On the Welfare Cost of Business Cycles: the Role of Labor-Market Heterogeneity*

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Abstract

Are business cycles equally beneficial or harmful to consumers? Has welfare inequality increased or decreased due to rise of income/wealth inequality? By utilizing a heterogeneous agent RBC model with endogenous labor supply, this paper aims to answer these questions. We first show that while technology-driven business cycles are beneficial on average, a finding that is consistent with the recent literature, the welfare gain is not equally distributed; it is beneficial (resp. harmful) for agents who are relatively rich (resp. poor). The key to understanding the monotonic relationship between welfare gain from the business cycles and wealth level, a finding different from the previous literature that argues that there is a non-monotonic relationship between the two, is shown to be endogenous labor supply. Finally, we analyze the short run consequence of rising income/wealth inequality on welfare cost.

JEL classification: E24, E30, E32

Keywords: Heterogenous agent model, Welfare cost of business cycles, Endogenous labor supply, Rising inequality

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1 Introduction

Are business cycles beneficial or harmful to consumers? If so, how much is the cost? After the seminal work by Lucas (1987) that first addresses these questions, literature on the welfare cost of business cycles has flourished and has been developed in many dimensions: See Barlevy (2004); Ramey and Ramey (1995); Otrok (2001); Storesletten, Telmer, and Yaron (2001) among many others. This paper aims to contribute to this literature by merging two different strands of the research in this area with a unified framework.

The first strand of the literature that we consider is relatively new; two recent papers, Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014), find that business cycles are welfare-improving rather than welfare-detrimental in the class of representative real business cycles (henceforth RBC) model when production factors are endogenously determined.[1] This is because agents can take advantages of business cycles in favor for them by varying production factors, and that benefit generally dominates the cost associated with the fluctuations due to risk-aversion. The second strand of the literature is to study heterogeneous aspects of the welfare cost of business cycles across agents (see Krusell, Mukoyama, Şahin, and Smith (2009) and Mukoyama and Şahin (2006)): While the average welfare cost might not be sizable as shown by Lucas (1987), the cost might be more substantial for agents with low income while be less substantial for agents with high income. For instance, Mukoyama and Şahin (2006) found that the welfare cost is much greater for unskilled workers.

On the one hand, while previous papers with heterogeneous agent model have documented the possible heterogeneity in the welfare cost across agents, to our best knowledge, none of them explicitly consider the role of endogenous labor supply: As is emphasized by Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014), however, endogenous labor supply is particularly important in studies of the welfare cost. On the other hand, these two recent papers only consider the aggregate welfare cost, and hence has neglected the role of heterogeneity among workers. We try to synthesize these two perspectives by introducing endogenous labor choice into otherwise standard Aiyagari type model.

Our model framework is a version of Chang and Kim (2007) except that we mainly consider the intensive margin of labor.[2] Heterogeneity is introduced in the form of idiosyncratic labor productivity

[1]While Heiberger and Maußner (2020) argue non-robustness of Cho, Cooley, and Kim (2015)'s finding to an alternative solution algorithm with log-level specification of the exogenous shock, Kim and Shim (2020) show that Cho, Cooley, and Kim (2015)'s finding is still preserved in the model in which the exogenous shock is specified in level.

[2]Extensive margin of labor is further considered in Section 3.4.

across the workers. We assume that there is an incomplete asset market and hence wealth distribution is non-degenerate. The most important distinction between our model and the model introduced in Krusell, Mukoyama, Şahin, and Smith (2009) and Mukoyama and Şahin (2006) is that hours worked is endogenously determined by the household. A representative firm exists in the economy following the convention and all markets are perfectly competitive. Importantly, the source of business cycles is the shock to aggregate TFP, which enables our analysis to be comparable with the previous literature.

Main findings can be summarized as follows. First, while there exists welfare gain at the aggregate level even in the economy with heterogenous agents, the welfare is not evenly distributed across the agents; the welfare gain increases monotonically and convexly with wealth level. In particular, agents with more wealth (the rich) would prefer the economic fluctuations while those with less wealth (the poor) would hate them. This is particularly interesting when compared to the previous findings: With the heterogenous agent model with exogenous labor supply, Mukoyama and Şahin (2006) and Krusell, Mukoyama, Şahin, and Smith (2009) found that there exists an inverse-U relationship between wealth level and welfare gains. In their model, welfare gain is relatively high for the agents at the middle of the wealth distribution and is relatively low for the agents at the top and bottom of the wealth distribution. We argue, by decomposing the welfare gain into the mean effect and the fluctuations effect at the individual level, that this discrepancy arises from the fact that our model allows the agents to utilize the business cycles as they want with flexible labor supply: Agents with more assets have higher labor productivity and hence the mean effect, the channel through which each agent can increase consumption level under uncertainty (Cho, Cooley, and Kim (2015)) and hence enjoys higher utility, from the fluctuations is greater for them. Agents with less assets do not have high labor productivity as well as enough assets to enjoy higher interest rate in such an economy and hence would have lower mean effect. This channel shuts down (or is lower) in the model economy without endogenous labor.

We then show that the dimension of inequality matters when analyzing the extent to which rising inequality affects the distribution of the welfare cost. In particular, we first consider rising *income* inequality by introducing mean-preserving spread into idiosyncratic labor productivity and then introduce rising *wealth* inequality by allowing households to have different discount factors, following Krusell, Mukoyama, Şahin, and Smith (2009). While the main finding that there is a monotonic relationship between wealth level and the welfare cost are preserved in any cases, the way considering rising inequality has strikingly different welfare consequences: Greater income inequality dampens the degree of convexity while greater wealth inequality tends to enhance it. We argue that such a difference results from the fact that mean effect is reinforced in the presence of rising wealth inequality while becomes lower in the presence of rising income inequality.

Lastly, we further show that our main findings are preserved under various circumstances: (1) Introducing labor indivisibility, (2) alternative value for borrowing limit, (3) alternative Frisch labor supply elasticity, (4) different magnitude of TFP shocks, and (5) persistence of idiosyncratic labor productivity.

There are two main contributions of this paper to the literature. First, our work is the first paper to analyze the welfare consequences of the business cycles with (1) endogenous labor supply and (2) worker heterogeneity in a unified framework. Second, we unveil the extent to which rising inequality, the long-run phenomenon, affect the welfare cost of business cycles, the short-run welfare consequences, which has not been investigated before.

Remaining sections are organized as follows. Section 2 introduces the model and then Section 3 studies the role of labor market heterogeneity in determining the welfare cost of business cycles. Section 4 concludes.

2 The Model

This section introduces the model, which is quite standard in the sense that it is an extended version of Aiyagari (1994) model by endogenizing labor supply choice of a household.

2.1 Setup

The model economy consists of households with total measure of one and a representative firm. All markets are assumed to be perfectly competitive.

Household. There is a continuum of households distributed on the unit interval whose preferences are identical with each other. While households are ex-ante identical, they are ex-post heterogeneous because of idiosyncratic productivity x. The idiosyncratic productivity is assumed to follow a Markov process with transition probability distribution function $\pi_x(x'|x) = Pr(x_{t+1} \leq x'|x_t = x)$, where x_t is assumed to follow an AR (1) process: $\ln x_{t+1} = \rho_x \ln x_t + \varepsilon_t^x$. ε_t^x is assumed to be drawn from a normal distribution with mean zero and variance σ_x^2. Faced with idiosyncratic productivity shock x_t and market-determined equilibrium wage rate w_t, each household intensively chooses to work h_t hours which is divisible to get labor income $w_t x_t h_t$ in the given period.

In addition, there exists only one asset in the form of physical capital, a_t, since financial market is incomplete. We further introduce borrowing constraint: $a_t \geq \bar{a}$ with $\bar{a} \leq 0$. The asset a_t yields the net real interest rate, r_t. Then each household maximizes the following life-time utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \tag{2.1}$$

subject to $c_t + a_{t+1} = w_t x_t h_t + a_t(1+r_t)$ where $\beta \in (0,1)$ is the discount factor, $\sigma > 0$ is the relative risk aversion, $B > 0$ is disutility from labor, $\gamma > 0$ is the Frisch labor supply elasticity, and c_t is consumption at time t.

Representative firm. There exists a representative firm that produces final goods according to the Cobb-Douglas production function in the economy. The firm employs capital K_t and effective labor L_t as inputs to produce the final goods. The source of the economic fluctuations is aggregate productivity shock z_t that follows the AR (1) process. Capital is depreciated at the rate of $\delta \in (0,1)$ and the production function takes the following form: $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$.

Recursive representation. The value function V for the household with asset a and productivity level x is given as:

$$V(a, x; z, \mu) = \max_{c,a} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - B \frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta \mathbb{E}[V(a', x'; z', \mu') | x, z] \right\} \tag{2.2}$$
$$s.t \; c + a' = wxh + a(1+r)$$

where $\ln x' = \rho_x \ln x + \varepsilon^x$ with $\varepsilon^x \sim N(0, \sigma_x^2)$, $a' \geq \bar{a}$, and $\mu' = T(z, \mu)$. Here, $'$ denotes future value and T denotes a transition operator which defines the law of motion for the distribution of households $\mu(a, x)$ when total factor productivity is given as z. In our benchmark analysis, we do not allow borrowing ($\bar{a} = 0$) and will check later if prohibiting borrowing drives our main findings.

2.2 Equilibrium

A steady-state equilibrium[3] consists of value functions for individuals with different state spaces $V(a, x)$; a set of optimal decision rules for consumption, savings, and labor supply decisions for households $\{c(a, x), a'(a, x), h(a, x)\}$; aggregate effective labor and capital, L and K, wage

[3]Equilibrium with the fluctuations can be similarly defined.

rate w and interest rate r; and the invariant distribution of individuals $\mu(a,x)$, such that:

1. Households' utility is maximized: Given prices, the households' optimal decision rules $c(a,x), a'(a,x), h(a,x)$ solve the value functions for households.

2. Representative firm's profit is maximized: $w = (1-\alpha)(K/L)^{\alpha}$ and $r = \alpha(K/L)^{\alpha-1} - \delta$

3. Goods market clears: $\int \{a'(a,x) + c(a,x)\}\, d\mu = K^{\alpha}L^{1-\alpha} + (1-\delta)K$.

4. Factor markets clear: $L = \int x h(a,x) d\mu$, $K = \int a d\mu$.

5. Individual and aggregate behaviors are consistent: For all $A^0 \subset A$ and $X^0 \subset X$,
$\mu'(A^0, X^0) = \int\limits_{A^0, X^0} \left\{ \int\limits_{A,X} \mathbf{1}_{a'=a'(a,x)} d\pi_x(x'|x) d\mu \right\} da' dx'$.

2.3 Parameterization We calibrate parameters in line with the previous literature to make our results comparable with previous findings.

Time is quarter. Share of capital income, α, is 0.36, and depreciation rate of capital, δ, is set to be 2.5% (10% annually). Relative risk aversion parameter, σ, is set to be 1 to satisfy the balanced growth path (King, Plosser, and Rebelo (1988)) and Frisch labor supply elasticity, γ, is set to be 1, which is so-called macro elasticity of labor (Chang and Kim (2006)). Since the estimates for the elasticity varies from literature to literature[4], we will check the robustness of our findings with alternative values for the Frisch labor supply elasticity.

For parameters specific to individuals, we also followed the literature that estimates idiosyncratic productivity shock process.[5] In particular, we set baseline parameters for idiosyncratic productivity process, ρ_x and σ_x, as 0.975 and 0.165 respectively. For standard deviation of shocks to productivity, σ_x, we would set different values for the parameter since it could largely determine wage inequality in the model.

For total factor productivity, $\ln z' = \rho_z \ln \lambda + \varepsilon^z$ with $\varepsilon^z \sim N(0, {\sigma_z}^2)$, we assume $\rho_z = 0.95$ and $\sigma_z = 0.007$ following Kydland and Prescott (1982). Disutility of labor, B, and discount factor, β, are calibrated to target average hours worked to be 1/3 and real interest rate to be 1%, which are quite general target statistics in the literature (e.g. Chang and Kim (2006)). Table 2.1 summarizes the parameter values for the benchmark calibration.

[4] For instance, Chetty, Guren, Manoli, and Weber (2012) suggested the value to be between 0.4 and 0.5 while Rogerson and Wallenius (2013) and Rogerson and Wallenius (2016) argue that the values could be 1 or larger.

[5] See Flodén (2001), Chang and Kim (2006), Chang, Kim, Kwon, and Rogerson (2019), and Heathcote, Storesletten, and Violante (2008) as examples.

Table 2.1: Calibration

Parameter	Value	Description
α	0.36	Capital share
σ	1.0	Relative risk aversion
β	0.9813	Discount factor
δ	0.025	Depreciation rate
γ	1.0	Frisch labor elasticity
B	7.005	Disutility of labor
ρ_x	0.975	Persistence of log idiosyncratic productivity
σ_x	0.165	Standard deviation of idiosyncratic shocks
ρ_z	0.95	Persistence of log total factor productivity
σ_z	0.007	Standard deviation of total factor productivity shocks

2.4 Computation of Welfare cost

This section describes how we calculate the welfare costs of business cycles.

We first define the value function for living in the steady-state economy for an individual with asset a and productivity x in period 0 as follows:

$$V^{SS}(a_0, x_0, K_{ss}, 1) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t(a_t, x_t, K_{ss}, 1), h_t(a_t, x_t, K_{ss}, 1)\right) \tag{2.3}$$

where $c_t, h_t, K_{ss}, 1$, and u denote consumption, hours worked, steady-state aggregate capital, steady-state TFP and utility function, respectively. Note that consumption and hours worked are not fixed even in the steady-state since each household faces uninsurable idiosyncratic productivity shock.

Then the value of living in a non-fluctuating economy adjusted with compensating variation λ is defined as follows:

$$V^{SS,\lambda}(a_0, x_0, K_{ss}, 1) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left((1+\lambda)c_t(a_t, x_t, K_{ss}, 1), h_t(a_t, x_t, K_{ss}, 1)\right) \tag{2.4}$$

We can also define a value function for individuals with the same state spaces, a, x, K_{ss}, and TFP of 1, in period 0 as follows:

$$V^{F}(a_0, x_0, K_{ss}, 1) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u_t\left(c_t(a_t, x_t, K_t, z_t), h_t(a_t, x_t, K_t, z_t)\right) | K_0 = K_{ss}, z_0 = 1\right] \tag{2.5}$$

Hence, λ is the compensating variation that measures the percentage of consumption that has to be changed for the consumer living in the steady-state to be indifferent to living in the fluctuating economy. In particular, λ is the solution to the following equation:

$$V^{SS,\lambda} = V^{F} \tag{2.6}$$

When relative risk aversion parameter is 1, $\lambda = \exp((V^F - V^{SS})(1-\beta)) - 1$ and hence economic fluctuations are welfare-improving (resp. welfare-detrimental) if $\lambda > 0$ (resp. $\lambda < 0$). For the welfare costs for groups of individuals, the value functions are replaced by the average of those for individuals under the spirit of utilitarianism.

3 Welfare Cost of Business Cycles: the Role of Labor Market Heterogeneity

According to Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014), business cycles are welfare-improving in the representative RBC model in which production factors are endogenously chosen. In this section, we verify whether such an observation held at the aggregate level applies to the individual level.

3.1 Who Prefers Business Cycles? We first note that at the aggregate level, business cycles are still welfare-improving in the benchmark economy. On average, welfare gain from the fluctuations is about 0.003%, which is in line with Cho, Cooley, and Kim (2015) (figure 3 of their paper). Are business cycles then beneficial to all agents? Figure 3.1 provides an answer to this question by showing the welfare costs for agents with different levels of asset holdings; welfare costs for each group are obtained by solving the equation (2.6).[6]

Figure 3.1 clearly shows that benefits from the fluctuations are not equally-distributed: Rich agents (who hold more assets) enjoy welfare gains while poor agents suffer from the business cycles. Most interestingly, welfare gain from the business cycles strictly increases with agents' wealth level in the benchmark economy: In contrast to the finding by the previous literature that there is a gain at the

[6] In this paper, nth percentile means that individuals belong to the $n-1\%$ to $n\%$. For instance, 10th asset percentile means the individuals with bottom 9% to 10% in terms of asset level.

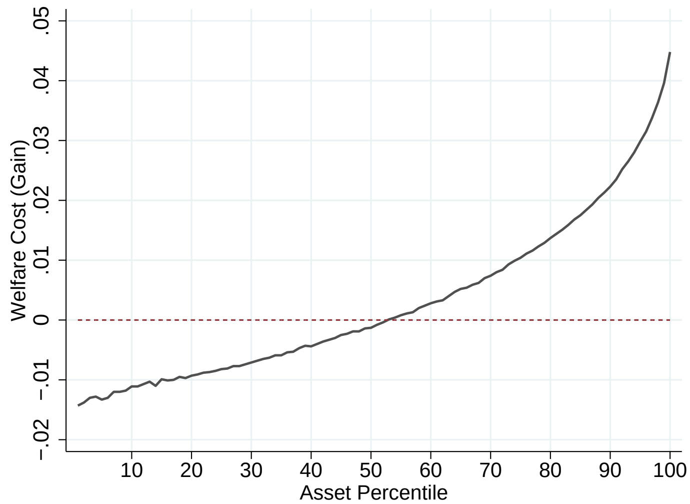

Figure 3.1: Welfare Cost in the Benchmark Economy

Note: Horizontal axis represents asset percentile for individuals. For instance, if the value is 10, it represents the average welfare costs for individuals with asset percentiles from bottom 9% to 10%. Note that this holds for other figures in the paper. Solid line represents welfare costs corresponding to asset levels.

aggregate/average level by introducing uncertainty into the otherwise tranquil economy, welfare in the fluctuating economy is lower than that in the steady-state economy for more than half of the agents, bottom 52.2%.

Then why is the average welfare gain positive while more than half of agents suffer from the fluctuations? This is because the welfare gain is not just increasing in wealth percentile but is convex. For instance, while the welfare gain for the top 10% of the wealth distribution is about 0.02%, that for the top 1% is about 0.044% and that for the top 0.1% is greater than 0.05%. Due to the convex relationship between welfare cost and asset percentile, the welfare gain at the aggregate level is 0.0028% although more than half experience welfare loss.

To further investigate the extent to which income inequality and wealth inequality might have heterogenous effects on the welfare gain, we plot Figure 3.2. This figure plots the welfare costs according to asset quintile when income quintile is fixed. Each line indicates the same income level and it is easy to observe that the welfare gain strictly increases as asset quintile increases, a consistent finding with Figure 3.1. In addition, convexity of welfare cost function with respect to asset quintile is preserved. This is quite an interesting result when compared to Mukoyama and Şahin (2006) and Krusell, Mukoyama, Şahin, and Smith (2009) that also studied the welfare cost of business cycles with heterogenous agent model; they show that the shape of the welfare cost function takes an inverse-U pattern[7]: The welfare cost for agents at the middle of the wealth distribution are low when compared to that for the poor and the rich. We will discuss more about this discrepancy below. Figure 3.3 shows the welfare costs of individuals with different levels of income as well as asset levels in 3-dimensional space.

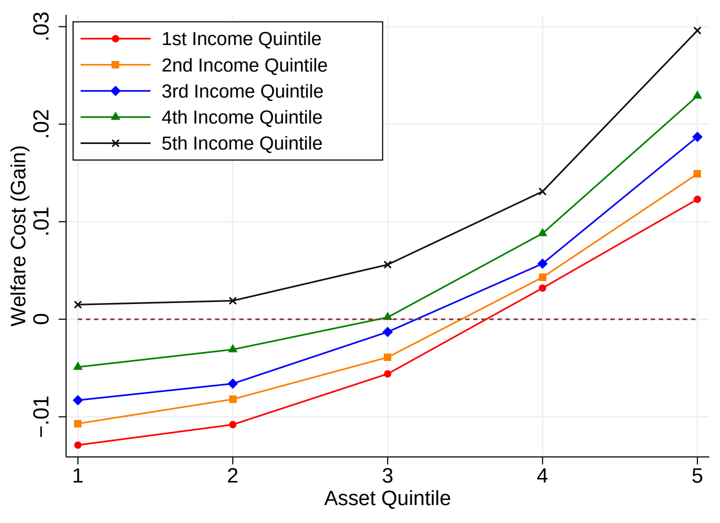

Figure 3.2: Welfare Costs by Income and Wealth Quintile

3.2 Mechanism: Role of Endogenous Labor In the previous section, we showed that welfare costs of business cycles are unevenly distributed across agents: Agents with more wealth prefer business cycles while agents with low wealth hates them. What is the mechanism behind this result? In order to answer this question, we closely follow Cho, Cooley, and Kim (2015) by decomposing the welfare gain into the mean effect and fluctuations effect.[8] Mean effect refers to benefits from the fluctuations with higher expected consumption level in the presence of business cycles when consumers can take advantage of the cycles by "working harder and investing more" when TFP is high. As is shown in Cho, Cooley, and Kim (2015), this effect becomes greater when production factors are endogenously determined. On the contrary, fluctuations effect denotes the welfare loss from the fluctuations due to concavity of utility function that makes consumers to prefer smooth consumption. When the former effect dominates the latter, business cycles can be welfare-improving in contrast to the usual belief that

[7]Since those studies calculated the welfare gain from eliminating the fluctuations, the function is U-shaped in their context.

[8]For the decomposition exercise, they adopted the approach by Flodén (2001) and Heathcote, Storesletten, and Violante (2008).

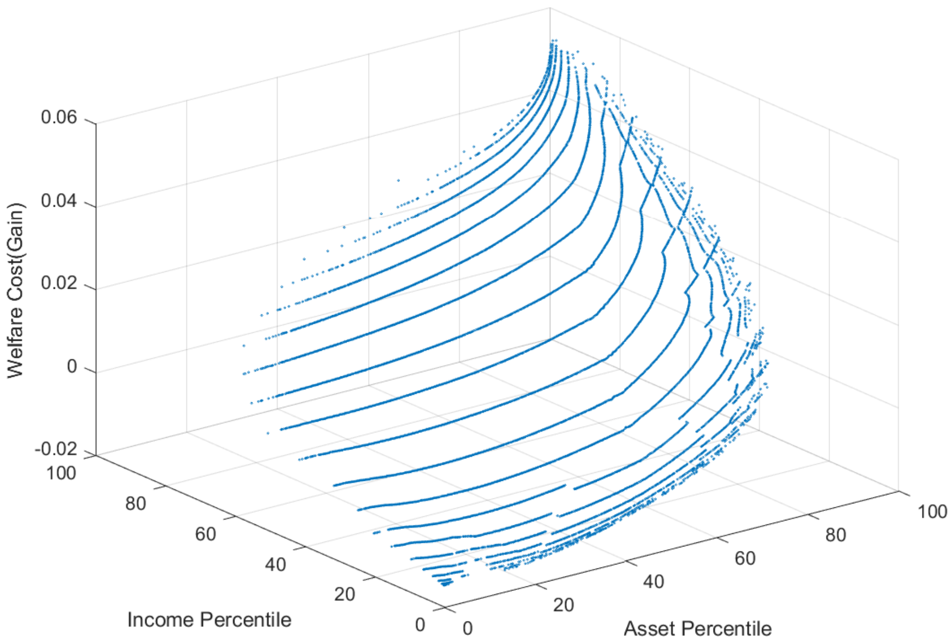

Figure 3.3: Welfare Costs across Income and Wealth Distribution

Note: Each point of the graph indicates a result for an individual in simulation consisting of 50,000 consumers.

business cycles are welfare-detrimental.

We first note that in contrast to the representative model analyzed by Cho, Cooley, and Kim (2015), decomposition of the welfare cost is not a trivial task as wealth distribution is non-degenerate. Hence, we first describe in detail how we compute the mean effect, which is also the contribution of this paper in the dimension of computation, and then discuss the mechanism behind our finding.

Decomposition of individual welfare cost. In order to compute the individual welfare cost, we adopted the approach by Heathcote, Storesletten, and Violante (2008) that studied welfare effects of labor market uncertainty at the aggregate level. The main difference between our exercise and Cho, Cooley, and Kim (2015) is that we use individual conditional expectations for future consumption and hours worked that vary over time and state spaces while they used unconditional expectations for consumption and hours that are fixed.

We first define the cost of individual uncertainty, p_{ss}, in the economy without business cycles using conditional expectations as follows[9]:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t(a_t,x_t), h_t(a_t,x_t)\right) = \sum_{t=0}^{\infty} \beta^t u\left((1-p_{ss})\mathbb{E}_0[c_t(a_t,x_t)|a_0,x_0], \mathbb{E}_0[h_t(a_t,x_t)|a_0,x_0]\right) \tag{3.1}$$

Value of p_{ss} is equal to 0 in the representative model since there is no individual uncertainty that each agent faces. Hence, p_{ss} means the amount that agents are willing to forgive to make the welfare in the economy with fluctuations be equivalent to the welfare in an economy where conditional consumption and hours worked are guaranteed. Since utility function is concave, p_{ss} is always positive.

Similarly, the cost of aggregate uncertainty[10], under economy with fluctuations can be defined as follows:

$$\begin{aligned}&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t(a_t,x_t,K_t,z_t|K_{ss},1), h_t(a_t,x_t,K_t,z_t|K_{ss},1)\right)\\ &= \sum_{t=0}^{\infty} \beta^t u\left((1-p_f)\mathbb{E}_0[c_t(a_t,x_t,K_t,z_t)|a_0,x_0,K_0=K_{ss},z_0=1], \mathbb{E}_0[h_t(a_t,x_t,K_t,z_t)|a_0,x_0,K_0=K_{ss},z_0=1]\right)\end{aligned} \tag{3.2}$$

Then, following Heathcote, Storesletten, and Violante (2008), we can define the fluctuations effect, λ_f, as follows:

$$(1+\lambda_f)(1-p_{ss}) = 1-p_f \tag{3.3}$$

The fluctuations effect, λ_f, is approximately similar to $p_{ss}-p_f$, which measures the increased cost of uncertainty under fluctuations. Since the economy with fluctuations are more volatile, p_f is larger than p_{ss} ($\lambda_f < 0$).

The mean effect[11], λ_m, is defined as follows, in a consistent manner with Cho, Cooley, and Kim

[9] The conditions for steady-state economy, that aggregate capital is equal to K_{ss}, and TFP is 1, are omitted for simplicity.
[10] This value is always negative since in the economy with business cycle, p_f, the dispersion of the variables are more severe.
[11] This value is always positive because agents faced with multiplicative shocks endogenously change labor supply and savings in favor of fluctuations.

(2015):

$$\sum_{t=0}^{\infty} \beta^t u\left((1+\lambda_m)\mathbb{E}_0[c_t(a_t, x_t)|a_0, x_0], \mathbb{E}_0[h_t(a_t, x_t)|a_0, x_0]\right)$$
$$= \sum_{t=0}^{\infty} \beta^t u\left(\mathbb{E}_0[c_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1], \mathbb{E}_0[h_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1]\right) \tag{3.4}$$

λ_m measures the proportion of consumption that makes welfare in the economy with conditional means for steady-state is equivalent to the economy with conditional means under fluctuations. Here, the mean effect is always positive.

By the proposition 1 of Flodén (2001), $\lambda_m + \lambda_f \simeq \lambda$ because the utility function in the benchmark model satisfies $u(\omega c, h) = f(\omega)u(c, h) + g(\omega)$ for any ω.[12] Since conditional expectations given state spaces for each individual are hard to compute analytically, we used the simulation approach described in the appendix to compute the conditional expectations.

Individual-level mean effect vs. fluctuations effect. Based on the computational procedure introduced above and in the appendix, we compute the individual-level mean effect and fluctuations effect for the benchmark economy and plot them in Figure 3.4. The figure clearly shows that the mean effect tends to increase monotonically with asset level.[13] In contrast, fluctuations effect do not vary much for most of the asset percentile. Given that the mean effect reflects the relationship between wealth level and the welfare cost and the fluctuations effect is computed as residual, we will focus on the mean effect in the subsequent analysis.

The mean effect arises as there is a convex relationship between shocks to TFP and hours worked: Note that hours worked increases when TFP increases and hence consumption can increase more than TFP increases, which enables consumers to enjoy higher consumption level than that can be obtained in the steady-state. While the opposite holds during the recession, the former dominates the latter, and hence average consumption level achieved in the volatile economy yields positive mean effect as is observed in Figure 3.4. In order to examine why mean effect is increasing in the wealth level, we compute the changes in hours worked from the steady-state counterpart when shock to TFP is either highest ($\lambda_t \fallingdotseq 1.07$, 7% higher than the steady-state value) or lowest ($\lambda_t \fallingdotseq 0.93$, 7% lower than the

[12]One can easily show that the sum of two effects are approximately equal to welfare cost when log utility is assumed.

[13]Wiggles are a quite common feature in the results calculated by the numerical method. In this case, discretization of the continuous variables like productivity level and some random draw of simulation could have resulted in wiggles.

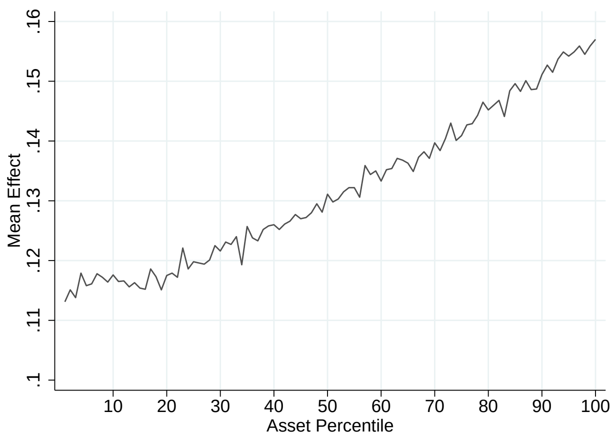

Figure 3.4a: Mean Effect

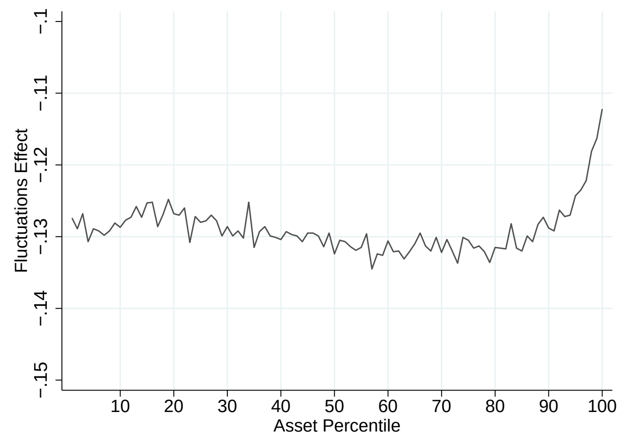

Figure 3.4b: Fluctuations Effect

Figure 3.4: Individual Level Mean Effect vs. Fluctuations Effect

steady-state value) and plot them in Figure 3.5. Solid blue (resp. dotted red) line indicates response of labor to a positive (resp. negative) shock to TFP for each asset percentile.

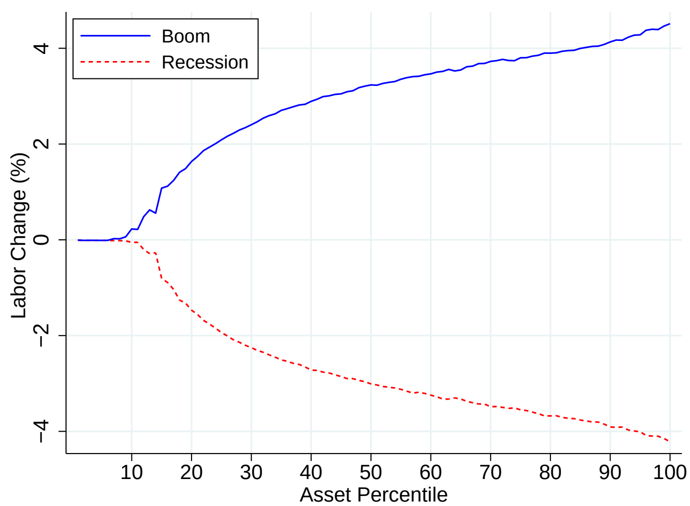

Figure 3.5: Response of Hours Worked to Shock to TFP

Note: Horizontal axes represent asset percentile for individuals. Solid (resp. dotted) line shows response of hours worked to positive (resp. negative) TFP shock.

We can first observe that most of the variations in hours worked come from relatively more rich agents. Note that more labor supply together with greater wage rate generates higher labor income[14] compared to the steady-state when TFP increases, implying greater mean effect for agents with more assets. Given that uncertain environment raises interest rate paid to capital, greater saving of these type

[14] This is because idiosyncratic productivity and asset percentiles are highly correlated in the model. For instance, average productivity of 10th percentile is 0.5344, 1.1769 for 50th percentile, and 2.2544 for 90th percentile in the model.

would also make them happier in the economy with the economic fluctuations. Overall, the wealthy in our benchmark economy would benefit from the business cycles through two channels (labor and capital channel).

On the contrary, agents with less assets do not change their labor supplies much: In particular, labor supply of agents up to 10th wealth percentile hardly changes in any circumstance. This generates lower mean effect and hence is dominated by the fluctuations effect (Figure 3.4), implying business cycles are welfare-detrimental to consumers with relatively low wealth. Since these are the workers whose savings are not sufficiently accumulated, higher interest rate in the volatile economy is not helpful for them.

Role of endogenous labor. While there is a monotonic relationship between the welfare cost of business cycles and wealth distribution in our benchmark economy, Mukoyama and Şahin (2006) and Krusell, Mukoyama, Şahin, and Smith (2009) showed that the relationship is non-monotone in the Ayagari type model in which labor is exogenously determined. In order to verify the role of endogenous labor, we prohibit workers to flexibly adjust their labor supply; hours worked are fixed at average levels of hours worked, 1/3, with same parameters as in the benchmark model for comparison. After solving the alternative model with exogenous labor supply, we compute the welfare costs accordingly.

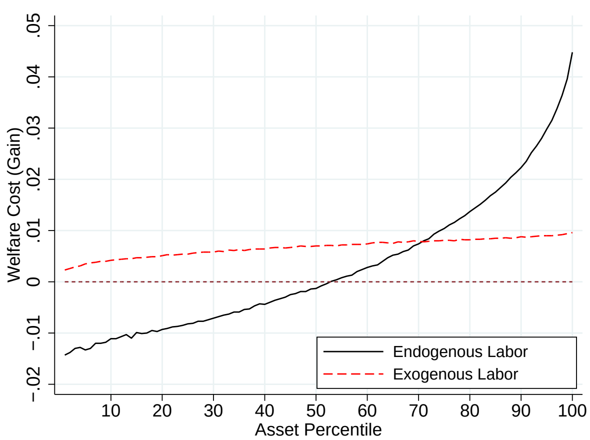

Figure 3.6a: Costs by wealth level

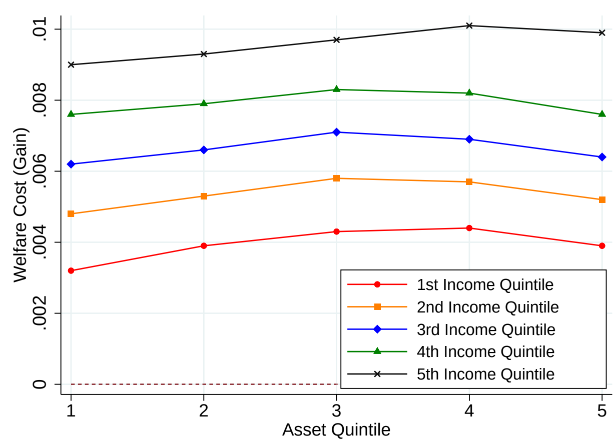

Figure 3.6b: Costs by wealth and income quintiles

Figure 3.6: Welfare Cost with Exogenous Labor

Left panel of Figure 3.6 (Figure 3.6a) compares the welfare costs by wealth levels between our benchmark model (solid black line) and the model with exogenous labor (dotted red line). The most notable observation is that welfare gain is increasing in wealth level in both cases but the slope is much lower in the model with exogenous labor. Given that the only difference between the benchmark economy and our model economy is the endogeneity of labor supply, it seems to be a correct guess that the convex relationship we find in Figure 3.2 is driven by the mean effect due to endogenous labor supply.[15]

Furthermore, different from what we observed in Figure 3.3, Figure 3.6b confirms that there exists an inverse-U shaped welfare cost, which is in line with previous studies that assumed endogenous labor. Although the degree of the inverse-U shape is relatively small, we can observe that the welfare gain for the middle class (3th and 4th asset quintiles) is the highest when income level is fixed. This figure implies that the inverse-U shape of the previous literature could have derived from the assumption of exogenous labor in some degree.

3.3 Effect of Rising Inequality on the Welfare Cost

The property of our model that it can generate inequality between consumers enables us to further explore if the rise of inequality during the several decades have any impacts on the distribution of welfare cost. In doing so, we first examine the welfare consequences of rise of *income* inequality on the welfare cost by varying the dispersion of the shock to idiosyncratic productivity. We then address if the rise of *wealth* inequality has any impact on the welfare cost by introducing discount factor heterogeneity á-la Krusell and Smith (1999).

Rise of income inequality. In order to generate more or less equal income distribution, we noticed the role of the standard deviation of the individual productivity: Recall that $\ln x' = \rho_x \ln x + \varepsilon_x$ with $\varepsilon_x \sim N(0, \sigma_x^2)$ governs individual worker's labor productivity. Hence, σ_x governs the extent to which labor productivity is dispersed across the workers, resulting in more or less equal income distribution.[16] In particular, increases in the standard deviation of the idiosyncratic shock increases income inequality as it yields more dispersed realization of individual productivity. For the quantitative exercise, we consider five values for the standard deviations; $\sigma_x = 0.0825, 0.12375, 0.165$ (Benchmark), $0.20625, 0.2475$. These

[15] The finding that the welfare gain in the economy with exogenous labor supply is positive is not that counterintuitive as it seems. At the aggregate level, this finding is in line with Cho, Cooley, and Kim (2015): Figure 4 of Cho, Cooley, and Kim (2015) shows that business cycles are welfare-improving regardless of the value of Frisch labor supply elasticity. At the micro level, this finding can be explained by Figure 3.5: Notice first that the gain from the economic fluctuations becomes greater (resp. lower) for poor (resp. rich) agents as we move from the benchmark economy with endogenous labor supply to the alternative economy with exogenous labor supply. This comes from the general equilibrium effect (Krusell, Mukoyama, Şahin, and Smith (2009) and Mukoyama and Şahin (2006)); aggregate capital is accumulated more in the fluctuating economy because of precautionary savings motive. This raises wage level as marginal product of labor increases in capital level. This is good news for poor agents since their labor income becomes higher in the fluctuating economy so that mean level of consumption arises. This is possible because labor supply is not flexibly adjusted for these agents even in the economy with flexible labor supply (Figure 3.5). On the contrary, higher wage does not benefit rich agents as they cannot adjust labor supply as they desire; they enjoy business cycles by changing labor supply as they want (Figure 3.5) but it is not available any more. And hence welfare gain from the fluctuations becomes lower for rich agents.

[16] In the quantitative exercise, other parameters are kept same to match the same target as in the baseline calibration.

values generate income GINI of $0.279, 0.387, 0.482, 0.567, 0.638$, respectively.[17] Then we could do the same exercise as in the benchmark case to examine the consequences of changing income inequality on welfare costs.

Figure 3.7 displays the welfare costs by wealth level under economies with different degree of income inequality. We first note that the monotonic relationship between wealth level and individual welfare cost is preserved. More interestingly, the figure indicates the existence of a tendency that (1) the welfare cost becomes more dispersed along the wealth distribution and (2) the convexity becomes more pronounced as income equality becomes smaller. In particular, compared to the benchmark case with $\sigma_x = 0.165$, welfare cost becomes more flat for a more unequal society ($\sigma_x = 0.2475$) while it becomes more steep for a more equal economy ($\sigma_x = 0.0825$).

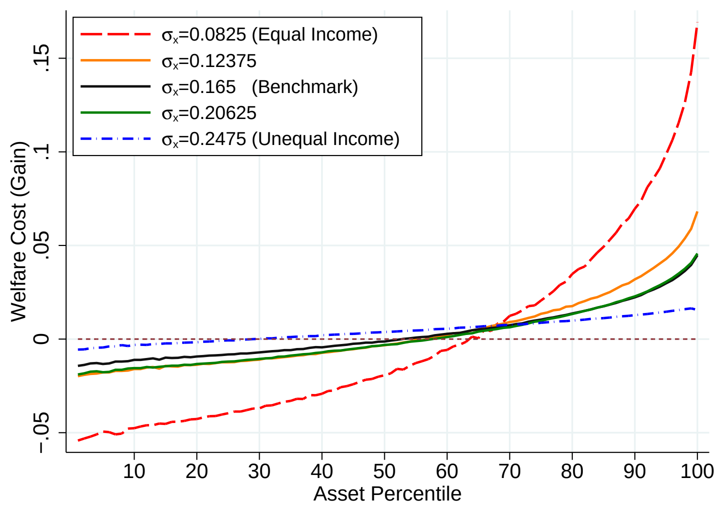

Figure 3.7: Welfare Cost with Different Income Inequality

Figure 3.8 further shows the distribution of welfare costs by both income and asset levels with the same scale for welfare costs.[18] It confirms that the welfare costs are more dispersed in the economy with more equal income distribution (Figure 3.8a) evidently: Agents who benefit the most could increase their welfare by more than 0.2% while the poor could lose about 0.1% of welfare compared to the steady-state. In the economy with less equal income distribution, the welfare costs varies from -0.01% to 0.02% according to wealth and income levels.

[17]For wealth GINI, these values generate $0.632, 0.667, 0.693, 0.715, 0.716$, respectively.

[18]Note that although the right panel of Figure 3.8 looks relatively flat, it displays some degree of convexity when z-axis is rescaled to match the range of the costs.

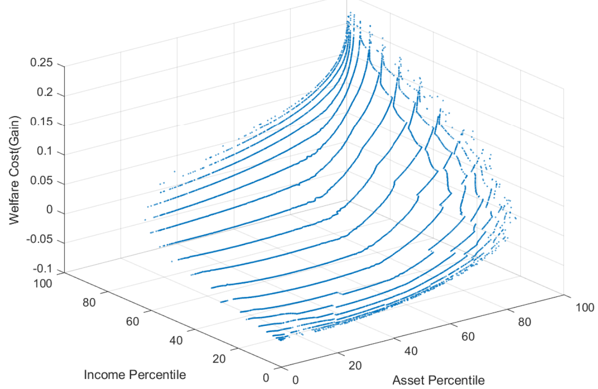

Figure 3.8a: More Equal Income ($\sigma_x = 0.0825$)

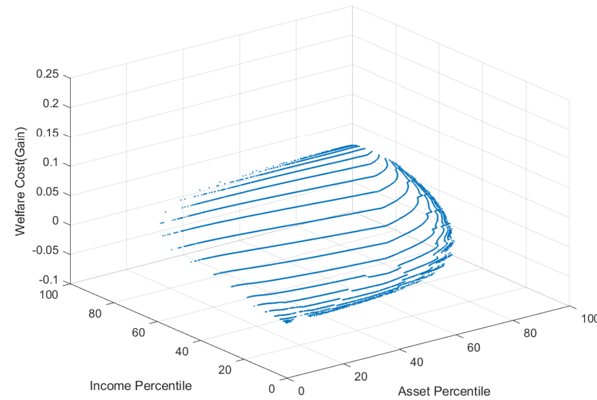

Figure 3.8b: Less Equal Income ($\sigma_x = 0.2475$)

Figure 3.8: Role of Income Inequality

This observation that greater (resp. lower) income inequality results in more (resp. less) equal distribution of the welfare cost of business cycles across agents seems puzzling at the first glance. In order to understand this phenomenon, we report several important statistics generated from the model in Table 3.1. The first and second rows present changes in aggregate capital and wage rate in the economy with the fluctuations from that without the business cycles, respectively.

Table 3.1: Change in statistics by different std of idiosyncratic shock

	$\sigma_x = 0.0825$	$\sigma_x = 0.12375$	$\sigma_x = 0.165$	$\sigma_x = 0.20625$	$\sigma_x = 0.2475$
Increase in aggregate capital (%)	0.518	0.183	0.164	0.228	0.036
Increase in wage rate(%)	0.179	0.052	0.038	0.053	0.0005
Social welfare cost (%)	-0.0016	0.0031	0.0028	0.0007	0.0043

We first notice that the key to generate the mean effect is that aggregate production (and hence consumption) should be large enough in the fluctuating economy compared to that in the steady-state economy. Consider the economy with more equal income distribution ($\sigma_x = 0.0825$). In this economy, dispersion in labor income is not large so that aggregate capital in the steady-state equilibrium would be small as the ability of consumers with high income to accumulate capital is restricted. As a result, adding aggregate shocks to this economy will have a greater impact on the capital and hence on wage rates due to the complementarity in capital and labor. This raises the mean effect, and thus the inequality in terms of the welfare cost of business cycles would become severe. In the economy with less equal income distribution, the opposite would hold: Greater income inequality enables high-income earners to accumulate more capital at the steady-state and hence the difference between capital in the steady-state economy and that in the volatile economy becomes smaller (0.036% in the most extreme case) and hence the mean effect becomes the smallest.

Rise of wealth inequality. We now address the extent to which greater *wealth* inequality impacts the distribution of the welfare cost across agents. In order to generate more or less unequal wealth distribution without affecting the income inequality, we adopted the strategy taken by Krusell, Mukoyama, Şahin, and Smith (2009) and Mukoyama and Şahin (2006); preference heterogeneity in the form of heterogenous discount factor, β, is introduced. Compared to other ways to obtain greater inequality at the top (Quadrini (1999) or Kaymak and Poschke (2016)), allowing preference heterogeneity has two main advantages. It is relatively simple to implement in the computational procedure and enables us to keep the same structure of the model.

In particular, consumers are further divided by patience level: A consumer with higher β is more patient than a consumer with lower β. We particularly assume that β follows a three-state Markov process.

The welfare of each individual consumer in the modified model is now defined as follows:

$$V \equiv \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\prod_{j=0}^{t} \beta_j) \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \right] \tag{3.5}$$

subject to the budget constraint.

Then the recursive representation is similar to the benchmark model:

$$V(a, x, \beta; z, \mu) = \max_{c,h,a'} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - B \frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta E[V(a', x', \beta'; z', \mu')|x, z] \right\} \tag{3.6}$$

subject to the budget constraint.

The three-state Markov transition process is assumed as in Mukoyama and Şahin (2006).[19] Since discount factors are not the same over time, welfare costs can be computed by the following formula with simple calculations: $\lambda = \exp((V^F - V^{SS})/D)$, where $D = \mathbb{E}_0[\sum_{t=0}^{\infty} (\prod_{j=0}^{t} \beta_j)]$.[20]

We calibrate parameters by targeting wealth GINI; 0.71 (similar to the benchmark model), 0.80

[19] Detailed explanation on the transition probability matrix and assumptions on the process is described in the appendix of Mukoyama and Şahin (2006).

[20] The way to compute of these values are described in the appendix of Krusell, Mukoyama, Şahin, and Smith (2009).

(similar to the data), and 0.84 (higher than the data).[21] We take the same steps to compute the welfare gains from the business cycles under each scenario and plot them in Figure 3.9.

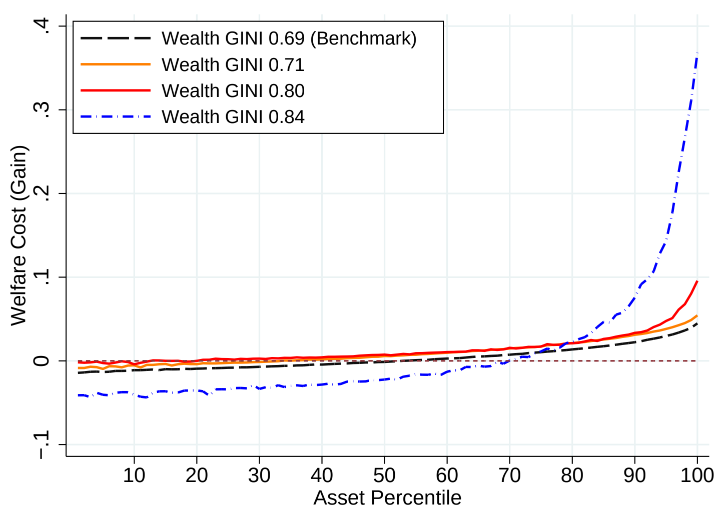

Figure 3.9: Effect of Rising Wealth Inequality on the Welfare Cost

Note: Horizontal axes represent asset percentile for individuals. For instance, if the value is 10, it represents the average welfare costs for individuals with asset percentiles from bottom 9% to 10%.

The first important observation is that the finding that the welfare gain monotonically increases with wealth level still holds in the model with more unequal wealth distribution. Importantly, the convexity becomes more evident when the wealth inequality rises; the convexity is the most (resp. the least) evident in the economy that generates wealth GINI 0.84 (resp. 0.69). For example, in the most unequal economy (blue dotted line), the top 1% could increase their welfare in the fluctuating economy by more than 0.35% while 70% of the agents suffers from the fluctuations and the welfare loss for the least poor agent is about -0.05%. On the contrary, in the most equal economy (benchmark economy; black dotted line), the top 1% could increase their welfare in the fluctuating economy by about 0.04% while about 50% of the agents suffers from the fluctuations and the welfare loss for the least poor agent is about -0.01%.

Interestingly, the effect of rising wealth inequality on the distribution of the welfare cost of business cycles across agents is the opposite to that of rising income equality: As is reported in Figure 3.7, greater income inequality generated by greater dispersion in idiosyncratic productivity is associated with a less convex welfare cost function with respect to the asset distribution. In contrast, greater

[21] However, this wealth GINI could be similar or lower than the data nowadays (e.g. Wolff (2017)).

wealth inequality generated by discount factor heterogeneity is associated with a more convex welfare cost function. This is because the change assumed in the economy with greater wealth inequality does not change the way mean effect affects the welfare cost. Notice that changes in the dispersion of labor income was a key to understand Figure 3.7; this is not the case anymore as the dispersion of labor income per se is preserved. Instead, preference heterogeneity only affects the amount of capital that each household can accumulate, which implies that the mean effect due to saving becomes exaggerated and hence the convexity of the welfare cost with respect to the wealth level becomes more evident.

Hence, the above quantitative exercises provide a lesson that it is crucial to identify the source of rising (or non-rising) inequality: Whether the changes in inequality structurally affect the mean effect or not will give different results on the way such changes affect the distribution of the welfare cost of business cycles.

3.4 Robustness Checks In this subsection, we conduct a battery of robustness analysis to check if our previous findings, both monotonic and convex relationship between the welfare cost and wealth percentile, are preserved.[22] In particular, we analyze if our results are robust to (1) indivisibility of labor, (2) alternative value for borrowing limit, (3) alternative Frisch labor supply elasticity, (4) different magnitude of TFP shocks, and (5) persistence of idiosyncratic productivity.

Indivisible labor. We first address if divisibility assumption of hours worked in our benchmark model drives our findings; as is argued by Chang and Kim (2007), considering extensive margin of labor might be more appropriate in explaining labor market dynamics. In doing so, we use the Aiyagari model with indivisible labor suggested by Chang and Kim (2007): The model shares the benchmark model specifications in most dimensions. The only difference comes from the assumption that workers are allowed to choose employment status; whether to work ($h_t = \bar{h} > 0$) or not ($h_t = 0$) to maximize their lifetime utilities. Now the value function for agents when working (V^E) is defined as follows:

$$V^E(a,x;z,\mu) = \max_{c,a'} \left\{ \frac{c^{1-\sigma}-1}{1-\sigma} - B\frac{\overline{h}^{1+1/\gamma}}{1+1/\gamma} + \beta\mathbb{E}\left[\max\left\{V^E(a',x';z',\mu'), V^N(a',x';z',\mu')|x,z\right\}\right]\right\} \tag{3.7}$$

subject to $c + a' = wx\bar{h} + a(1+r)$.

[22]Targets and other parameters are the same to the benchmark model if not specified. In addition, the benchmark model would be represented by black solid line in each figures reported in this section.

The value function for unemployed is defined similarly:

$$V^N(a, x; z, \mu) = \max_{c,a'} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} + \beta \mathbb{E}[\max\{V^E(a', x'; z', \mu'), V^N(a', x'; z', \mu')|x, z\}] \right\} \tag{3.8}$$

subject to $c + a' = a(1 + r)$.

Then, the value function for each consumer could be defined as:

$$V(a, x; z, \mu) = \max_{h \in \{0, \bar{h}\}} \left\{ V^E(a, x; z, \mu), V^N(a, x; z, \mu) \right\}. \tag{3.9}$$

The other parts of the models including idiosyncratic risks, parameters as well as calibration targets are the same to those in the benchmark model. The only additional calibration target is the employment rate (70%).[23]

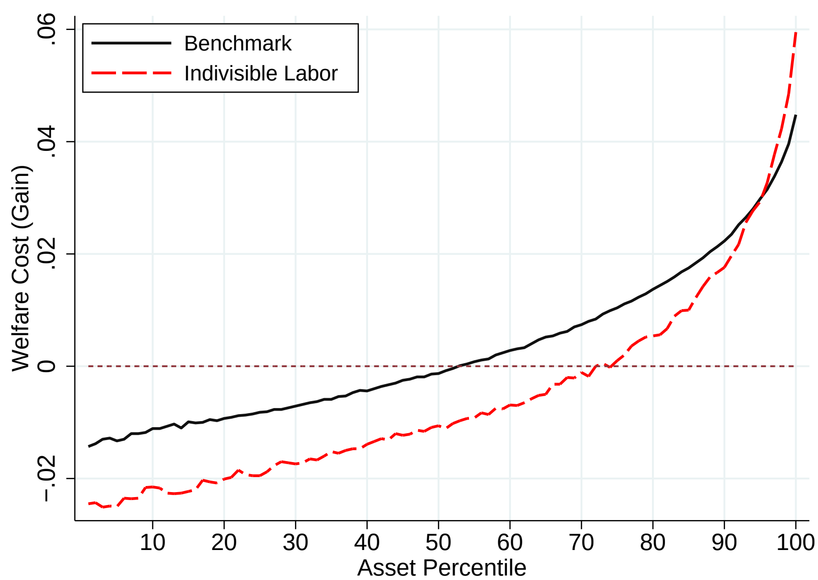

Figure 3.10: Welfare Costs with Indivisible Labor

Figure 3.10 plots the welfare costs for the benchmark model (solid black line) and those for the alternative model (dotted red line, model with indivisible labor). The figure clearly shows that both

[23] $\bar{h}$ is assumed to be 1/3 following Chang and Kim (2007).

monotonicity and convexity of the welfare cost function are preserved in the alternative framework, confirming the robustness of our findings. Note that the welfare gain from the fluctuations becomes smaller in most of region except the agents with sufficiently accumulated assets, an intuitive result as labor adjustment is not easy when compared to the model without indivisibility.

Alternative borrowing limit. One might raises an issue on our benchmark result as we did not allow borrowing ($\bar{a} = 0$). Given that the wealth distribution is affected by the choice of value for borrowing limit, it is natural to further study if allowing borrowing would alter our findings. In particular, we consider borrowing up to -2, which is about 150% of the average income in the benchmark model.[24] Since model features are exactly equivalent to the benchmark model, introduction of model is omitted. In Figure 3.11, we present the welfare costs with zero borrowing limit (solid black line), those with alternative borrowing limit (dotted red line), those with greater wealth inequality (dotted black line), and those with both alternative borrowing limit and greater wealth inequality (dotted blue line).

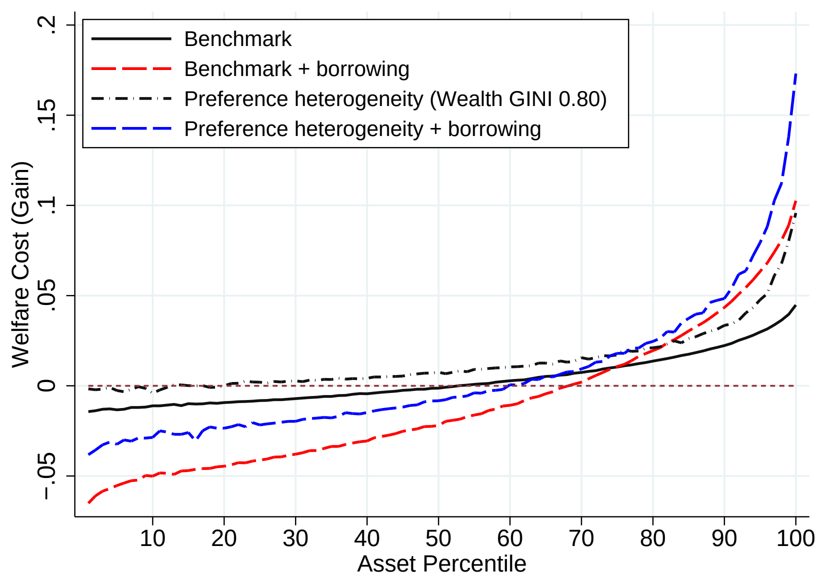

Figure 3.11: Welfare Cost with Alternative Borrowing Limit

Several observations are noteworthy. First, the property of the welfare cost function (monotonicity and convexity) is well preserved when borrowing limit changes. In particular, it reinforces the convexity; given that borrowing means that consumers with low labor productivity would hold less (negative) asset, the negative effect from the economic fluctuations would be greater for them. Also, the wealth

[24]This is also the borrowing limit used by An, Chang, and Kim (2009).

distribution would become more unequal, making the wealthier consumers to exploit the business cycles more, which is in line with the findings in Section 3.3. Second, this property would still hold in the economy with more unequal wealth distribution. As a result, when we jointly consider rising wealth inequality and less tight borrowing constraint (dotted blue line), the convexity would become more evident than other cases.

Alternative Frisch labor supply elasticity. We then test if different values for Frisch labor supply elasticity might affect our findings on the welfare costs. In doing so, we vary the elasticity from 0.25 to 2.[25] Again, other parameters are in line with the benchmark analysis. Figure 3.12 plots the relationship between the welfare gain and the wealth percentile without borrowing (left panel) and with borrowing (right panel).

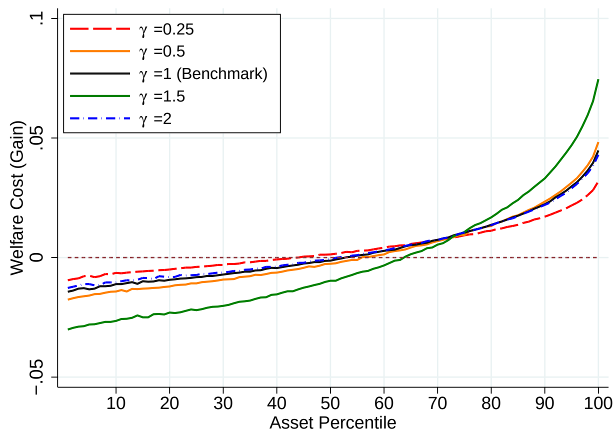

Figure 3.12a: without borrowing

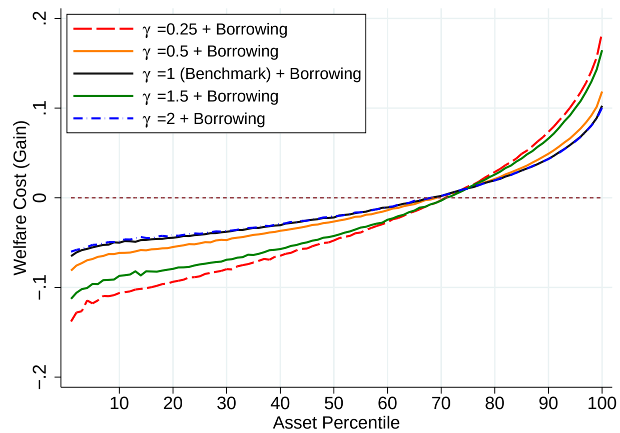

Figure 3.12b: with Borrowing

Figure 3.12: Frisch Elasticity combined with Different Borrowing Limits

It is easy to check that both monotonicity and convexity of the relationship between the two variables are preserved in any cases, confirming the robustness of our findings.

Magnitude of the TFP shocks. We additionally check if the level of the standard deviation used for the TFP shock, which is assumed to be $\sigma_z = 0.007$ in the benchmark analysis following previous literature (e.g. Hansen (1985)), drives our findings. In doing so, we consider different values for σ_z that ranges from 0.0035 to 0.018, which covers most of the values used in the literature. Figure 3.13 shows the results. Again, our findings on monotonicity and convexity still hold in any circumstances. In addition, the welfare gain becomes greater as the standard deviation becomes greater; this is the

[25]This is a quite reasonable range because in microeconomics the value is usually about 0.5 while it is about 1 in macroeconomics.

result of amplified mean effect, a consistent result with our main argument.

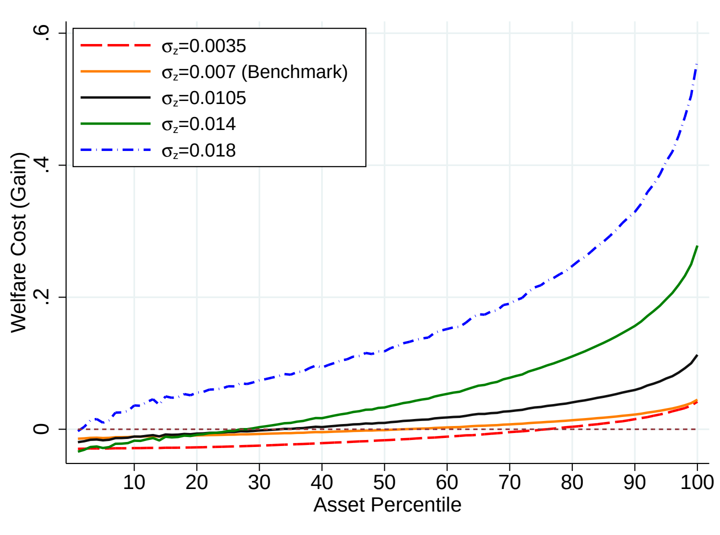

Figure 3.13: Varying Standard Deviation of TFP shocks

Other parameters. We then further check if (1) varying persistency of idiosyncratic labor income process, ρ_x, and (2) adjusting mean of the shock to TFP change the result of our finding. Results are presented in Figure 3.14. First, persistency parameter might affect our finding as it affects the probability of a worker to earn more or less, hence resulting in different wealth distribution. In particular, we change the values from 0.95 to 0.99. Second, as is discussed in Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014), adjustment of the mean of TFP level is required when the shock is specified in log-level since adding shock leads to higher TFP in mean. We hence use $-\frac{\sigma_z^2}{2(1+\rho_z)}$ as mean value for ε^z to make unconditional mean of the aggregate shock to be equal to 1.

Reassuringly, our findings are robust in any alternatives: Business cycles are beneficial for the rich while are welfare-detrimental for the poor; convexity of the welfare cost function with respect to asset distribution is preserved. While the overall welfare gain becomes slightly smaller when we adjust the mean of the TFP (dotted blue line), the finding that only a fraction of rich agents are benefited from the economic fluctuations is preserved and is actually more pronounced.

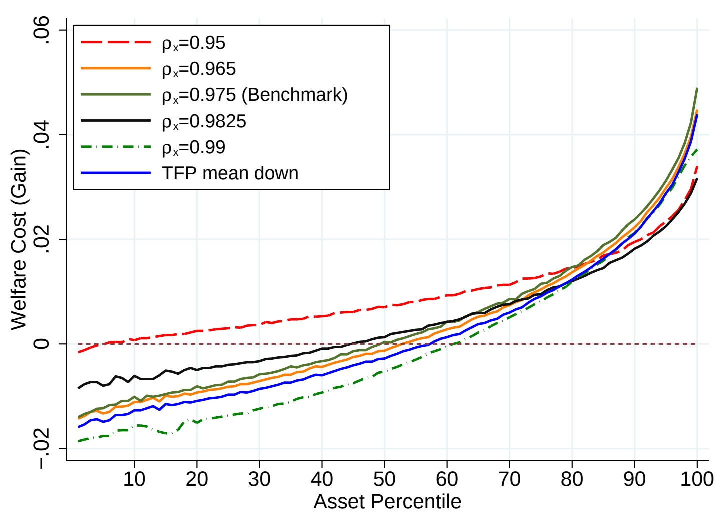

Figure 3.14: Persistency of Idiosyncratic Shocks and Adjustment of Mean of TFP

Note: Horizontal axes represent asset percentile for individuals. For instance, if the value is 10, it represents the average welfare costs for individuals with asset percentiles from bottom 9% to 10%.

4 Concluding Remark

This paper utilizes a heterogeneous agent RBC model framework with endogenous labor supply to provide insights on the distribution of the welfare cost across consumers. We first show that while technology-driven business cycles are beneficial on average, a consistent finding with the recent literature, the welfare gain is not equally distributed across consumers; it is beneficial only for agents who are relatively rich. We also show that the key to understand the monotonic relationship between welfare gain from the business cycles and wealth level, a finding different from the previous literature that argues that there is a non-monotonic relationship between the two, is endogenous labor supply. In particular, we compute the mean effect at the individual level to show that the main driver of our finding is the mean effect, a channel through which agents can exploit the economic fluctuations. Finally, we analyze the short run welfare consequence of rising income/wealth inequality.

Our findings provide several important policy implications. First, stabilization policies might still matter even in the economy that is well-approximated by the RBC model in the presence of income and wealth inequality; according to previous findings (Cho, Cooley, and Kim (2015) as an example), business cycles are welfare-improving and hence there is no role for stabilization policy as it lowers the mean effect. However, even when the business cycles might not be that welfare-detrimental at the aggregate level, negative welfare effect of the fluctuations can be still sizable for poor agents. Hence, such a policy can be welfare improving if it can sufficiently improve the welfare of the poor. Second, it is important to identify the sources for the rise of inequality; depending on the sources, effects on the distribution of the welfare gain can be dramatically different. Third, a well-designed redistribution policy can be welfare-improving; consider a government transfer that is financed through lump-sum tax on rich households. Such tax would not affect labor decision of the rich and hence its impact on the mean effect would not be substantial while the transfer can improve the welfare of the poor by lowering the fluctuations effect. Design of such (optimal) policy is interesting but is beyond the scope of this paper and hence we leave it as a future work.

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A Appendix. Computational Procedures

A.1 Computational procedures for competitive equilibrium with aggregate Shocks Computational procedures for the economy with aggregate risk require us to keep track of the measure of workers and aggregate productivity shocks over time in the list of state variables. We assume that agents make use of mean assets only in predicting the distribution of workers along the time following Krusell and Smith (1998). This makes computing the equilibrium equivalent to finding the decision rules, value functions, and predictions for the aggregate capital and wage are represented by log-linear functions in K and z.

Then the procedure consists of two steps. First, we solve policy functions for state variables, (a, x, w, K, z), given the forecasting rules (*the inner loop*). Then, we update the forecasting rules from the simulations of the economy using the individual policy functions (*the outer loop*). Iterating the two steps continues until the forecasting rules converge. That is, the difference between the forecasting rules used in the inner loop and the new forecasting rules calculated in the outer loop is small enough. Note that we use five state variables, (a, x, w, K, z), when deriving individual policy functions. Although this is computationally burdensome, this guarantees the labor market clear in the outer loop. Omitting the labor market clear in the outer loop could lead to misleading results as in Takahashi (2014) and we find that it could result in serious errors when computing mean effects and fluctuations effects. The procedures for the model with preference heterogeneity is similar except we use six state variables, (a, x, β, w, K, z), in this case.

Inner loop. We solve for the value functions $V(a, x, \widetilde{w}, K, z)$ and $V(a, x, K, z)$ in the inner loop. We use non-evenly spaced grid for a and evenly spaced grid for w and K. Aggregate capital and wage range from $[0.9K_{ss},\ 1.1K_{ss}]$ and $[0.9w_{ss},\ 1.1w_{ss}]$ respectively. In our simulations, aggregate capital and wage never reached the lower or upper bound. We approximate idiosyncratic productivity x and TFP shock z by Tauchen' algorithm (Tauchen (1986)). In addition, we use $301, 21, 13, 11, 11$ grids for state variables (a, x, w, K, z) respectively. When choosing the ith asset grid among n numbers of grids, we used minimum value of asset grid+maximum value of asset grid$(\frac{i-1}{n-1})^2$ following Chang and Kim (2014). The minimum and maximum value of asset grid is 0 and 300 respectively in the baseline model.

For the value function, we first solve the auxiliary value function like below, $\widetilde{V}(a, x, \widetilde{w}, K, z)$, which depends on an arbitrary wage.

$$\widetilde{V}(a, x, \widetilde{w}, K, z; g, m) = \max_{c,a'} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - B\frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta \mathbb{E}\left[V(a', x', K', z'; g, m)|x, z\}\right] \right\} \tag{A.1}$$

subject to

$$
\begin{aligned}
c + a' &= (1 + \widetilde{r}(\widetilde{w}, K, z))a + \widetilde{w}xh \\
K' = m(K, z) &= \exp(m_0^0 + m_1^0 \ln K + m_2^0 \ln z) \\
w = g(K, z) &= \exp(g_0^0 + g_1^0 \ln K + g_2^0 \ln z) \\
\widetilde{r}(\widetilde{w}, K, z) &= z^{1/\alpha}\alpha(\widetilde{w}/(1-\alpha))^{(\alpha-1)/\alpha} - \delta \\
V(a, x, K, z; g, m) &= \widetilde{V}(a, x, g(K, z), K, z; g, m)
\end{aligned}
\tag{A.2}
$$

To evaluate the conditional expectation, we compute value functions not on the grid points of (K', a') by polynomial interpolation. By solving these problems, we obtain the decision rules for hours $h(a, x, \widetilde{w}, K, z; g, m)$ and savings $a'(a, x, \widetilde{w}, K, z; g, m)$ as the maximizer of the problem (A.1).

Outer loop. In the outer loop, we simulate the model economy based on the policy functions obtained in the inner loop. As already noted, we check the labor market-clearing in each period during the simulation. In this simulation, L and K is constructed by aggregating individual labor supply, which is same to the labor demand of the representative firm by labor market-clearing, and asset holdings.

Labor market-clearing wage rate in each period can be found by following procedure. First, we assume that wage rate ranges from $[0.9w_{ss}, 1.1w_{ss}]$. Then, given wage rate $\widetilde{w}$, we can find the aggregate labor supply of individuals, $L^s = \int xh(a, x, \widetilde{w}, K, z; g, m)d\mu$. Labor demand of the representative firm is given by: $L^d = z^{1/\alpha}((1-\alpha)/\widetilde{w})^{1/\alpha}K$. We find the equilibrium wage rate w which makes $L^s = L^d$ by Brent method. Note that using forecasting rules in the inner loop $g(K, z)$ is not exact enough to clear the market, accumulating the errors in the simulation over the time. With the labor market-clearing condition, we can find exact simulation results of the model economy.

Based on the simulation, we then generate a set of artificial time-series for K_t and w_t by aggregating the individuals' decisions over time. To be specific, we simulate 50,000 individuals for 3,500 periods and discard the first 500 periods in order to minimize the effect of initial condition following Chang and Kim (2007). By OLS from the artificial data, we obtain new values for the coefficients m^1's and g^1's. If the new coefficients are close enough to the old coefficients in the inner loop, m^0's and g^0's, we have found the forecasting rules. Otherwise, we update the coefficients by adequately combining the old and new coefficients and repeat the whole procedure for the inner and outer loops until the coefficients converge. The estimated coefficients of forecasting rules and their accuracy for baseline model are as follows.

$$
\begin{aligned}
\ln K' &= 0.1309 + 0.9555 \ln K + 0.0940 \ln z, R^2 = 0.999946 \\
\ln w &= -0.3669 + 0.4185 \ln K + 0.8553 \ln z, R^2 = 0.997780
\end{aligned}
$$

A.2 Computational procedures for decomposing individual welfare cost We suggested the way to decompose the welfare cost of individual in section 3.2. In this subsection, we describe how we computed mean effect and thus fluctuations effect practically. We first show the formula for deriving the mean effect λ_m defined by the equation (3.4). For convenience, we denote the relevant part for calculation as follows.

$$\sum_{t=0}^{\infty} \beta^t u\left(\mathbb{E}_0[c_t(a_t, x_t)|a_0, x_0], \mathbb{E}_0[h_t(a_t, x_t)|a_0, x_0]\right) = V_{m,ss} \tag{A.3}$$
$$\sum_{t=0}^{\infty} \beta^t u\left(\mathbb{E}_0[c_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1], \mathbb{E}_0[h_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1]\right) = V_{m,f}$$

Although the formula for mean effect, $\lambda_m = \exp((V_{m,f} - V_{m,ss})(1-\beta)) - 1$, is clear, it is impossible to calculate $V_{m,ss}$ and $V_{m,f}$ directly. This is because the conditional expectations over time are included. For instance, it is nearly impossible to calculate $\mathbb{E}_0[c_{1000}(a_{1000}, x_{1000})|a_0, x_0]$ while it is possible for $\mathbb{E}_0[c_1(a_1, x_1)|a_0, x_0]$. Considering the 1000 periods of future idiosyncratic productivity shocks entails 21^{1000} calculations. It is more complex for $\mathbb{E}_0[c_{1000}(a_{1000}, x_{1000}, K_{1000}, z_{1000})|a_0, x_0, K_0 = K_{ss}, z_0 = 1]$. Therefore, we substitute the sample means from the simulations described below for conditional expectations. We also approximate $V_{m,ss}$ and $V_{m,f}$ by aggregating the utility up to 1000 periods as β^t becomes nearly zero far in the future. The procedures behind shows the way to approximately evaluate $V_{m,f}$.

1. Generate 50,000 individuals with different asset a_0 and productivity x_0 from the steady-state distribution.

2. Draw $i = 1, ..., N_{sim}$ sets of exogenous random values for idiosyncratic productivity and aggregate productivity, each of which have $t = 1, ..., 1000$ periods. We use a random draw sampling with Markov chains.

3. For each set of i, simulate the economy under fluctuations which starts with the distribution of 50,000 individuals in step1. In each period of the simulations, it is important to check the labor market-clearing as in the outer loop of appendix A.1. Ignoring market-clearing leads to serious errors when deriving the mean effects and fluctuations effects. The simulation proceeds from $t = 1$ to $t = 1,000$.

4. Sample means for conditional expectations at time t for individuals with a_0, x_0 are given by averaging the results at time t in simulations $i = 1, ..., N_{sim}$:

$$\widehat{\mathbb{E}_0}[c_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1] = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} c_t(a_t, x_t, K_t, z_t)$$
$$\widehat{\mathbb{E}_0}[h_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1] = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} h_t(a_t, x_t, K_t, z_t)$$

5. Based on the samples means derived in step 4 of 50,000 individuals for 1,000 periods, we approx-

imate $V_{m,f}$ for all consumers as follows:

$$V_{m,f} \approx \sum_{t=0}^{1000} \beta^t u\left(\widehat{\mathbb{E}_0}[c_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1], \widehat{\mathbb{E}_0}[h_t(a_t, x_t, K_t, z_t)|a_0, x_0, K_0 = K_{ss}, z_0 = 1]\right)$$

The procedures for deriving $V_{m,ss}$ is similar to the above process except labor market-clearing is not considered in step 3. We set $N_{sim} = 1,000$ and fluctuations effect λ_f is derived by the formula $\lambda \approx \lambda_m + \lambda_f$.