposition, we use male pronouns for the agent.

[4]Note that the precision parameters (α_x^t, α_p), as well as public information p, are common knowledge to the agents when choosing their actions.

[5]We assume that r is the same across all agents, and $r < 1$ to guarantee uniqueness of equilibrium.

We define $\delta^H \equiv \frac{\alpha_x^H}{\alpha_x^H+\alpha_p}$ and $\delta^L \equiv \frac{\alpha_x^L}{\alpha_x^L+\alpha_p}$, which measure the relative precisions of private information for the two types of agents. Then we can rewrite (1) as

$$\lambda^H = \frac{(1-r)\delta^H}{1-r\bar{\delta}} \quad \text{and} \quad \lambda^L = \frac{(1-r)\delta^L}{1-r\bar{\delta}}, \tag{2}$$

where $\bar{\delta} \equiv \chi\delta^H + (1-\chi)\delta^L$ is the weighted average of δ^H and δ^L in the population.

III. Results

3.1. Equilibrium Behavior

We first consider the effects of changes in α_p, χ, and μ on the agents' equilibrium behavior captured by λ^H and λ^L.

Proposition 2. $\frac{\partial\lambda^t}{\partial\alpha_p} < 0$ *for all* $t \in \{H, L\}$.

Proposition 2 is consistent with Morris and Shin (2002). If public information is more precise, the agents rely more on public information than on private information in equilibrium.

Let λ be the sensitivity of the equilibrium actions to private information relative to public information in the model of Morris and Shin (2002), where $\mu = 1$ in terms of our notation:

$$\lambda \equiv \frac{(1-r)\alpha_x}{(1-r)\alpha_x + \alpha_p}.$$

We obtain the following comparison, which is useful for understanding subsequent results.

Lemma 1. $\lambda^H > \lambda^L > \lambda$ *if* $r > 0$ *and* $\lambda^H > \lambda > \lambda^L$ *if* $r < 0$.

Lemma 1 differs from the related discussion in James and Lawler (2012) in two aspects. First, the existence of two agent types causes the high-type agents to rely more on private information in our model, relative to the Morris and Shin case in which all agents are of the same type; whereas the high-type agents rely more on public information in James and Lawler's (2012, p.347) counterpart. The reason is because the perspective of modelling heterogeneity in private information quality is different. While our model can be thought of as taking a fraction of agents in Morris and Shin's (2002) setting and making their private information quality better, James and Lawler (2012) make it worse. So the results, rather than being contradictory, supplement each other.[6] Second, we consider the case where actions are strategic substitutes ($r < 0$) that James and Lawler (2012) do not. For such case, the high-type agents use more private information while the low-type agents use less private information, relative to the Morris and Shin case.

Proposition 3. *$\frac{\partial \lambda^t}{\partial \chi} \gtreqless 0$ if and only if $r \gtreqless 0$ for all $t \in \{H, L\}$.*

James and Lawler (2012) assume that each of the two types comprises one-half of the population, that is, $\chi = 1/2$ in terms of our notation. This restriction may simplify the analysis, but relaxing this parameterization renders an interesting result. A higher proportion of privately better-informed (or high-type) agents in the population induces both types of agents to respond more (resp. less) strongly to private information when $r > 0$ (resp. $r < 0$).

The technical insights can be gained by re-expressing λ^L and λ^H as follows:

$$\lambda^L = \lambda + \underbrace{\frac{\lambda r \chi(\delta^H - \delta^L)}{1 - r\bar{\delta}}}_{(CE^L)} \quad \text{and} \quad \lambda^H = \lambda + \underbrace{\frac{(\lambda r \chi + 1 - r)(\delta^H - \delta^L)}{1 - r\bar{\delta}}}_{(CE^H)}, \tag{3}$$

where (CE^L) and (CE^H) measure the "residual" sensitivity of equilibrium actions to private information for the low- and high-type agents in the presence of informational gap. The term (CE^L) is positive if $r > 0$ and negative if $r < 0$, while (CE^H) is always positive (Lemma 1). Noting that $(1 - r\bar{\delta})$ decreases (resp. increases) in χ if $r > 0$ (resp. $r < 0$), we can see that both (CE^L) and (CE^H) increase in χ if $r > 0$ and decrease in χ if $r < 0$.

The driving force behind Proposition 3 is that the average quality of private information in the entire population increases when relatively more agents receive better-quality private information. In terms of our parameters, this key channel can be represented by

[6] James and Lawler's (2012) model is qualitatively equivalent to our model with $\mu < 1$. All the results in the present paper for such modified model are briefly reviewed and compared in Appendix B.2.

the average relative precision of private information in the population, $\bar{\delta}$, which increases with χ. The intuition can be explained as follows. When actions are strategic complements ($r > 0$), agents want to align their actions with the average action of all agents. An improvement in the average quality of private information, because more agents are better-informed, induces agents to rely more on private information because each agent recognizes that other agents "on average" will now respond more strongly to private information. In the case of strategic substitutes ($r < 0$) where agents want to differentiate, an improvement in the average quality of private information compounds to the tendency for agents to react less on private information. This is attributable to the fact that each agent realizes that other agents "on average" will respond more to private information.

Proposition 4. $\frac{\partial \lambda^H}{\partial \mu} > 0$*; whereas* $\frac{\partial \lambda^L}{\partial \mu} \gtreqless 0$ *if and only if* $r \gtreqless 0$*.*

James and Lawler (2012) also discuss the effects of changes in private information quality on the equilibrium weights on information for the case when actions are strategic complements. They conclude that the less pronounced the informational gap, the more the agents rely on private information (James and Lawler 2012, p.347). This may seem to contradict Proposition 4 because the informational gap is smaller with a lower μ in our setting. But because some fraction of the population is informationally disadvantaged relative to the Morris and Shin case in James and Lawler's (2012) setting, narrowing the information gap between the two groups means making the informationally disadvantaged group better informed; whereas it means making the informationally advantaged group worse informed in our setting. So their analysis nor Proposition 4 should not be interpreted as a causal effect of the informational gap on the agents' reliance on information. Rather, the results can be integrated, together implying that if private information gets better for either group of agents, the agents rely more on private information when actions are strategic complements.

Further, Proposition 4 is interesting in the sense that the two types of agents behave in an opposite manner particularly when actions are strategic substitutes. To understand the intuition, let us decompose the residual term (CE^H) in (3) as a sum comprised of

(CE^L) plus an additional term:

$$(CE^H) = \underbrace{\frac{\lambda r\chi(\delta^H - \delta^L)}{1 - r\bar{\delta}}}_{(CE^L)} + \underbrace{\frac{(1 - r)(\delta^H - \delta^L)}{1 - r\bar{\delta}}}_{(IE)}. \tag{4}$$

It can be easily verified that $\frac{\delta^H - \delta^L}{1 - r\bar{\delta}}$ increases in μ. Then we can see that when $r > 0$, both (CE^L) and (CE^H) increase in μ. When $r < 0$, (CE^L) decreases in μ, but (CE^H) increases in μ because $\lambda r\chi + 1 - r = 1 - r(1 - \lambda\chi) > 0$.

As with the intuition behind Proposition 3, the key channel is the average quality of private information, represented by $\bar{\delta}$, which increases in μ. By an improvement in the quality of high-type agents' private information, the average quality of private information in the population improves as well. When actions are complements ($r > 0$), both types of agents, desiring to align with what others do on average, would respond more strongly to private information. When actions are substitutes ($r < 0$), both types of agents, desiring to do the opposite of what others do on average, would respond less strongly to private information. This common effect is captured by (CE^L) for both types of agents. In addition, the high-type agents recognize that the low-type agents would necessarily respond less strongly to private information than the high-type agents, simply because low-type private information is now comparatively poorer; a consideration that induces the high-type agents to respond more strongly to private information. This positive effect for the high-type agents is captured by (IE), which dominates the common negative effect measured by (CE^L) when $r < 0$.

3.2. Social Welfare

We now address the welfare consequences of the precision of public information (in the presence of information inequality), the proportion of high-type agents in the population, and the precision of high-type agents' private information.

Social welfare evaluated at equilibrium is defined as the normalized average of individual utilities:

$$W(a,\theta) \equiv \frac{1}{1-r}\int_0^1 u_i d_i = -\int_0^1 (a_i-\theta)^2 di$$
$$= -\left[\int_0^{\chi} (a_i^H-\theta)^2 d_i + \int_{\chi}^1 (a_i^L-\theta)^2 d_i\right].$$

Then equilibrium expected welfare, conditional on θ, is given by:

$$\mathbb{E}(W|\theta) = \chi\mathbb{E}(U^H|\theta) + (1-\chi)\mathbb{E}(U^L|\theta),$$

where $\mathbb{E}(U^t|\theta)$ denotes the ex-ante expected utility for the agent of type $t \in \{H, L\}$, defined as:

$$\mathbb{E}(U^t|\theta) \equiv -\mathbb{E}\left((a_i^t-\theta)^2|\theta\right) = -\left[\left(\lambda^t(\alpha_x^t)^{-1/2}\right)^2 + \left((1-\lambda^t)(\alpha_p)^{-1/2}\right)^2\right].$$

The following result identifies necessary and sufficient conditions for $\mathbb{E}(U^H|\theta)$ and $\mathbb{E}(U^L|\theta)$ to decrease in the precision of public information.

Proposition 5. *(i)* $\frac{\partial \mathbb{E}(U^H|\theta)}{\partial \alpha_p} < 0$ *if and only if*

$$\alpha_p\left[\left(1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)^3 + (1-r)\mu\alpha_x\left(1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right]$$
$$< (1-r)\mu\alpha_x\left(2r-1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)\left(1+\alpha_x\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right). \tag{5}$$

The set of parameter values that satisfy condition (5) exists if

$$r > \frac{1}{2+(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p}}. \tag{6}$$

(ii) $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} < 0$ *if and only if*

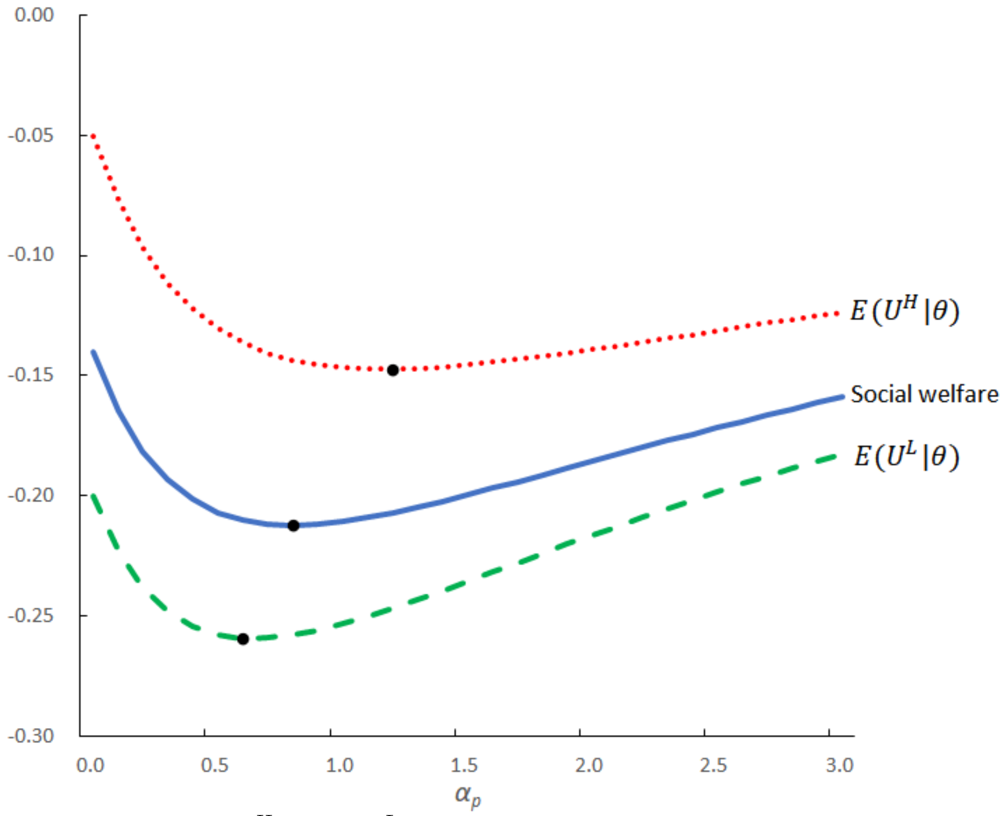
$$\alpha_p\left[\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)^3 - (1-r)\alpha_x\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)\frac{r\chi(\mu-1)\alpha_x}{(\mu\alpha_x+\alpha_p)^2}\right]$$
$$< (1-r)\alpha_x\left(2r-1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)\left(1-\alpha_x\frac{r\chi(\mu-1)\mu\alpha_x}{(\mu\alpha_x+\alpha_p)^2}\right). \tag{7}$$

The set of parameter values that satisfy condition (7) exists if

$$r > \frac{1}{2 - \chi(\mu - 1)\frac{\alpha_x}{\mu\alpha_x + \alpha_p}}. \tag{8}$$

Figure 1 plots the expected utilities for the agents of type H and L as well as social welfare with respect to α_p. As can be seen from the figure, there are ranges of parameter values where the expected utilities for both types are decreasing in the precision of public information. Another observation is that when the low-type agents' utility is decreasing, the high-type agents' utility is necessarily decreasing, but not vice versa. This observation indicates that there are some ranges of cases where more public information benefits one type of agents while harming the other type.

Figure 1. Expected utilities for high- and low-type agents, and social welfare



Note: The figure plots $\mathbb{E}(U^H|\theta)$, $\mathbb{E}(U^L|\theta)$, and $\mathbb{E}(W|\theta)$ with respect to α_p for the parameter values of $r = 0.8$, $\alpha_x = 5$, $\mu = 4$, and $\chi = 0.4$.

Corollary 1 (Extension of Morris and Shin (2002))**.** *In the presence of information inequality, there exists a set of parameter values for which more precise public information* *lowers social welfare. If actions are strategic substitutes or strategic complements with the degree of complementarity weak enough, then more precise public information increases social welfare.*

This corollary parallels the findings of Morris and Shin (2002) and James and Lawler (2012) that with strategic complementarity, it is not always the case that greater precision of public information is socially desirable. The necessary and sufficient condition under which welfare decreases with public information is somewhat complex to state here. Instead, we can still apply Morris and Shin's (2002) intuitions behind the condition. By letting $\mu = 1$ in our model, we retrieve their conditions: $r > \frac{1}{2}$ from conditions (6) and (8), and $\alpha_p < (2r-1)(1-r)\alpha_x$ from conditions (5) and (7). Morris and Shin (2002) show that if $r > \frac{1}{2}$, then there are ranges of parameter values, in particular, $\alpha_p < (2r-1)(1-r)\alpha_x$, where more precise public information lowers social welfare. That is, with strong complementarity such that $r > \frac{1}{2}$, if the agents' private information is very precise (so that α_x is sufficiently high), then more precise public information is harmful. Their immediate implication that "the greater the precision of the agents' private information, the more likely it is that increased provision of public information lowers social welfare" (Morris and Shin 2002, p.1522) extends to our setting. With sufficiently strong complementarity, the greater the precision of the low- or high-type agents' private information relative to public information, either in terms of α_x or μ, the more likely the public information reduces welfare. The intuition is that, given that some fraction of the population is informationally advantaged, and therefore has less to gain in terms of forecast accuracy, when public information is more precise, the coordination motive generated by strong complementarity magnifies the (inefficient) overweighting of public information than required by social efficiency. This is discussed further following the next two propositions.

Proposition 6. *Social welfare increases with an increase in χ.*

Proposition 6 subsumes as a special case one of Morris and Shin's findings that an improvement in the precision of private information is beneficial to welfare. The two $\chi = 0$ and $\chi = 1$ extreme cases correspond exactly to the Morris and Shin (2002) scenario in which all agents' precision of private information increases from α_x to $\mu\alpha_x$. It is to be expected that relatively more (higher χ) agents with better-quality private information ($\mu\alpha_x$) in the population will also improve welfare. The proof also shows that each agent's equilibrium expected utility also increases in χ. An increase in χ represents an environment whereby the average quality of private information improves in the population, which is essentially responsible for an improvement in social welfare.

Proposition 7. *Social welfare necessarily increases with an increase in μ.*

Also as an extension of Morris and Shin's (2002) finding that more precise private information is beneficial to welfare, Proposition 7 entails three things. First, the result holds regardless of the size of χ. Second, the equilibrium expected utility for each type of agent increases with an increase in μ, which is shown in the proof. In these two senses, Proposition 7 asserts that more precise private information *only for some agents* is always to the benefit of all agents in the population, regardless of whether those agents getting better private information is very small or large.

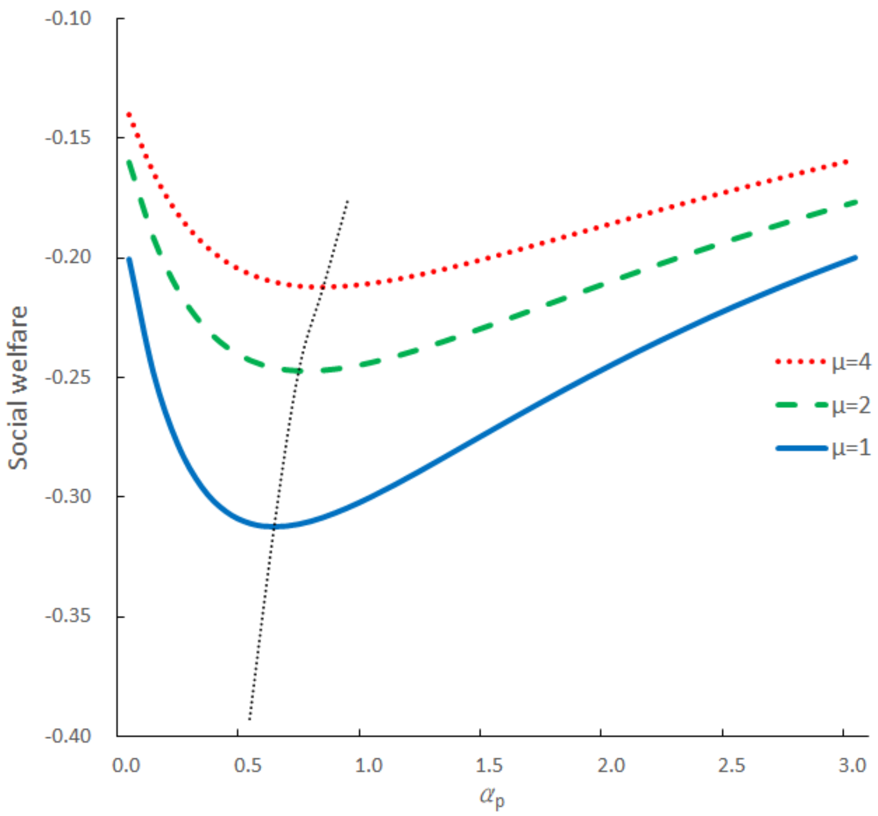
Third, the result holds also for $\mu < 1$, which is shown in Appendix B.2. An increase $\mu > 1$ represents increased inequality in private information, whereas an increase in $\mu < 1$ describes decreased inequality. Regardless of whether $\mu \gtrless 1$, the immediate effect of an increase in μ is an improvement in the average relative quality of private information, $\bar{\delta}$, which is the key conduit for increased welfare.[7] Proposition 7 does not suggest a causal effect of greater information inequality on social welfare. Rather, it signifies that more precise private information for some agents is beneficial to welfare, *regardless of whether the group of agents getting better private information is already better-informed or not.* While this result is consistent with one of the findings of James and Lawler (2012) that an improvement in the precision of private signals is beneficial to welfare, their interpretation entails that a narrowing of information inequality is welfare-improving. In the case of $\mu > 1$ that we focus on, the result implies that social welfare can improve even if it

[7] We can confirm that $\bar{\delta}$ is the key channel of our result by considering the welfare effect when μ increases, holding $\bar{\delta}$ fixed by changing χ simultaneously. In such exercise, we find that social welfare does not change, the formal proof of which is available upon request. This confirms that $\bar{\delta}$ is exactly the channel through which welfare is affected by a change in μ.

is accompanied by greater information inequality. Propositions 6 and 7 together imply that when the privately better-informed agents get even better private information or the proportion of those agents grow, social welfare increases.

Figure 2 graphically illustrates that over some ranges, increased precision of public information lowers social welfare (Corollary 1); and that for any given α_p, social welfare is greater for a higher μ (Proposition 7). In the figure, we observe that the contour of minimum welfare levels for different μ's (represented by the black dotted line) is upward-sloping, which holds generally in our setting and is formalized by the following result.

Figure 2. Social welfare for various values of μ



Note: The figure plots $\mathbb{E}(W|\theta)$ with respect to α_p for $\mu = 1, 2, 4$, for given parameter values of $r = 0.8$, $\alpha_x = 5$, and $\chi = 0.4$.

Corollary 2. *Relative to the Morris and Shin case of $\mu = 1$, social welfare is decreasing in the precision of public information for a broader set of parameter configurations.*

The intuition follows from that of Corollary 1. The existence of privately better-informed agents exacerbates the overweighting of public information. In our setting, the more pronounced is the difference between the two types' private information (due to an improvement in the quality of private information for the already better-informed agents), the more likely that public information reduces welfare. On the other hand, James and Lawler (2012, p.346) assert that the set of parameter configurations under which better public information reduces welfare is smaller when a second type exists than in the Morris and Shin framework. We can reconcile these seemingly contrasting conclusions because James and Lawler (2012) view the problem in the opposite side where the second group of agents is privately less well-informed, i.e., $\mu < 1$ in terms of our notation. Thus, their interpretation only pertains to the lower part below the "$\mu = 1$ line" and left of the upward-sloping dotted line in Figure 2. The narrowing of the informational gap per se does not cause the beneficial forecasting-accuracy effect to be dominant or not; the more appropriate interpretation is that, in the presence of information inequality in private information, the detrimental welfare effect of public information is more likely when any group of agents becomes privately better-informed. In this sense, we contribute to providing an integrated analysis with more consistent interpretations than do James and Lawler (2012).

IV. Conclusion

Our analysis has an important policy implication for the debate on public disclosures by government agencies or central banks. With larger information disparities among economic agents, there is greater possibility that public information may be detrimental to welfare. Distinct from James and Lawler (2012) who model public disclosure in terms of the proportion of agents to whom public information is released, we model it in terms of the degree of transparency of public information released to all agents. While Morris and Shin (2002) have already shown that increased public disclosures (in our sense) can be detrimental to social welfare, this paper highlights that such detrimental effect of public information is more likely in the presence of information inequality when the already privately better-informed agents are getting even better information. Svensson (2006) argued, in response to Morris and Shin (2002), that social welfare is unlikely to decrease under reasonable assumptions on parameters. While Svensson's (2006) argument questions the quantitative significance of Morris and Shin's (2002) results, "the question of whether the public signal is *sufficiently* precise to justify disclosure" (Morris and Shin 2006, p.453) remains. Similarly in terms of our results, the response of $\arg\min_{\alpha_p} \mathbb{E}(W|\theta)$ to a change in μ may seem visually and quantitatively small. Whether this quantitative prediction has a qualitative meaning for disclosure policies would require a more systematic study for general cases or an empirical analysis depending on the economic context, which is beyond the scope of this paper. Nonetheless, given the theoretical underpinning of the paper, we conclude that in formulating disclosure policies for government agencies or central banks on how much they should disclose, it is important to carefully identify the informational environment that economic agents face. Otherwise, increased transparency through disclosures can render socially undesirable outcomes counter to the policy maker's goals.

Appendix A. Proofs

Proof of Proposition 1. After observing the realization of x_i^t and y, each agent chooses a_i^t so as to maximize the expected payoff $\mathbb{E}[u_i^t|x_i^t, y]$. So the best response of agent i of type $t \in \{H, L\}$ is determined by the first order conditions (FOCs):

$$a_i^t(x_i^t, y) = (1-r)\mathbb{E}(\theta|x_i^t, y) + r\Big(\int_0^{\chi} \mathbb{E}(a_j^H|x_i^t, y)dj + \int_{\chi}^1 \mathbb{E}(a_j^L|x_i^t, y)dj\Big), \; \forall x_i^t.$$

Note that $\mathbb{E}(\theta|x_i^t, y) = \frac{\alpha_x^t}{\alpha_x^t+\alpha_p}x_i^t + \frac{\alpha_p}{\alpha_x^t+\alpha_p}p$ is linear in (x_i^t, p). So given the linearity of FOCs and the normality of posterior beliefs about θ, it is natural to look for an equilibrium strategy that is linear in x_i^t and p so that $a_i^H = \kappa_0^H x_i^H + \kappa_1^H p$ and $a_i^L = \kappa_0^L x_i^L + \kappa_1^L p$, where κ_0^t and κ_1^t are constants determined in equilibrium.[8] Note that $\mathbb{E}[x_j^s|x_i^t, y] = \mathbb{E}[\theta|x_i^t, y] = \frac{\alpha_x^t}{\alpha_x^t+\alpha_p}x_i^t + \frac{\alpha_p}{\alpha_x^t+\alpha_p}p$ for all $t, s \in \{H, L\}$; so we have $\mathbb{E}[a_j^s|x_i^t, y] = \kappa_0^s\mathbb{E}(x_j^s|x_i^t, y) + \kappa_1^s p$ for all

[8] The quadratic structure of utility function ensures the linearity of best response functions.

$t, s \in \{H, L\}$. Then FOC for agent i of type $t \in \{H, L\}$ reduces to

$$
\begin{aligned}
a_i^t =&(1-r)\Big[\frac{\alpha_x^t}{\alpha_x^t+\alpha_p}x_i^t + \frac{\alpha_p}{\alpha_x^t+\alpha_p}p\Big] \\
&+ r\Big[\chi\Big(\kappa_0^H\big(\frac{\alpha_x^t}{\alpha_x^t+\alpha_p}x_i^t + \frac{\alpha_p}{\alpha_x^t+\alpha_p}p\big) + \kappa_1^H p\Big) \\
&\quad + (1-\chi)\Big(\kappa_0^L\big(\frac{\alpha_x^t}{\alpha_x^t+\alpha_p}x_i^t + \frac{\alpha_p}{\alpha_x^t+\alpha_p}p\big) + \kappa_1^L p\Big)\Big].
\end{aligned}
$$

Comparing coefficients, it follows that $a_i^H = \kappa_0^H x_i^H + \kappa_1^H p$ and $a_i^L = \kappa_0^L x_i^L + \kappa_1^L p$ constitute a linear equilibrium if and only if $\kappa_0^H = \frac{\alpha_x^H}{\alpha_x^H+\alpha_p}\frac{\alpha_x^L+\alpha_p}{\alpha_x^L}\kappa_0^L$, $\kappa_0^L = \frac{(1-r)\alpha_x^L}{\alpha_x^L+\alpha_p-\alpha_x^L r\left(\chi\frac{\alpha_x^H}{\alpha_x^H+\alpha_p}\frac{\alpha_x^L+\alpha_p}{\alpha_x^L}+1-\chi\right)}$, $\kappa_1^H = 1-\kappa_0^H$, and $\kappa_1^L = 1-\kappa_0^L$. Let $\kappa_0^t = \lambda^t$ and $\kappa_1^t = 1-\lambda^t$ for $t \in \{H, L\}$; then we obtain formulas in (1). Clearly, this is the unique linear equilibrium, and following the same argument as in Morris and Shin (2002), when best responses are linear, there do not exist equilibria other than the linear one. □

Proof of Proposition 2. $\frac{\partial\lambda^H}{\partial\alpha_p} \propto -(1-r)\left[1-\frac{\alpha_x}{\alpha_x+\alpha_p}\left(-r(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p}\right)\right]$, which is always negative given Condition 1. $\frac{\partial\lambda^L}{\partial\alpha_p} \propto -(1-r)\left[1-r\chi\frac{(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\frac{\mu\alpha_x}{\mu\alpha_x+\alpha_p}\right] < 0$ because $r\chi\frac{(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\frac{\mu\alpha_x}{\mu\alpha_x+\alpha_p} < 1$. □

Proof of Lemma 1. $\lambda^H > \lambda^L$ if and only if $\delta^H > \delta^L$, which always holds with $\mu > 1$. Also, $\lambda^H > \lambda$ if and only if $(\delta^H - \delta^L)(1 - r(1-\chi)\delta^L) > 0$, which always holds. Lastly, $\lambda^L < \lambda$ if and only if $0 < -r\chi(\mu-1)\frac{\alpha_x\alpha_p}{\mu\alpha_x+\alpha_p}$, which holds if and only if $r < 0$. □

Proof of Proposition 3. Note that $\frac{\partial\bar{\delta}}{\partial\chi} = \delta^H - \delta^L > 0$. Then $\frac{\partial\lambda^H}{\partial\chi} = \frac{r(1-r)\delta^H}{(1-r\bar{\delta})^2}\frac{\partial\bar{\delta}}{\partial\chi} \gtreqless 0$ if and only if $r \gtreqless 0$, and $\frac{\partial\lambda^L}{\partial\chi} = \frac{r(1-r)\delta^L}{(1-r\bar{\delta})^2}\frac{\partial\bar{\delta}}{\partial\chi} \gtreqless 0$ if and only if $r \gtreqless 0$. □

Proof of Proposition 4. $\frac{\partial\lambda^L}{\partial\mu} = \frac{r(1-r)\chi\frac{\alpha_x}{\alpha_x+\alpha_p}\frac{\alpha_x\alpha_p}{(\mu\alpha_x+\alpha_p)^2}}{\left(1-r\left(\chi\frac{\mu\alpha_x}{\mu\alpha_x+\alpha_p}+(1-\chi)\frac{\alpha_x}{\alpha_x+\alpha_p}\right)\right)^2} \geq 0$ if and only if $r \geq 0$ (with equality at $r = 0$). Also, $\frac{\partial\lambda^H}{\partial\mu} = \frac{(1-r)\left(1-\frac{r(1-\chi)\alpha_x}{\alpha_x+\alpha_p}\right)\frac{\alpha_x\alpha_p}{(\mu\alpha_x+\alpha_p)^2}}{\left(1-r\left(\chi\frac{\mu\alpha_x}{\mu\alpha_x+\alpha_p}+(1-\chi)\frac{\alpha_x}{\alpha_x+\alpha_p}\right)\right)^2} > 0$. □

Proof of Proposition 5. Plugging in λ^H and λ^L, the ex-ante expected utility for the agents of type H and L can be rewritten as

$$
\begin{aligned}
\mathbb{E}(U^H|\theta) &= -\Big[(1-r)^2\mu\alpha_x + \big(1+r(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p}\big)^2\alpha_p\Big]/[H(\alpha_p)]^2; \\
\mathbb{E}(U^L|\theta) &= -\Big[(1-r)^2\alpha_x + \big(1-r\chi(\mu-1)\frac{\alpha_x}{\mu\alpha_x+\alpha_p}\big)^2\alpha_p\Big]/[L(\alpha_p)]^2,
\end{aligned}
$$

where $H(\alpha_p) \equiv \mu\alpha_x + \alpha_p - \mu\alpha_x r\left(\chi + (1-\chi)\frac{\mu\alpha_x+\alpha_p}{\mu(\alpha_x+\alpha_p)}\right) = (1-r)\mu\alpha_x + \left(1 + \frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)\alpha_p$ and $L(\alpha_p) \equiv \alpha_x + \alpha_p - \alpha_x r\left(\chi\frac{\mu(\alpha_x+\alpha_p)}{\mu\alpha_x+\alpha_p} + 1 - \chi\right) = (1-r)\alpha_x + \left(1 - \frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)\alpha_p$.

(i) The derivative of $\mathbb{E}(U^H|\theta)$ with respect to α_p is:

$$\begin{aligned}\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\alpha_p} &= \frac{1}{[H(\alpha)]^3}\Bigg[\alpha_p\left(1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)^3 \\ &+ (1-r)\mu\alpha_x\alpha_p\left(1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2} \\ &- (1-r)\mu\alpha_x\underbrace{\left(2r-1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)}_{(A)}\left(1+\alpha_x\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right)\Bigg],\end{aligned} \tag{9}$$

where the term in the bracket can be rewritten as

$$\begin{aligned}\alpha_p\left(1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right)^3 &+ (1-r)\mu\alpha_x\Bigg\{2\left[1+\alpha_x\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right](1-r) \\ &- \left[1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right]\left(1+r(1-\chi)(\mu-1)\frac{\alpha_x(\alpha_x-\alpha_p)}{(\alpha_x+\alpha_p)^2}\right)\Bigg\}.\end{aligned}$$

Note that $\left[1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right] > 0$ and $\left[1+\alpha_x\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right] > 0$ by Condition 1. With $r \le 0$, $\left[1+\alpha_x\frac{r(1-\chi)(\mu-1)\alpha_x}{(\alpha_x+\alpha_p)^2}\right] \ge \left[1+\frac{r(1-\chi)(\mu-1)\alpha_x}{\alpha_x+\alpha_p}\right]$ and $(1-r) \ge \left(1+r(1-\chi)(\mu-1)\frac{\alpha_x(\alpha_x-\alpha_p)}{(\alpha_x+\alpha_p)^2}\right)$, where the equality holds iff $r=0$. So if $r \le 0$, then $\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\alpha_p} > 0$. One can also easily see from (9) that if $r > 0$ and $(A) \le 0$, then $\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\alpha_p} > 0$. But if $r > 0$ and $(A) > 0$, then there are ranges of parameters for which $\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\alpha_p} < 0$. The condition $(A) > 0$ reduces to $r > \frac{1}{2+(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p}}$, which is the sufficient condition (6) in Proposition 5 for there to be parameter configurations for which $\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\alpha_p} < 0$. We can obtain a necessary and sufficient condition for $\mathbb{E}(U^H|\theta)$ to decrease by letting (9)< 0, resulting in condition (5).

(ii) The derivative of $\mathbb{E}(U^L|\theta)$ with respect to α_p is:

$$\begin{aligned}\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} &= \frac{1}{[L(\alpha_p)]^3}\Bigg[\alpha_p\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)^3 \\ &- (1-r)\alpha_x\alpha_p\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)\frac{r\chi(\mu-1)\alpha_x}{(\mu\alpha_x+\alpha_p)^2} \\ &- (1-r)\alpha_x\underbrace{\left(2r-1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)}_{(B)}\left(1-\alpha_x\frac{r\chi(\mu-1)\mu\alpha_x}{(\mu\alpha_x+\alpha_p)^2}\right)\Bigg].\end{aligned} \tag{10}$$

If $r \leq 0$, then $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} > 0$, because $\left(2r-1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right) < 0$. Let us consider the case when $r > 0$. First if $(B) \leq 0$, the first and third terms in the bracket above are positive, while the second term is not; but by grouping up the first and the second terms in the bracket, we can see that $\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right)^2 - (1-r)\alpha_x\frac{r\chi(\mu-1)\alpha_x}{(\mu\alpha_x+\alpha_p)^2} > 0$ because $\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right) > (1-r)$ and $\left(1-\frac{r\chi(\mu-1)\alpha_x}{\mu\alpha_x+\alpha_p}\right) > \alpha_x\frac{r\chi(\mu-1)\alpha_x}{(\mu\alpha_x+\alpha_p)^2}$. (The latter inequality holds because $1 > r\chi\frac{(\mu-1)(\mu+1)\alpha_x^2+(\mu-1)\alpha_x\alpha_p}{(\mu\alpha_x+\alpha_p)^2}$ where $r\chi < 1$ and $\frac{(\mu-1)(\mu+1)\alpha_x^2+(\mu-1)\alpha_x\alpha_p}{(\mu\alpha_x+\alpha_p)^2} < 1$.) So if $r > 0$ and $(B) \leq 0$, then $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} > 0$. But if $r > 0$ and $(B) > 0$, then there are ranges of parameters for which $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} < 0$. The condition $(B) > 0$ reduces to $r > \frac{1}{2-\chi(\mu-1)\frac{\alpha_x}{\mu\alpha_x+\alpha_p}}$, which is the sufficient condition (8) in Proposition 5 for there to be parameter configurations for which $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} < 0$. We can obtain a necessary and sufficient condition for $\mathbb{E}(U^L|\theta)$ to decrease by letting (10)< 0, resulting in condition (7). □

Proof of Corollary 1. Condition (8), $r > \frac{1}{2-\chi(\mu-1)\frac{\alpha_x}{\mu\alpha_x+\alpha_p}}$, is stronger than condition (6), $r > \frac{1}{2+(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p}}$. It can be verified that condition (7) is stronger than condition (5). So when $r > \frac{1}{2-\chi(\mu-1)\frac{\alpha_x}{\mu\alpha_x+\alpha_p}}$, under the range of parameters for which $\frac{\partial \mathbb{E}(U^L|\theta)}{\partial \alpha_p} < 0$, it will also be $\frac{\partial \mathbb{E}(U^H|\theta)}{\partial \alpha_p} < 0$. Hence, there exists a set of parameter values for which a higher α_p lowers $\mathbb{E}(W|\theta)$. Recalling that $\mathbb{E}(W|\theta) = \chi\mathbb{E}(U^H|\theta) + (1-\chi)\mathbb{E}(U^L|\theta)$, the necessary and sufficient condition for $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$ can be exactly characterized by $\chi \times (9) + (1-\chi) \times (10) < 0$. □

Proof of Proposition 6. The derivative of $\mathbb{E}(W|\theta)$ with respect to χ is $\frac{\partial \mathbb{E}(W|\theta)}{\partial \chi} = \mathbb{E}(U^H|\theta) -$

$\mathbb{E}(U^L|\theta) + \chi\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\chi} + (1-\chi)\frac{\partial\mathbb{E}(U^L|\theta)}{\partial\chi}$. First, $\mathbb{E}(U^H|\theta) - \mathbb{E}(U^L|\theta) > 0$ if and only if

$$\begin{aligned}
&\left(\lambda^H(\alpha_x^H)^{-1/2}\right)^2 + \left((1-\lambda^H)(\alpha_p)^{-1/2}\right)^2 < \left(\lambda^L(\alpha_x^L)^{-1/2}\right)^2 + \left((1-\lambda^L)(\alpha_p)^{-1/2}\right)^2 \\
\leftrightarrow &\frac{(1-r)^2\delta^H}{\alpha_x^H+\alpha_p} < \frac{(1-r)^2\delta^L}{\alpha_x^L+\alpha_p} + \frac{(1-r\bar{\delta}-(1-r)\delta^L+1-r\bar{\delta}-(1-r)\delta^H)(1-r)(\delta^H-\delta^L)}{\alpha_p} \\
\leftrightarrow &(1-r)\delta^H(1-\delta^H) - (1-r\bar{\delta}-(1-r)\delta^H)\delta^H \\
&\quad < (1-r)\delta^L(1-\delta^L) - (1-r\bar{\delta}-(1-r)\delta^L)\delta^L + (1-r\bar{\delta})(\delta^H-\delta^L) \\
\leftrightarrow &(-r+r\bar{\delta})(\delta^H-\delta^L) < (1-r\bar{\delta})(\delta^H-\delta^L),
\end{aligned}$$

which holds because $-r(1-\bar{\delta}) < 1-r\bar{\delta}$ for all $r \in (-1,1)$ and $\delta^H > \delta^L$. Second, $\frac{\partial\mathbb{E}(U^H|\theta)}{\partial\chi} = -2\frac{\partial\lambda^H}{\partial\chi}\left[\frac{\lambda^H}{\alpha_x^H} - \frac{(1-\lambda^H)}{\alpha_p}\right] \geq 0$ because $\frac{\partial\lambda^H}{\partial\chi} \gtreqless 0$ and $\frac{\lambda^H}{\alpha_x^H} \lesseqgtr \frac{(1-\lambda^H)}{\alpha_p}$ iff $r \gtreqless 0$. Third, $\frac{\partial\mathbb{E}(U^L|\theta)}{\partial\chi} = -2\frac{\partial\lambda^L}{\partial\chi}\left[\frac{\lambda^L}{\alpha_x^L} - \frac{(1-\lambda^L)}{\alpha_p}\right] > 0$ because $\frac{\partial\lambda^L}{\partial\chi} \gtreqless 0$ and $\frac{\lambda^L}{\alpha_x^L} \lesseqgtr \frac{(1-\lambda^L)}{alpha_p}$ iff $r \gtreqless 0$. Therefore, it follows that $\frac{\partial\mathbb{E}(W|\theta)}{\partial\chi} > 0$ for any $r \in (-1,1)$. □

Proof of Proposition 7. The derivative of $\mathbb{E}(W|\theta)$ with respect to μ is

$$\begin{aligned}
\frac{\partial\mathbb{E}(W|\theta)}{\partial\mu} = &-\chi\left[\frac{2\lambda^H\frac{\partial\lambda^H}{\partial\mu}\mu\alpha_x - (\lambda^H)^2\alpha_x}{(\mu\alpha_x)^2} - \frac{2(1-\lambda^H)\frac{\partial\lambda^H}{\partial\mu}\alpha_p}{(\alpha_p)^2}\right] \\
&-(1-\chi)\left[\frac{2\lambda^L\frac{\partial\lambda^L}{\partial\mu}\alpha_x}{(\alpha_x)^2} - \frac{2(1-\lambda^L)\frac{\partial\lambda^L}{\partial\mu}\alpha_p}{(\alpha_p)^2}\right].
\end{aligned} \tag{11}$$

Here, the terms in the first big bracket are

$$\frac{2\lambda^H\frac{\partial\lambda^H}{\partial\mu}\mu\alpha_x - (\lambda^H)^2\alpha_x}{(\mu\alpha_x)^2} - \frac{2(1-\lambda^H)\frac{\partial\lambda^H}{\partial\mu}\alpha_p}{(\alpha_p)^2} = -2\frac{\partial\lambda^H}{\partial\mu}\frac{r(1-\bar{\delta})}{\alpha_p(1-r\bar{\delta})} - \frac{(\lambda^H)^2\alpha_x}{(\mu\alpha_x)^2}. \tag{12}$$

If $r \geq 0$, then it is easy to see that (12) is negative. For the case of $r < 0$, using $\frac{\partial\lambda^H}{\partial\mu} = \lambda^H\frac{(1-r(1-\chi)\delta^L)\alpha_p}{\mu(1-r\bar{\delta})(\mu\alpha_x+\alpha_p)}$, (12) can be rewritten as

$$\begin{aligned}
&-\frac{\lambda^H}{\mu(\mu\alpha_x+\alpha_p)(1-r\bar{\delta})^2}\left[2r(1-r(1-\chi)\delta^L)(1-\bar{\delta}) + (1-r)(1-r\bar{\delta})\right] \\
&= -\frac{\lambda^H}{\mu(\mu\alpha_x+\alpha_p)(1-r\bar{\delta})^2}\left[(1-r(1-\chi)\delta^L)(1+r-2r\bar{\delta}) - r(1-r)\chi\delta^H\right],
\end{aligned}$$

which is negative (noting that $r > -1$). The terms in the second big bracket in (11) are

$$\frac{2\lambda^L \frac{\partial \lambda^L}{\partial \mu}\alpha_x}{(\alpha_x)^2} - \frac{2(1-\lambda^L)\frac{\partial \lambda^L}{\partial \mu}\alpha_p}{(\alpha_p)^2} = \frac{2\frac{\partial \lambda^L}{\partial \mu}\left(\lambda^L \alpha_p - (1-\lambda^L)\alpha_x\right)}{\alpha_x \alpha_p} = -2\frac{\partial \lambda^L}{\partial \mu}\frac{r(1-\bar{\delta})}{\alpha_p(1-r\bar{\delta})} < 0,$$

because $\frac{\partial \lambda^L}{\partial \mu} \gtreqless 0$ iff $r \gtreqless 0$. Therefore, it follows that $\frac{\partial \mathbb{E}(W|\theta)}{\partial \mu} > 0$ for any $r \in (-1, 1)$. □

Proof of Corollary 2. A necessary and sufficient condition for $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$ in Morris and Shin case is $\frac{\alpha_p}{\alpha_x} < (2r-1)(1-r)$, where $\alpha_x \equiv \alpha_x^L = \alpha_x^H$ in terms of our notation. When $r > 1/2$, if there are two differing types such that $\alpha_x = \alpha_x^L < \alpha_x^H = \mu\alpha_x$, then $\frac{\alpha_p}{\alpha_x^H} < \frac{\alpha_p}{\alpha_x^L} < (2r-1)(1-r)$ becomes a sufficient condition for $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$. Given the two differing types, the possibility arises that $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$ will occur when either $\frac{\alpha_p}{\alpha_x^H} < (2r-1)(1-r) < \frac{\alpha_p}{\alpha_x^L}$ or $(2r-1)(1-r) < \frac{\alpha_p}{\alpha_x^H} < \frac{\alpha_p}{\alpha_x^L}$ hold. For example, for a set of parameter values $\{r = 0.8, \chi = 0.4, \mu = 1.25, \alpha_x = 4, \alpha_p = 0.5\}$ so that $\frac{\alpha_p}{\alpha_x^H} < (2r-1)(1-r) < \frac{\alpha_p}{\alpha_x^L}$, the possibility of $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$ arises; $\{r = 0.8, \chi = 0.4, \mu = 1.025, \alpha_x = 4, \alpha_p = 0.5\}$ is an example in which $\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha_p} < 0$ arises when $(2r-1)(1-r) < \frac{\alpha_p}{\alpha_x^H} < \frac{\alpha_p}{\alpha_x^L}$. Thus, the set of parameter configurations under which better public information reduces welfare is larger when $\alpha_x^L < \alpha_x^H$ is the case. □

Appendix B. Technical Derivations

B.1. Derivation of Condition 1

By examining the formulas in (2), it is easy to see that $\lambda^H > 0$ and $\lambda^L > 0$. Also, $\lambda^L < 1$ if and only if $(1-r)\delta^L < 1 - r(\chi\delta^H + (1-\chi)\delta^L)$, which reduces to $r\chi\frac{\mu\alpha_x - \alpha_x}{\mu\alpha_x + \alpha_p} < 1$; this is always satisfied. $\lambda^H < 1$ if and only if $(1-r)\delta^H < 1 - r(\chi\delta^H + (1-\chi)\delta^L)$. This inequality reduces to $-r(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p} < 1$, which is Condition 1.

Condition 1 always holds if $r > 0$; but may fail if $r < 0$ and fails only if $\mu > 2$. A specific parameterization that results in $-r(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p} > 1$ is

$$\{r = -0.9, \chi = 0.1, \mu = 4, \alpha_x = 2, \alpha_p = 1\},$$

while

$$\{r = -0.9, \chi = 0.1, \mu = 1.5, \alpha_x = 2, \alpha_p = 1\}$$

is an instance in which $-r(1-\chi)(\mu-1)\frac{\alpha_x}{\alpha_x+\alpha_p} < 1$. Roughly stated, the more likely it is that Condition 1 fails the greater the magnitude of information inequality, the more severe the agents wish to differentiate from one another, and/or the smaller the fraction of high-type agents. For example, if a very small fraction of agents have access to much better private information than the rest of the population with strong substitutability in actions, then that small group of agents would find it optimal to rely only on private information in equilibrium; we do not consider such cases in the present paper.

B.2. Model with $\mu < 1$

Fixing Morris and Shin's (2002) model as a reference point, our model with $\mu > 1$ postulates a situation where a χ-fraction of the population has become privately better informed, whereas James and Lawler's (2012) model takes a χ-fraction of the population to be privately less well-informed (with $\chi = 1/2$ for simplicity). So their model would be equivalent to our model with a modification that $\mu < 1$. While the results for $\mu < 1$ would be dual to those for $\mu > 1$, it is useful to reformulate them and give appropriate interpretations in relation to James and Lawler (2012).

With $\mu < 1$, type H agents are now privately less well-informed than type L agents. To avoid confusion, we will refer to them with a small letter h rather than H.

Condition 1 is always satisfied with $\mu < 1$. Proposition 1 remains the same. So we can use the same formulas for λ^t in (1) or (2) as:

$$\lambda^h = \frac{(1-r)\delta^h}{1-r\bar{\delta}} \quad \text{and} \quad \lambda^L = \frac{(1-r)\delta^L}{1-r\bar{\delta}},$$

where $\delta^h \equiv \frac{\mu\alpha_x}{\mu\alpha_x+\alpha_p} < \frac{\alpha_x}{\alpha_x+\alpha_p} \equiv \delta^L$, and $\bar{\delta} \equiv \chi\delta^h + (1-\chi)\delta^L$.

Proposition 2 holds; Propositions 5 and Corollary 1 remain qualitatively intact.

Lemma 1 for $\mu < 1$ is as follows: $\lambda^h < \lambda^L < \lambda$ if $r > 0$ and $\lambda^h < \lambda < \lambda^L$ if $r < 0$. When

$r > 0$, the high-type (now type L) agents rely more on public information, concurring with James and Lawler (2012) about the overweighting of public information.

Propositions 3 and 6 for $\mu < 1$ are reversed: $\frac{\partial \lambda^t}{\partial \chi} \lesseqgtr 0$ if and only if $r \gtreqless 0$ for all $t \in \{h, L\}$; Social welfare decreases with an increase in χ. The reversals are simply because a higher χ now refers to a situation where there are more privately less well-informed agents.

Proposition 4 holds for $\mu < 1$. For the case of $r > 0$, James and Lawler (2012) interprets their corresponding result as follows: The less pronounced the informational gap, the more the agents rely on private information. In their setting, an increase in the precision of low-type agents' private information (i.e., an increase in $\mu < 1$ for type h in terms of our notation) represents a narrowing of the informational gap. Proposition 4 for $\mu > 1$ asserts that an increase in the precision of high-type agents' private information (i.e., an increase in $\mu > 1$ for type H), which represents a widening of the informational gap, also increases the agents' reliance on private information. In fact, James and Lawler's (2012) interpretation should be amended because a narrowing of the informational gap is not the cause for the agents' higher reliance on private information. But apart from interpreting, we can integrate James and Lawler's (2012) relevant analysis and our Proposition 4 as follows. Let us refer to the type with a higher precision of private information in both settings as "the rich" and the type with a lower precision of private information as "the poor." In the case of strategic complements ($r > 0$), the agents rely more on private information either when the rich get richer or the poor get richer. The former situation is accompanied by greater information inequality while the latter is associate with lesser information inequality.

Proposition 7 remains the same for $\mu < 1$, implying that more precise private information some agents is beneficial to welfare, regardless of whether the rich is getting richer or the poor is getting richer. James and Lawler (2012), comparing their result with Morris and Shin (2002), deduce that the set of parameter configurations under which decreasing welfare with public information arises is smaller when there are two types of private information quality. In terms of our model, their perspective is equivalent to reducing $\mu < 1$, so their results is actually consistent with our Corollary 2. Again, the interpretation should not imply a causal relation between a narrowing or widening of the informational gap and social welfare.

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