Does Higher Firm Profit Dispersion Reflect Greater Micro Uncertainty?

Jin Yeub Kim[a,*] and Myungkyu Shim[b]

[a] *Department of Economics, University of Nebraska–Lincoln, 730 N. 14th Street, Lincoln, NE 68588, USA*
[b] *School of Economics, Sogang University, 35 Baekbeom-Ro, Mapo-Gu, Seoul, 04107, South Korea*

October 20, 2018

Abstract

Countercyclical dispersion of firm outcomes (micro dispersion) is commonly used as a proxy for micro uncertainty. In this paper, we characterize conditions under which micro dispersion and micro uncertainty co-move positively in the context of a large Cournot economy with dispersed information and a financial market that aggregates private information. We also show that the parameter region supporting the positive co-movement shrinks when (1) public signal is endogenous through financial asset prices or (2) strategic substitutability in firms' output decisions is weak. Our analysis raises a cautionary note on using micro dispersion as a measure of uncertainty shocks.

JEL classification: D82; D83; E32; G12; G14

Keywords: Firm profit dispersion; Micro uncertainty; Cournot market; Dispersed Information; Endogenous public signal; Strategic substitutability

* Corresponding author.
E-mail addresses: shiningjin@gmail.com (J. Y. Kim), audrb917@gmail.com (M. Shim).

1. Introduction

Uncertainty shocks—changes in the second moment of economic variables—may play a role for explaining aggregate fluctuations (Bloom, 2014). The volatility of stock market returns (VIX), the dispersion of survey-based forecasts, and the cross-sectional dispersion in firm-level outcomes have all been widely used as measures of uncertainty shocks. A growing body of literature is concerned with how closely these uncertainty measures are linked to true economic uncertainty (e.g. Jurado, Ludvigson and Ng, 2015; Kozeniauskas, Orlik and Veldkamp, 2018). This paper focuses on the dispersion of firm profits and examines its adequacy as a proxy for micro uncertainty—the uncertainty that firms have about their own economic outcomes.

One potential source of micro uncertainty shocks is a change in the variance of private signals that firms receive. Cross-sectional dispersion of firm profits captures idiosyncratic differences in information signals across firms, so it may serve as a sensible proxy for micro uncertainty that arises from noisy private signals. However, as Kozeniauskas, Orlik and Veldkamp (2018) point out, noisier signals are weighted less in firms' decisions, which may lead to a lower micro dispersion; hence, more dispersed firm profits may not reflect greater micro uncertainty. This paper complements Kozeniauskas, Orlik and Veldkamp (2018) by examining *when* micro dispersion could be tightly linked to true micro uncertainty.[0]

In doing so, we consider a large Cournot economy with strategic substitutability in which each firm receives two signals about a demand shock: a private signal, and a public signal that is endogenously determined through a financial market (á la Rondina and Shim (2015)). For such economies, we exactly characterize the necessary and sufficient conditions under which higher micro uncertainty (formally, an increase in private signal variance) generates higher micro dispersion. We show that the presence of an endogenous public signal reduces the set

[0]While Kozeniauskas, Orlik and Veldkamp (2018) analyze the relationship between different types of uncertainty measures due to various uncertainty shocks, this paper focuses on micro uncertainty and micro dispersion.

of parameter values over which firm profit dispersion increases with private signal variance. The parameter set is further reduced for economies with weaker strategic substitutability.

Our results identify characteristics of economies where the micro uncertainty and dispersion measures co-vary positively or negatively. In terms of the parameter range, it is more likely for micro dispersion to be lower due to greater micro uncertainty when the public signal is endogenous through financial asset prices compared to when the public signal is exogenous; as well as when strategic substitutability in firms' output decisions is weaker. An implication is that the variation in cross-sectional firm-level outcomes is not necessarily driven by micro uncertainty shocks. Our analysis emphasizes that micro dispersion may or may not be tightly linked to true micro uncertainty. In this sense, this paper suggests that there is a strong caution to be added to the approach that uses dispersion of firm-level outcomes to infer micro uncertainty.

2. The Model

Our analysis builds on the model and results of Rondina and Shim (2015), which we briefly review here. There are two markets: a Cournot market and a financial market.

The Cournot market is populated by a measure-one continuum of firms, indexed by $i \in [0, 1]$, all producing a homogeneous output good. Each individual firm chooses an output level q, facing the inverse demand curve $z = \theta - aQ$ and a quadratic cost function $c(q) = \frac{1}{2}cq^2$, where z is the unitary price of the firm's output, θ is a demand shock common across firms, Q is aggregate output, $a \geq 0$, and $c > 0$. Then the individual firm's profit is given by

$$\Pi(q, Q, \theta) \equiv zq - c(q) = (\theta - aQ)q - \frac{1}{2}cq^2. \tag{1}$$

The demand shock parameter θ is an exogenous random variable drawn from the (improper) uniform distribution over the real line.[1] When output decisions are made, each firm

[1] This is a reduced form approach in line with Leduc and Liu (2016).

does not know θ but instead observes noisy private and public signals about θ. A private signal is given by $x_i = \theta + \varepsilon_i$, where idiosyncratic noises ε_i are normally distributed, independent of θ and across firms, with mean zero and variance σ_x^2. A public signal is given by $p = \theta + \xi$, where common noise ξ is normal, independent of θ and all ε_i, with mean zero and variance σ_p^2.

The equilibrium output strategy of firm i in the Cournot market is characterized as

$$q^*(x_i, p) = \rho\big[\lambda x_i + (1-\lambda)p\big], \tag{2}$$

where $\rho = 1/(a+c)$, $\lambda = \frac{\sigma_x^{-2}}{\sigma_x^{-2} + \frac{1}{1-r}\sigma_p^{-2}}$, and $r = -a/c$. The parameter r measures the degree of substitutability between individual and aggregate outputs in the Cournot market. We restrict attention to $r \in (-1, 0]$. The parameter λ measures the sensitivity of equilibrium outputs to private information relative to public information.

In the financial market, a large number of traders take positions on a risky asset, the payoff of which is a function of the aggregate output of the Cournot market. We interpret p as the price of the risky asset, which aggregates disperse private information in the financial market, acting as a public signal. Then the variance of public signal σ_p^2 (or, equivalently, the precision of the asset price σ_p^{-2}) is endogenously determined by trades in the financial market.[2] Focusing on equilibrium asset prices of the form $p = \theta + \xi$, we can solve for σ_p^2 in a Rational Expectations equilibrium in the financial market. Proposition 1 of Rondina and Shim (2015) asserts that in the stable equilibrium, the precision of the financial asset price is decreasing in the precision of firms' private signals:

$$\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}} < 0. \tag{3}$$

The intuition for this result is straightforward: When private signals become more precise,

[2] One might treat σ_p^2 as the VIX, which measures the future volatility of the stock market, by taking the financial asset to be a stock market index.

firms in the Cournot market find it optimal to rely more on their private information, which makes their output decisions less predictable from the perspective of traders in the financial market. As traders perceive the asset payoff as riskier, they reduce trading volumes; so less private information is aggregated into the asset price, resulting in less precise pubic signal.

3. Results

Changes in the variance of private signals (σ_x^2) is one potential source of changes in uncertainty that firms have about their own economic outcomes, which Kozeniauskas, Orlik and Veldkamp (2018) call micro uncertainty. Because data on firms' beliefs about future outcomes is rare, dispersion of firm-level outcomes (micro dispersion) is often used as a proxy for micro uncertainty. We examine how micro dispersion and micro uncertainty are related in the context of our model—that is, what happens to micro dispersion when private signal variance rises.

We define micro dispersion to be the cross-sectional dispersion of firm profits, measured by the variation in the equilibrium profits across firms for any given realizations of (θ, p):

$$\mathcal{MD} \equiv Var\left(\Pi^*|\theta, p\right) \propto \lambda^4 \sigma_x^4, \tag{4}$$

the derivation of which is given in Appendix A.1.

The following proposition provides a necessary and sufficient condition for micro dispersion to increase with micro uncertainty. The proof is in Appendix A.2.

Proposition 1. *The dispersion of firm profits $\mathcal{MD}$ increases with the variance of private signals σ_x^2 if and only if*

$$\lambda\left(1 + \frac{1}{1-r}\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}}\right) > \frac{1}{2}, \tag{5}$$

where $\lambda = \frac{\sigma_x^{-2}}{\sigma_x^{-2} + \frac{1}{1-r}\sigma_p^{-2}}$.

This result asserts that the micro uncertainty and dispersion measures can be either positively or negatively correlated depending on two opposing effects arising from noisier private signals. The direct effect (captured by the term σ_x^4 in (4)) is that firms will have more dispersed beliefs about their profits, which raises micro dispersion. The indirect effect (captured by the term λ^4 in (4)) is that firms will assign less weight on their private signals, which lowers micro dispersion. For a large enough sensitivity of firm outputs to private information, the indirect effect is secondary so that micro dispersion will co-move positively with micro uncertainty. But when firms are assigning a sufficiently small weight on their private signals relative to the publicly observable financial price, the indirect effect dominates and micro dispersion will decrease even with greater micro uncertainty.

Importantly, the indirect effect is aggravated when the public signal p is endogenously determined in the financial market. When firms' private signals become less precise (a higher σ_x^2), the financial asset price becomes more accurate (a lower σ_p^2). If this deceasing reaction of asset price variance is strong enough (a sufficiently high $-\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}}$), then more noise in private signals always lowers micro dispersion. The following proposition gives a sufficient condition for micro dispersion to decrease with micro uncertainty.

Proposition 2. *If $-\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}} > \frac{1-r}{2}$, then the dispersion of firm profits $\mathcal{MD}$ unambiguously decreases with an increase in σ_x^2.*

Proof. Plugging in $\lambda = \frac{\sigma_x^{-2}}{\sigma_x^{-2} + \frac{1}{1-r}\sigma_p^{-2}}$ and after rearranging, we can rewrite (5) as:

$$\sigma_x^{-2} > \frac{\sigma_p^{-2}}{1-r} + \frac{2\sigma_x^{-2}}{1-r}\left[-\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}}\right]. \tag{6}$$

The right-hand-side of (6) is strictly greater than its left-hand-side if $\frac{2}{1-r}\left[-\frac{\partial \sigma_p^{-2}}{\partial \sigma_x^{-2}}\right] > 1$; so it is a sufficient condition under which $\frac{d\mathcal{MD}}{d\sigma_x^2} < 0$. $\square$

When public signal p is exogenous, the dispersion of firm profits $\mathcal{MD}$ increases with an increase in σ_x^2 if and only if $\sigma_x^{-2} > \frac{\sigma_p^{-2}}{1-r}$. Comparing this condition with (6), we can see the role of the endogenous price. The following corollary is immediate.

Corollary 1. *Given* $r \in (-1, 0]$*, the term* $\frac{2\sigma_x^{-2}}{1-r}\left[-\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right]$ *in* (6) *captures the extent to which the endogeneity of the public signal impacts the range of parameters for which micro dispersion decreases with greater micro uncertainty.*

In the presence of a financial market where public signal is endogenous through asset prices, the micro uncertainty and dispersion measures are more likely to be negatively correlated, compared to when public information is exogenous; more likely in the sense of the larger parameter range for which micro dispersion falls with micro uncertainty due to the endogenous price impact.

We now analyze how the degree of strategic substitutability in the Cournot market, captured by $r \in (-1, 0]$, affects our findings. We can see from the closed-form expression for $\left[-\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right]$ derived in Rondina and Shim (2015) that $\left[-\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right]$ is increasing in r. Thus the right-hand-side of (6) increases with a higher r. That is, the range of parameter values for which micro dispersion and micro uncertainty co-move positively shrinks as firms' output decisions are less substitutable with each other. This proves the following corollary.

Corollary 2. *Given the financial market that endogenously determines public signal, the weaker the degree of strategic substitutability in the Cournot market, the more likely that micro dispersion decreases with greater micro uncertainty.*

This result is intuitive. When substitutability in firms' output decisions becomes weaker, firms would rely more on public information than on private information. In our model with endogenous price, the public signal becomes more informative by more dispersed private signals, inducing firms to rely more on public information. So a weaker degree of strategic substitutability amplifies the effect generated by endogenous public signal. As a result, the parameter region in which micro dispersion and micro uncertainty move in the opposite direction is enlarged.

4. Conclusion

In this paper, we assert that higher micro uncertainty (represented by more dispersed private signals) may lead to lower micro dispersion. That is, it is possible for the dispersion of firm profits, which is a commonly used measure for micro uncertainty, to decrease and at the same time true micro uncertainty to increase. This is more likely in economies with an endogenous public signal through financial markets than in economies with an exogenous public signal; or in economies with weaker strategic substitutability in Cournot markets. An implication is that the observation of a higher profit dispersion is not a sufficient statistic or indicator for greater uncertainty that firms have about their outcomes.

A. Appendix

A.1. Derivation of Profit Dispersion

Given the equilibrium firm strategy in (2), aggregate output in equilibrium is $Q^*(\theta, p) = \rho\,[\lambda\theta + (1-\lambda)p]$. Plugging q^* and Q^* into (1), the expected value of firm profit in equilibrium for any given (θ, p) is:

$$
\begin{aligned}
\mathbb{E}[\Pi^*(q^*, Q^*, p)|\theta, p] &= (\theta - aQ^*)\mathbb{E}[q^*|\theta, p] - \frac{1}{2}c\mathbb{E}[q^{*2}|\theta, p] \\
&= (\theta - aQ^*)Q^* - \frac{1}{2}(Var(q^*|\theta, p) + Q^{*2})
\end{aligned}
$$

The deviation of the equilibrium profit level from the mean is:

$$
\begin{aligned}
\Pi^* - \mathbb{E}[\Pi^*|\theta, p] &= (q^* - Q^*)\Big(\theta - aQ^* - \frac{1}{2}c(q^* + Q^*)\Big) + \frac{1}{2}cVar(q^*|\theta, p) \\
&= \rho\lambda\varepsilon_i\Big(-(1-\lambda)\xi - \frac{1}{2}c\rho\lambda\varepsilon_i\Big) + \frac{1}{2}cVar(q^*|\theta, p)
\end{aligned}
$$

Then the equilibrium level of cross-sectional firm profit dispersion for any given (θ, p) is:

$$
\begin{aligned}
Var(\Pi^*|\theta,p) &= \mathbb{E}\left[(\Pi^* - \mathbb{E}[\Pi^*|\theta,p])^2|\theta,p\right] \\
&= \frac{1}{4}c^2\rho^4\lambda^4\mathbb{E}[\varepsilon_i^4|\theta,p] - \frac{1}{2}c^2\rho^2\lambda^2\mathbb{E}[\varepsilon_i^2|\theta,p]Var(q^*|\theta,p) + \frac{1}{4}c^2Var(q^*|\theta,p)^2 \\
&= \frac{1}{2}c^2\rho^4\lambda^4\sigma_x^4
\end{aligned}
$$

where the last equality follows from $\mathbb{E}[\varepsilon_i^4|\theta,p] = 3\sigma_x^4$, $\mathbb{E}[\varepsilon_i^2|\theta,p] = \sigma_x^2$, and $Var(q^*|\theta,p) = \rho^2\lambda^2\sigma_x^2$. Because $c > 0$ and $\rho = 1/(a+c)$ are positive constants, we arrive at (4).

A.2. Proof of Proposition 1

Differentiating $\lambda^2\sigma_x^2$ with respect to σ_x^2 gives

$$
\begin{aligned}
\frac{d\lambda^2\sigma_x^2}{d\sigma_x^2} &= \lambda^2 + 2\lambda\left[\frac{-\sigma_x^{-4}\left(\sigma_x^{-2} + \frac{1}{1-r}\sigma_p^{-2}\right) - \sigma_x^{-2}\left(-\sigma_x^{-4} + \frac{1}{1-r}\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}(-\sigma_x^{-4})\right)}{\left(\sigma_x^{-2} + \frac{1}{1-r}\sigma_p^{-2}\right)^2}\right]\sigma_x^2 \\
&= \lambda^2 + 2\lambda\left[-\lambda(1-\lambda) + \lambda^2\frac{1}{1-r}\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right] \\
&= \lambda^2\left[2\lambda\left(1 + \frac{1}{1-r}\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right) - 1\right].
\end{aligned}
$$

Because $\mathcal{MD} \propto (\lambda^2\sigma_x^2)^2$, we obtain that $\frac{d\mathcal{MD}}{d\sigma_x^2} > 0$ if and only if $2\lambda\left(1 + \frac{1}{1-r}\frac{\partial\sigma_p^{-2}}{\partial\sigma_x^{-2}}\right) - 1 > 0$, which is equivalent to (5).

Acknowledgments

The authors thank the anonymous referee for helpful suggestions. Jin Yeub Kim appreciates the hospitality of School of Economics at Sogang University; this research was done during her summer visit there.

Declaration of Interest

The authors declare that there are no financial support and competing interests to disclose for this article.

References

Bloom, Nicholas. 2014. "Fluctuations in Uncertainty." *Journal of Economic Perspectives* 28(2):153–175.

Jurado, Kyle, Sydney C. Ludvigson and Serena Ng. 2015. "Measuring Uncertainty." *American Economic Review* 105(3):1177–1216.

Kozeniauskas, Nicholas, Anna Orlik and Laura Veldkamp. 2018. "What Are Uncertainty Shocks?" *Journal of Monetary Economics* . Forthcoming.

Leduc, Sylvain and Zheng Liu. 2016. "Uncertainty Shocks are Aggregate Demand Shocks." *Journal of Monetary Economics* 82:20–35.

Rondina, Giacomo and Myungkyu Shim. 2015. "Financial Prices and Information Acquisition in Large Cournot Markets." *Journal of Economic Theory* 158 (Part B):769–786.