Ex-ante Information Heterogeneity in Global Games Models with Application to Team Production*

Seongkyun Kim† Myungkyu Shim‡ Doyoung Song§

September 19, 2023

Abstract

This paper introduces ex-ante information heterogeneity between players into an otherwise standard global game so that both a type with superior private information and that with inferior private information coexist. After proving that there exists a unique perfect Bayesian threshold equilibrium in which both types follow their own threshold strategies, several comparative statics results in the context of team production are established. Particularly, we identify the conditions under which (*i*) more-informed workers tend to put more efforts than less-informed ones and (*ii*) success probability of the team production becomes higher. We then derive implications on optimal information allocation problem.

JEL classification: D80, D83, G14

Keywords: Global games, unique equilibrium, information heterogeneity, comparative statics, Optimal Information

*We are grateful to David M. Frankel for his helpful and constructive comments. We would also like to thank seminar and conference participants at Yonsei University and Korean Econometric Society Summer Meeting. Seoyoon Jeong provided excellent research assistance. Shim acknowledges the financial support from Yonsei University and Yongwoon Scholarship Foundation (Yonsei-Yongwoon Research Grant No. 2021-11-0410).

†School of Economics, Yonsei University. Email: ksk7757@yonsei.ac.kr. Postal Address: 50 Yonsei-ro Seodaemun-gu, Seoul 03722, South Korea

‡School of Economics, Yonsei University. Email: myungkyushim@yonsei.ac.kr. Postal Address: 50 Yonsei-ro Seodaemun-gu, Seoul 03722, South Korea

§Department of Economics, University of California, San Diego. Email: dos006@ucsd.edu. Postal Address: 9500 Gilman Dr. La Jolla, CA 92093.

1 Introduction

Since the seminal work by Carlsson and van Damme (1993) and Morris and Shin (1998), global games models have been a widely used tool to explain various economic circumstances with strong complementarity such as bank-run, financial crisis, and collapse of exchange rates. This class of models is particularly characterized by incomplete information on the fundamental value, denoted by θ, which is introduced in the form of dispersed information; instead of directly observing the fundamental value, each player is assumed to receive private information (and/or public information) with a noise, which yields *ex-post* information heterogeneity across the players. In reality, however, there also exists *ex-ante* information asymmetry (or inequality) across players; some players possess more precise information and/or they have comparative advantages in processing additional information than others. In addition, information inequality might have become widened in the last few decades as it is associated with income inequality (Kennedy and Prat (2019)), raising the importance of considering information inequality in the analysis. The extent to which this feature influences the equilibrium properties, however, has not yet been extensively studied in the context of global games. For instance, whether a more-informed agent is more aggressive/active in action than a less-informed one is an unexplored but important question from the perspective of a policymaker who would like to keep the status quo. Team production is another example: A worker might have different information on the success probability of the project from other workers, and this might result in coordination failure. Manager of the firm would then need to implement a strategy to avoid the coordination failure; suppose that the manager can control the precision of private information à-la Moriya and Yamashita (2020). In this case, what would be the solution to optimal information allocation problem in the context of global games? This paper aims to add to the literature by addressing the above issues.

Toward that end, we assume that there are two types of players in the game, especially in the form of a regime attack game; the first type is an "expert type," who receives an informative private signal with precision $\alpha_x^e > 0$. The other type is an "inexpert type," who receives a private information with precision $\alpha_x^i > 0$. In order to introduce ex-ante heterogeneity in information, we further assume that $\alpha_x^e = (1+\mu)\alpha_x^i > \alpha_x^i$ with $\mu > 0$ so that experts receive more informative private information. This captures the information asymmetry across the market participants. In this regard, three papers are closely related to our paper. First, James and Lawler (2012) consider a coordination game with two types of players who have different private information.

The major difference between our work and theirs is that we consider a model with strong complementarity that yields multiple equilibria while they consider a coordination game with weak complementarity and hence there is no such an issue. Second, Corsetti, Dasgupta, Morris, and Shin (2004) and Bannier (2005) consider global games with two types of players. In contrast to our framework, their main consideration was the size, not information heterogeneity; they study the role of "large investor" in determining the equilibrium properties while we assume that each type consists of infinitesimal agents. This enables us to isolate the informational effect from the size effect. While the symmetric approach introduced in Bannier (2005) yields a similar equilibrium outcome to ours, our finding is different in (1) the sufficient condition for uniqueness of equilibrium and (2) providing conditions under which equilibrium outcome alters when informational gap changes. In this sense, this paper complements and extends the insights of Bannier (2005) to an environment in which none of the players have advantages in size but in information. Furthermore, we apply the equilibrium properties to a different context, optimal information allocation problem.

We first prove the important property of this class of models[1] that there exists a unique symmetric perfect Bayesian threshold equilibrium. In doing so, we first show that there are two threshold levels of private signals under which each type attacks the regime when he/she receives a signal below the thresholds. In addition, we argue that there exists a unique threshold level for the fundamental value under which the regime collapses when the fundamental is lower than the threshold value. This is a different result from Corsetti, Dasgupta, Morris, and Shin (2004) that also consider two types of players, one large player and infinitely many small players: They show that there are two threshold values for the fundamental. The difference comes from the fact that we assume that all players are infinitesimal so that each player cannot significantly affect the size of aggregate attack. In the sense that we focus on informational gap across the agents, our work differentiates from theirs that study the role of large players in determining the equilibrium properties. With these two properties, it is proved that there exists a unique symmetric perfect Bayesian threshold equilibrium when private information of inexpert type (and hence expert type) is sufficiently precise, which nests Morris and Shin (1998) as a special case.[2]

[1]See Morris and Shin (1998), Morris and Shin (2003), and Angeletos and Werning (2006) for more detailed discussions on unique equilibrium.

[2]While Angeletos and Werning (2006) find that the unique equilibrium result is not robust to the introduction of endogenous public signal, Challe and Chrétien (2018) and Róndina and Shim (2021) show that such a conclusion is not robust to (1) market microstructure and (2) explicit distinction between traders who participate in the asset market in which public signal is endogenously determined and players who plays the coordination game,

We then exploit the virtue of the equilibrium uniqueness by establishing comparative statics. In particular, we consider team production problem following Moriya and Yamashita (2020) as an application of the general model discussed above: The economy consists of two types of workers: Each worker is either more-informed (expert type) or less-informed (inexpert type) on the success probability of the team production. A worker either works hard ($a_i = 1$) or not ($a_i = 0$) with i denoting each worker and the project in which both workers are involved succeeds when aggregate effort ($A = \int a_i di$) is greater than a threshold level (θ). In addition, there exists a manager who would like to exert the best efforts from workers to maximize A. The manager can control the information on the success probability of the project to both types.

Three key insights obtained from our analysis can be summarized as follows. To make each statement succinct, we will call the environment with (1) low cost of effort and (2) low realized public signal, which is observed by the players, "low-cost low-p" environment and focus on this case. Predictions from the opposite case ("high-cost high-p") are exactly the opposite.[3] First, the threshold level for more-informed worker is lower than that of less-informed worker in the low-cost low-p environment. As greater threshold value enlarges the range of private signal under which the worker puts effort, whether more-informed worker puts more effort or not crucially depends on the status of the underlying parameter values. If public signal is sufficiently low, all workers perceive the chance of success to be high. With low cost of effort, the less-informed worker would rely more on this public information than the more-informed ones and hence it increases their propensity to work compared to the latter.

Second, greater differences in private information between the more- and less-informed workers, measured by greater μ, lowers the threshold value regardless of the type in the low-cost low-p environment. Hence, both types become less active, resulting in low probability of project success. This extends the finding by Metz (2002) to a situation in which only a fraction of players in the economy receives unevenly better private information, which improves the average precision of private information in the economy. This result is preserved when μ is fixed while the measure of more-informed workers becomes greater. This finding provides an interesting policy implication: A policymaker has a policy tool to keep the status-quo in addition to pre-

which provides a relief on the model without endogenous public information.

[3]Other cases (low-cost high-p and high-cost low-p) also exist in the equilibrium. However, clear comparative statics cannot be obtained in such cases because the two assumptions have opposite effects on the equilibrium and we cannot determine which one dominates the other. Hence, we focus on the cases where we can derive clear predictions.

vious policies deciding only the extent of agents receiving better information: In the low-cost low-p environment, for example, it might exaggerate the informational gap across the agents by providing sophisticated information without changing the portion of more-informed workers. It only enhances precision of more-informed workers' private information and lowers the success probability.

Third, we can characterize the conditions under which the manager chooses an extreme information structure: In our model setup, it is optimal to provide all the workers with no-information (resp. full-information) when public information takes significantly low (resp. high) values. This implies that symmetric information allocation can be optimal, which is a finding different from Moriya and Yamashita (2020) who prove that full-information is never optimal while no-information and asymmetric information allocation can be optimal. In this regard, our finding confirms the idea that more work might be necessary to derive the optimal information allocation strategy along different dimensions of games (Moriya and Yamashita (2020)). Moreover, our finding implies that the extreme information structure can hold even if the information designer does not assume the worst scenario (Inostroza and Pavan (2022) and Morris, Oyama, and Takahashi (2022)).

Remaining sections are organized as follows. Section 2 introduces the model and then Section 3 provides the equilibrium properties of the model. We then do comparative statics in Section 4 by applying the model to optimal information allocation problem in the context of team production. Section 5 concludes.

2 The Model

In this section, we introduce the main model, the regime attack game, which is an extended version of Metz (2002).[4]

There are players who are uniformly distributed on $[0, 1]$. Each player maximizes his/her expected utility. In particular, each player i chooses either *to attack the status-quo* (denote as a_i $= 1$) or *not to attack* (denote as $a_i = 0$) and utility function is defined as

[4]The model introduced in Morris and Shin (1998) shares the same structure, but they assume that the shocks to signals are drawn from a uniform distribution.

$$U(a_i, A, \theta) = a_i(\mathbf{1}_{A \geq \theta} - c). \tag{1}$$

where $\mathbf{1}$ is an indicator function that takes the value of one (*resp.* zero) when the size of aggregate attack ($A = \int a_i di$) is greater (*resp.* lower) than the fundamental value (θ). We assume that θ is drawn from an improper distribution and is not directly observed by each player. $c \in (0, 1)$ is the cost of attacking.

Instead of directly observing θ, each player receives a (noisy) public signal and a (noisy) private signal. Public signal is described as follows.

$$p = \theta + \frac{1}{\sqrt{\alpha_p}}\varepsilon \tag{2}$$

where $\alpha_p > 0$ denotes precision of the public signal and $\varepsilon \sim N(0, 1)$.

Differently from the previous literature, we assume that there is an ex-ante heterogeneity in private information across the players. In doing so, we assume that players are divided into two types; an *expert type* with measure $\lambda \in [0,\ 1]$ and an *inexpert type* with measure $1 - \lambda$.

The inexpert type, distributed on $(\lambda, 1]$, is a player who is not specialized in accessing information on the fundamental.[5] One can think of small investors participating in coordination games, who do not have superior private information on the market. This type receives the private signal with precision $\alpha_x^i > 0$:

$$x_i = \theta + \frac{1}{\sqrt{\alpha_x^i}}\eta_i \tag{3}$$

where $\eta_i \sim N(0, 1)$ and $\eta_i \perp \varepsilon$.

Expert type, distributed on $[0, \lambda]$, denotes players who care more about economic situations and hence process more information on the fundamental. This type might include full-time investors and traders in the financial firms such as investment banks: Since income of these players rely more on the outcomes from the coordination game, this type would receive more private information. Or alternatively, one can consider a situation in which some players have

[5] This type is different from noisy traders usually assumed in the literature in which $\alpha_x^i \to 0$. We abstract from inclusion of noisy traders in the analysis because they do not have any important informational effect on the equilibrium.

better skills to exploit valuable information from the common sources of information; they might have greater capacity to process information (Sims (2003)) than others, resulting in better private information. Denoting precision of private signal received by this type as $\alpha_x^e > 0$, private signal can be described as follows:

$$x_e = \theta + \frac{1}{\sqrt{\alpha_x^e}}\eta_e \tag{4}$$

where $\eta_e \sim N(0,1)$ and $\eta_e \perp \eta_i \perp \varepsilon$.

The following equation captures the idea that this type receives more precise private information than the inexpert type:

$$\alpha_x^e = (1+\mu)\alpha_x^i > \alpha_x^i \tag{5}$$

where $\mu > 0$ measures the information discrepancy between the two types.

Notice that this model nests Morris and Shin (1998) and Metz (2002) as special cases when $\mu = 0$. Importantly, while the size of the expert type is fixed as λ, the measure of this type who chooses to attack the status-quo is not exogenously determined, which makes our model be distinguished from Corsetti, Dasgupta, Morris, and Shin (2004) and Bannier (2005). In their model, a size of the particular group (large investor) is predetermined and hence there is no strategic uncertainty around the size of the attack from the group. On the contrary, this feature, which is an important characteristic of this class of models, is preserved in our model.

3 Equilibrium Characterization and its Uniqueness

To be consistent with the previous literature (Metz (2002) and Bannier (2005) as examples), we consider a symmetric perfect Bayesian threshold equilibria in this paper.[6] In particular, inexpert (resp. expert) type would attack the regime if and only if the realization of her private signal is less than some threshold x_i^* (resp. x_e^*). In this section, we first describe how the threshold values are determined and then provide a sufficient condition for equilibrium uniqueness of the symmetric perfect Bayesian threshold equilibria. Under the assumption that all players take threshold strategies, we can first prove the following lemma.

Lemma 1 (Threshold value for θ)**.** *Let $\Phi(\cdot)$ be the cumulative density function of the standard normal (Gaussian) distribution. Given any x_i^* and x_e^*, there always exists at least one $\theta^* \in (0,1)$,*

[6] Here, symmetric equilibrium is defined as a strategy that is symmetric within a group that shares the same precision of private information.

which satisfies the fixed point condition $A(\theta^*) = \theta^*$ where A is an aggregate action. Formally, the following holds.

$$(1-\lambda)\Phi\left(\sqrt{\alpha_x^i}\,(x_i^* - \theta^*)\right) + \lambda\Phi\left(\sqrt{\alpha_x^e}\,(x_e^* - \theta^*)\right) = \theta^* \tag{6}$$

Proof. Aggregate action in this economy can be defined as follows.

$$\begin{aligned} A(\theta) &= (1-\lambda)Pr[x_i < x_i^*|\theta] + \lambda Pr[x_e < x_e^*|\theta] \\ &= (1-\lambda)\Phi\left(\sqrt{\alpha_x^i}\,(x_i^* - \theta)\right) + \lambda\Phi\left(\sqrt{\alpha_x^e}\,(x_e^* - \theta)\right) \end{aligned} \tag{7}$$

Notice that $A(0) \in (0,1)$ and $A(1) < A(0)$ since $\Phi(\cdot)$ is decreasing in θ. Continuity of $A(\theta)$ guarantees existence of at least one $\theta^* \in (0,1)$ such that $A(\theta^*) = \theta^*$ by the intermediate value theorem. □

Lemma 1 hence determines the threshold θ^*: The regime will survive only when its fundamental value, θ, is sufficiently high. We first note that this is a generalization of the previous findings with ex-ante homogenous players (Morris and Shin (1998)). Importantly, the above lemma is distinctive from that of Corsetti, Dasgupta, Morris, and Shin (2004): There are two threshold levels for θ in their model because the large investor can succeed in the attack even when there is no small investors who choose to attack. As a result, one additional threshold level would be obtained in the equilibrium. On the contrary, there also exists a strategic uncertainty among the expert type in our model framework. Hence, from the perspective of each individual player, it is required to consider one threshold level for θ when making a decision.

The equilibrium should ensure that two indifferent conditions under which both inexpert- and expert type would be indifferent between attack the regime and not to attack are met. Lemma 2 introduces the conditions under which expected utility of an agent is the same between the two strategies, attack and no attack.

Lemma 2 (Threshold values for private signals). *Define $\delta^i \equiv \frac{\alpha_x^i}{\alpha_x^i + \alpha_p}$ and $\delta^e \equiv \frac{\alpha_x^e}{\alpha_x^e + \alpha_p}$. If θ^* is determined as in Lemma 1, given any p, indifference conditions for each type are given as follows.*

$$\Phi\left(\sqrt{\alpha_x^i+\alpha_p}\left(\theta^*-(\delta^i x_i^*+(1-\delta^i)p)\right)\right)=c \tag{8}$$

$$\Phi\left(\sqrt{\alpha_x^e+\alpha_p}\left(\theta^*-(\delta^e x_e^*+(1-\delta^e)p)\right)\right)=c \tag{9}$$

Both x_i^* and x_e^* are threshold values of private signals for inexpert type and expert type, respectively.

Proof. We focus on the inexpert type in this proof. Expected utility of the inexpert type is $Pr[\theta<\theta^*(p)|x_i,p]-c=\Phi\left(\sqrt{\alpha_x^i+\alpha_p}\left(\theta^*(p)-(\delta^i x_i+(1-\delta^i)p)\right)\right)-c$. x_i^* should ensure the expected utility from attacking the status quo and that from not to attack of the inexpert type to be the same, which yields the condition (8). □

Hence, Lemma 2 is again a generalization of the previous results with only one type of players. With above lemmas, we can prove the following proposition, which is the first main result of our paper.

Proposition 1 (Sufficient Condition for Unique Equilibrium)**.** *For any realization of the public signal p, the regime attack game has a unique symmetric perfect Bayesian threshold equilibrium (θ^*, x_i^*, x_e^*) if*

$$(1-\lambda)\sqrt{\frac{1}{\alpha_x^i}}+\lambda\sqrt{\frac{1}{\alpha_x^e}}<\frac{\sqrt{2\pi}}{\alpha_p}\Leftrightarrow\frac{\alpha_p}{\sqrt{2\pi}}<\frac{\sqrt{\alpha_x^i}}{1-\lambda+\lambda\frac{1}{\sqrt{1+\mu}}}. \tag{10}$$

Proof. Rearranging (8) and (9), we obtain the following equations.

$$x_i^*=\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}\theta^*-\frac{\alpha_p}{\alpha_x^i}p-\frac{\sqrt{\alpha_x^i+\alpha_p}}{\alpha_x^i}\Phi^{-1}(c) \tag{11}$$

$$x_e^*=\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}\theta^*-\frac{\alpha_p}{\alpha_x^e}p-\frac{\sqrt{\alpha_x^e+\alpha_p}}{\alpha_x^e}\Phi^{-1}(c) \tag{12}$$

Substituting (11) and (12) into (6), we can get the following equation, which determines the threshold level θ^*.

$$(1-\lambda)\Phi\left(\frac{\alpha_p}{\sqrt{\alpha_x^i}}(\theta^*-p)-\sqrt{\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}}\Phi^{-1}(c)\right)+\lambda\Phi\left(\frac{\alpha_p}{\sqrt{\alpha_x^e}}(\theta^*-p)-\sqrt{\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}}\Phi^{-1}(c)\right)=\theta^* \tag{13}$$

Define $f(\theta^*) \equiv (1-\lambda)\Phi\left(\frac{\alpha_p}{\sqrt{\alpha_x^i}}(\theta^*-p)-\sqrt{\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}}\Phi^{-1}(c)\right)+\lambda\Phi\left(\frac{\alpha_p}{\sqrt{\alpha_x^e}}(\theta^*-p)-\sqrt{\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}}\Phi^{-1}(c)\right)$, which is the left hand side of the above equation. Then we can further define $g(\theta^*) = f(\theta^*) - \theta^*$. Since $\Phi(x) \in (0,1)$ for any finite x, $g(0) > 0$ and $g(1) < 0$ are satisfied for any given p. Also, $g(\theta^*)$ is a continuous function of θ^*. Hence, by the intermediate value theorem, there exists at least one $\hat{\theta} \in (0,1)$ such that $g(\hat{\theta}) = 0$ for any given p.

We now turn our focus on the uniqueness of the equilibrium. As $g(0) > 0$ and $g(1) < 0$, uniqueness of $\hat{\theta}$ is guaranteed if $g(\theta^*)$ is a strictly decreasing function of θ^*. In other words, we need to show that $g'(\theta^*) < 0$ holds for any given p to show that $\hat{\theta}$ is uniquely pinned down. Let $\phi(\cdot)$ be the probability density function of the standard normal (Gaussian) distribution. Then

$$\begin{aligned} g'(\theta^*) &= (1-\lambda)\underbrace{\phi(\beta^i)}_{\leq\frac{1}{\sqrt{2\pi}}}\frac{\alpha_p}{\sqrt{\alpha_x^i}}+\lambda\underbrace{\phi(\beta^e)}_{\leq\frac{1}{\sqrt{2\pi}}}\frac{\alpha_p}{\sqrt{\alpha_x^e}}-1 \\ &\leq \frac{\alpha_p}{\sqrt{2\pi}}\left((1-\lambda)\sqrt{\frac{1}{\alpha_x^i}}+\lambda\sqrt{\frac{1}{\alpha_x^e}}\right)-1 \end{aligned} \tag{14}$$

where $\beta^i \equiv \frac{\alpha_p}{\sqrt{\alpha_x^i}}(\theta^*-p)-\sqrt{\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}}\Phi^{-1}(c)$ and $\beta^e \equiv \frac{\alpha_p}{\sqrt{\alpha_x^e}}(\theta^*-p)-\sqrt{\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}}\Phi^{-1}(c)$.

Hence, $g'(\theta^*) < 0$ if $(1-\lambda)\sqrt{\frac{1}{\alpha_x^i}}+\lambda\sqrt{\frac{1}{\alpha_x^e}} < \frac{\sqrt{2\pi}}{\alpha_p}$. This completes the proof.

□

We first note that the above proposition is a condition different from what Bannier (2005) obtained in her paper. In page 1526, she argues that the sufficient condition for unique equilibrium is that precision of private information for both types should be simultaneously higher than $\frac{\alpha_p}{\sqrt{2\pi}}$, or equivalently $\min\{\alpha_x^i, \alpha_x^e\} > \frac{\alpha_p}{\sqrt{2\pi}}$. Suppose that $\alpha_p = 4$, $\alpha_x^i = 1$, $\alpha_x^e = 4$, and $\lambda = 0.8$. Then $\frac{\alpha_p}{\sqrt{2\pi}} \approx 1.6$ hence one cannot argue that there is a unique equilibrium based on the criterion provided by Bannier (2005) since $\alpha_x^i < \frac{\alpha_p}{\sqrt{2\pi}}$. However, Proposition 1 indicates that

$(1-\lambda)\sqrt{\frac{1}{\alpha_x^i}} + \lambda\sqrt{\frac{1}{\alpha_x^e}} = 0.6$ so that there does exist a unique equilibrium.

We then observe that Proposition 1 nests the sufficient condition for uniqueness result by Morris and Shin (1998) and Metz (2002) with no expert ($\lambda = 0$):

Corollary 1 (Special case with no expert)**.** *Suppose that $\lambda = 0$. Then for any realization of the public signal p, the regime attack game has a unique symmetric perfect Bayesian threshold equilibrium (θ^*, x_i^*) if*

$$\frac{\alpha_p}{\sqrt{2\pi}} < \sqrt{\alpha_x^i} \tag{15}$$

In our model, $\alpha_x^i \leq \alpha_x^e$ hence the condition for unique equilibrium obtained by Bannier (2005) is equivalent to Corollary 1, implying that the sufficient condition is in principle equivalent to Morris and Shin (1998) and Metz (2002).

It is immediate to further observe that the threshold level required to obtain unique equilibrium is a function of λ, measure of expert type, and μ, information difference between inexpert type and expert type:

Corollary 2 (Comparative statics with experts)**.** *Let $\alpha_x^{i,Expert} \equiv \left[\frac{\alpha_p}{\sqrt{2\pi}}\left(1-\lambda+\lambda\frac{1}{\sqrt{1+\mu}}\right)\right]^2$ be the minimum level of precision for private signal of inexpert type to achieve unique equilibrium by satisfying Proposition 1. This threshold level is decreasing in λ and μ.*

This is an intuitive result in the sense that higher λ and μ has a positive effect on overall precision of private information in this economy: An expert type tends to rely more on her private signal when making a decision than an inexpert type does as she receives more precise private signal. In other words, the role of public information acting as a coordination device decreases as either λ or μ increases. And hence, even when private information for the inexpert type is not precise enough, the overall precision of private information in this economy becomes better.

In Figure 1, we plot the parameter region at which the equilibrium is unique: Any region above each line exhibits unique equilibrium.[7] Figure 2 further verifies Corollary 2 that the

[7]Since Proposition 1 provides a sufficient condition for uniqueness, the region under each line does not necessarily yields multiple equilibria.

threshold level is decreasing in both λ and μ. Solid black line is the threshold level for Morris-Shin economy with $\alpha_p = 1$. With fixed α_p, higher λ and μ lowers the threshold level, which confirms the Corollary 2.

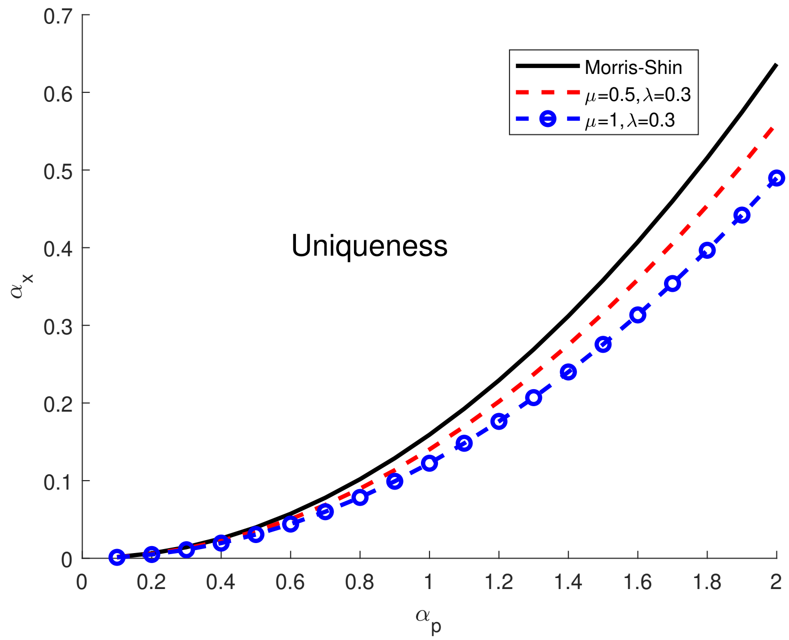

Figure 1: Uniqueness vs. Multiplicity

With a unique θ^*, equilibrium conditions (11) and (12) determine unique x_i^* and x_e^*. In the sense that the player attacks the regime only when her private signal is lower than the threshold level, we can say the player becomes more active (*resp.* passive) in attacking the regime when the threshold level increases (*resp.* decreases). With better private information, would the expert be more active or passive than the inexpert type? The following proposition summarizes the answer to this question.

Proposition 2 (Expert Type vs. Inexpert Type)**.** *Let $c \in [0, \frac{1}{2})$ (resp. $c \in [\frac{1}{2}, 1]$), $p \leq \theta^*$ (resp. $p > \theta^*$), and the sufficient condition for unique threshold equilibrium holds (Proposition 1). Then an expert type is relatively passive (resp. active) than the inexpert type. Formally, $x_e^* < x_i^*$ (resp. $x_e^* > x_i^*$).*

Proof. Substituting (5) into the equation (12), we can get

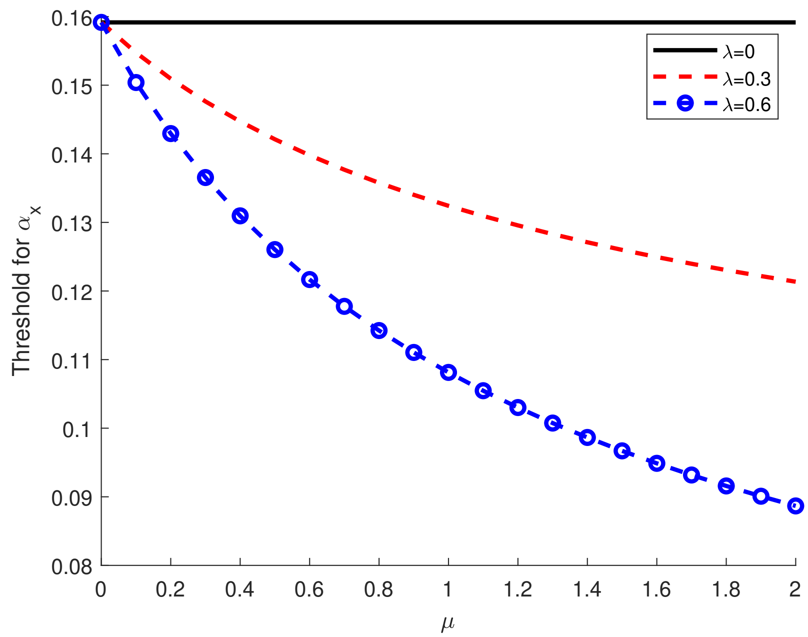

Figure 2: Uniqueness with Changes in μ and λ

$$x_e^* = \frac{(1+\mu)\alpha_x^i + \alpha_p}{(1+\mu)\alpha_x^i}\theta^* - \frac{\alpha_p}{(1+\mu)\alpha_x^i}p - \frac{\sqrt{(1+\mu)\alpha_x^i + \alpha_p}}{(1+\mu)\alpha_x^i}\Phi^{-1}(c)$$
$$= \theta^* + \frac{\alpha_p}{(1+\mu)\alpha_x^i}(\theta^* - p) - \sqrt{\frac{1}{(1+\mu)\alpha_x^i} + \frac{\alpha_p}{(1+\mu)^2(\alpha_x^i)^2}}\Phi^{-1}(c) \tag{16}$$

Define $g(\mu) = \theta^* + \frac{\alpha_p}{(1+\mu)\alpha_x^i}(\theta^* - p) - \sqrt{\frac{1}{(1+\mu)\alpha_x^i} + \frac{\alpha_p}{(1+\mu)^2(\alpha_x^i)^2}}\Phi^{-1}(c)$. Then,

$$g'(\mu) = -\underbrace{\frac{\lambda\frac{\alpha_p}{\sqrt{4(1+\mu)^3\alpha_x^i}}\left(\theta^* - p - \frac{1}{\sqrt{(1+\mu)\alpha_x^i+\alpha_p}}\Phi^{-1}(c)\right)}{1-(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{(1+\mu)\alpha_x^i}}\phi(\beta^e)}}_{\frac{\partial\theta^*}{\partial\mu}} - \frac{\alpha_p}{(1+\mu)^2\alpha_x^i}(\theta^* - p) + \frac{\frac{1}{(1+\mu)^2\alpha_x^i} + \frac{2\alpha_p}{(1+\mu)^3(\alpha_x^i)^2}}{2\sqrt{\frac{1}{(1+\mu)\alpha_x^i} + \frac{\alpha_p}{(1+\mu)^2(\alpha_x^i)^2}}}\Phi^{-1}(c) \tag{17}$$

Therefore $g(\mu)$ is decreasing in μ when $c \in [0, \frac{1}{2})$ (and hence $\Phi^{-1}(c) < 0$) and $p < \theta^*$. Thus $x_e^* = g(\mu) < g(0) = x_i^*$.

□

In order to get an economic intuition behind the above proposition, we consider a case in which (1) cost of attacking is low ($c \in [0, \frac{1}{2})$) and (2) the realized public signal is low enough ($p \leq \theta^*$), which is abbreviated as a "low-cost low-p" environment. We first note that all players perceive the chance of regime collapse high when public signal is sufficiently low. With low cost of attacking, an inexpert type would rely more on this public information than the expert type and hence it increases their propensity to attack the regime compared to the expert type, resulting in the inexpert type to be more active.[8] In contrast, the expert type knows more accurately about the fundamental. This enables them to rely less on public information even when p is sufficiently low. As a result, smaller size of expert type is required to make the regime collapse (in the "high-cost high-p" environment, on the contrary, there is an incentive for the expert type to act like a first penguin to enable regime collapse before the inexpert type based on more accurate private information, which is impossible without their action). This is an interesting result that provides potential implications on the market behavior of investors in the sense that informational advantage does not make the expert type always more or less aggressive/active than the inexpert type: More precise private information makes this type to be less dependent on the underlying fundamental (both of cost structure and the public signal) than the inexpert type. Hence, its position at the market (game) would be less volatile than that of the inexpert type, which can be possibly tested with the empirical data.

4 Comparative Statics: Application to Team Production

In this section, we study how the changes in key parameter values affect the equilibrium of this economy under the assumption that unique equilibrium is achieved. In other words, Proposition 1 is assumed to be satisfied. In order to highlight new findings due to ex-ante information heterogeneity, we present findings that extend previous results in the Appendix A.

We particularly consider a team production model à-la Moriya and Yamashita (2020) as an application of the general model discussed in the previous section. The economy consists of two types of workers who work together to make a project succeed: Each worker is either more-informed (expert type) or less-informed (inexpert type) on the success probability of the team production. A worker either works hard by providing enough efforts ($a_i = 1$) or not ($a_i = 0$)

[8] This is why the prediction on "high-cost low-p" (and hence "low-cost high-p") environment is indeterminate. Even when the public signal is realized to be low enough, high cost hinders the inexpert type to attack the regime and hence whether x_i^* is greater than x_e^* or not is unclear.

when i denotes each worker and the project in which both workers are involved succeeds only when the aggregate effort ($A = \int a_i di$) is greater than a random variable, θ, which is unknown to the workers. Providing effort is costly ($c > 0$). In addition, there exists a manager who would like to exert the best efforts from workers to maximize A. The manager's problem is to control the information on the success probability of the project to both types or one type or not in order to maximize the success probability of the team production.

We first address the extent to which ex-ante information heterogeneity affect the success probability in Proposition 3:

Proposition 3 (Comparative Statics on θ^*: Effect of λ and μ)**.** *Let $c \in [0, \frac{1}{2})$ (resp. $c \in [\frac{1}{2}, 1]$) and $p \leq \theta^*$ (resp. $p > \theta^*$), and the sufficient condition for unique threshold equilibrium holds (Proposition 1). Then the success probability becomes higher when (i) measure of more-informed workers becomes lower (resp. higher) and/or (ii) information gap between the two type becomes lower (resp. higher). Formally,*

$$\frac{\partial \theta^*}{\partial \lambda} < 0 \; (resp. > 0) \quad and \quad \frac{\partial \theta^*}{\partial (1+\mu)} < 0 \; (resp. > 0) \tag{18}$$

Proof. Again, we differentiate the equilibrium condition (13) with respect to λ and $1+\mu$:

$$\frac{\partial \theta^*}{\partial \lambda} = -\frac{\Phi(\beta^i) - \Phi(\beta^e)}{1 - (1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{(1+\mu)\alpha_x^i}}\phi(\beta^e)} \tag{19}$$

$$\frac{\partial \theta^*}{\partial (1+\mu)} = -\frac{\lambda\frac{\alpha_p}{\sqrt{4(1+\mu)^3\alpha_x^i}}\left(\theta^* - p - \frac{1}{\sqrt{(1+\mu)\alpha_x^i + \alpha_p}}\Phi^{-1}(c)\right)}{1 - (1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{(1+\mu)\alpha_x^i}}\phi(\beta^e)} \tag{20}$$

Define $g(\mu) = \frac{\alpha_p}{\sqrt{(1+\mu)\alpha_x^i}}(\theta^* - p) - \sqrt{\frac{(1+\mu)\alpha_x^i + \alpha_p}{(1+\mu)\alpha_x^i}}\Phi^{-1}(c)$. Then, $\beta^i = g(0)$ and $\beta^e = g(\mu)$. Since $g'(\mu) = -\frac{\alpha_p}{2\sqrt{(1+\mu)^3\alpha_x^i}}(\theta^* - p) - \frac{\lambda\frac{\alpha_p^2}{2(1+\mu)^2\alpha_x^i}\left(\theta^* - p - \frac{1}{\sqrt{(1+\mu)\alpha_x^i + \alpha_p}}\Phi^{-1}(c)\right)}{1-(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{(1+\mu)\alpha_x^i}}\phi(\beta^e)} + \frac{\frac{\alpha_p}{(1+\mu)^2\alpha_x^i}}{2\sqrt{\frac{(1+\mu)\alpha_x^i + \alpha_p}{(1+\mu)\alpha_x^i}}}\Phi^{-1}(c)$, $\beta^i > \beta^e$ when $c \in [0, \frac{1}{2})$ and $p \leq \theta^*$, implying $\frac{\partial \theta^*}{\partial \lambda} < 0$. Similarly, $\frac{\partial \theta^*}{\partial (1+\mu)} < 0$ when $c \in [0, \frac{1}{2})$ and $p \leq \theta^*$.

□

In order to understand the above proposition, we first focus on the effect of information gap (μ) on the threshold level of the more-informed worker using equation (12) in the low-cost low-p environment ($c \in [0, \frac{1}{2})$ and $p \leq \theta^*$). Given fixed θ^*, lower μ increases x_e^*. This is because lower private information makes the more-informed worker to rely more on public information, which signals that θ is likely to be low. Then the size of the aggregate effort increases, which will increase θ^* at the equilibrium. Hence, as more-informed one loses its superiority in private information, the likelihood of success would surge since both types rely more on the public signal that serves as a coordination device.

The effect of λ on θ^* can be understood by considering Proposition 2. In the low-cost low-p environment, $x_e^* < x_i^*$, implying that less-informed worker tends to put more effort. With lower λ, numbers of workers putting efforts increase while that of non-working workers decreases, resulting in high θ^*.

We can then characterize the extent to which λ and μ affect the threshold levels of private signals as presented in Corollary 3.

Corollary 3 (Comparative statics on x_i^* and x_e^*: Effect of λ and μ)**.** *Let $c \in [0, \frac{1}{2})$ (resp. $c \in [\frac{1}{2}, 1]$) and $p \leq \theta^*$ (resp. $p > \theta^*$) and there exists a unique threshold equilibrium as in Proposition 1. Then the threshold level of private signals, x_i^* and x_e^*, becomes higher (resp. lower) when (i) measure of more-informed workers becomes lower (resp. higher) and/or (ii) information gap between the two type becomes lower (resp. higher). Formally,*

$$\frac{\partial x_j^*}{\partial \lambda} < 0 \; (resp. > 0) \quad and \quad \frac{\partial x_j^*}{\partial (1+\mu)} < 0 \; (resp. > 0) \tag{21}$$

where $j = \{i, e\}$.

Proof. From equation (11) and (12), it is easy to obtain

$$\frac{\partial x_j^*}{\partial \lambda} = \left(1 + \frac{\alpha_p}{\alpha_x^j}\right) \frac{\partial \theta^*}{\partial \lambda} \tag{22}$$

where $j = \{i, e\}$.

Hence sign of the above term depends on the sign of $\frac{\partial\theta^*}{\partial\lambda}$, which follows Proposition 3. The proof for μ is similar, except that we have more terms that depend on c and p.

□

This is again the natural consequence of Proposition 3: With changes in θ^*, the threshold levels for private signals would subsequently change in the same direction. This implies that less workers will put their efforts when overall precision of the private information in the economy becomes greater.

The above finding provides a policy implication to keep the status-quo in addition to a usual policy to provide less public information in the low-cost low-p environment (Proposition A2). As the success probability becomes higher when informational gap between the agents and/or the numbers of the expert type is small in the low-cost low-p environment, the manager (policymaker) might increase the success probability by narrowing the information gap or lowering the number of more-informed workers. For instance, this could be achieved by providing less sophisticated information to the public so that greater fraction of workers can utilize it and hence resulting in smaller information gap. In the high-cost high-p environment, on the contrary, the manager might impose the opposite policy. This discussion leads to the following Corollary on the optimal information allocation problem, which is faced by the manager.

Corollary 4 (Optimal Information Allocation)**.** *Let* $c \in [0, \frac{1}{2})$ *(resp.* $c \in [\frac{1}{2}, 1]$*) and* $p \leq \theta^*$ *(resp.* $p > \theta^*$*), and the sufficient condition for unique threshold equilibrium holds (Proposition 1). Then the manager chooses* $\lambda = 0$ *and* $\mu = 0$ *(resp.* $\lambda = 1$ *and* $\mu = \infty$*).*

Proof. This directly follows from Proposition 3 and Corollary 3. □

Consider the low-cost low-p environment. As is shown in Corollary 3, more workers will work hard as λ and μ becomes lower. From the perspective of the manager, this implies that her optimal information allocation decision should be to make all the workers identical with respect to information. Since the opposite holds in the high-cost high-p environment, what is suggested to the manager is to choose an *extreme* information structure in some stylized parameters' region: The manager offers any workers no further private information in the low-cost low-p environment, while she offers every player perfectly precise private information in the high-cost high-p environment.[9] Such extreme information structure is also observed in Inostroza and Pavan (2022) and Morris, Oyama, and Takahashi (2022). They prove that the optimality and existence of binary signal structure which makes all players attack or not when the information designer assumes the worst scenario that agents choose a suboptimal action for the designer. While the manager in this paper does not assume the worst scenario, we show that extreme information structure can still be optimal. In this regard, our finding contributes to the literature on information design by proving that the extreme information structure robustly holds in global games. This finding is also important in the sense that it complements Moriya and Yamashita (2020): They show that providing full information to workers is always suboptimal. However, Corollary 4 implies that there is an equilibrium in which full information can also be optimal when both effort cost and public signal are substantially high.

5 Conclusion

In this paper, we extend the typical regime attack game to exhibit ex-ante information heterogeneity across players and characterize optimal information structure encouraging more players to attack the regime. Importantly, we prove that there exists a unique threshold equilibrium in this economy under a sufficient condition different from what was previously provided (Bannier (2005)). This uniqueness result allows us to do comparative statics in the context of team production: In particular, we analyze the extent to which ex-ante information heterogeneity (numbers of more-informed workers and information gap between the two types) influences the equilibrium result.

Our setting provides a tool to understand the behavior of small investors in the market, who possess less precise private information than professional traders. According to our model, informational superiority does not fix the position of the investor; depending on the market environment, small investors can be more or less active than professional ones. For instance, the rally of the stock price indices, including GameStop, in early 2021 were lead by the inexpert type, which can be interpreted as an attack to the regime in our setup. We also provide a tool to understand the optimal information allocation problem in the context of team production

[9]One might further consider the case in which the manager can also alter the cost from high- to low value or low- to high value in addition to controlling the precision of private signals. However, as is well-analyzed in Angeletos, Hellwig, and Pavan (2006), this setup yields multiple equilibria, which makes it difficult to analyze the equilibrium property.

model. We leave more applications of our model to different and interesting circumstances as future works.

References

Angeletos, G.-M., C. Hellwig, and A. Pavan (2006): "Signaling in a Global Game: Coordination and Policy Traps," *Journal of Political Economy*, 114(3), 452–484.

Angeletos, G. M., and I. Werning (2006): "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, 96(5), 1720–1736.

Bannier, C. E. (2005): "Big elephants in small ponds: Do large traders make financial markets more aggressive?," *Journal of Monetary Economics*, 52, 1517–1531.

Carlsson, H., and E. van Damme (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989–1018.

Challe, E., and E. Chrétien (2018): "Market Microstructure, Information Aggregation and Equilibrium Uniqueness in a Global Game," *European Economic Review*, 102, 82–99.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2004): "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders," *Review of Economic Studies*, 71(1), 87–113.

Inostroza, N., and A. Pavan (2022): "Adversarial Coodrination and Public Information Design," *Working Paper*.

James, J. G., and P. Lawler (2012): "Heterogeneous Information Quality; Strategic Complementarities and Optimal Policy Design," *Journal of Economic Behavior and Organization*, 83(3), 342–352.

Kennedy, P., and A. Prat (2019): "Where Do People Get Their News?," *Economic Policy*, 34(97), 5–47.

Metz, C. E. (2002): "Private and public information in self-fulfilling currency crises," *Journal of Economics*, 76(1), 65–85.

Moriya, F., and T. Yamashita (2020): "Asymmetric-information allocation to avoid coordination failure," *Journal of Economics & Management Strategy*, 29, 173–186.

Morris, S., D. Oyama, and S. Takahashi (2022): “Implementation via Information Design using Global Games,” *Working Paper.*

Morris, S., and H. S. Shin (1998): “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks,” *American Economic Review*, 88(3), 587–597.

——— (2003): “Global Games: Theory and Applications,” in *Advances in Economics and Econometrics*, pp. 56–114. Cambridge University Press.

Róndina, G., and M. Shim (2021): “Financial Prices and Equilibrium Uniqueness in Global Games Models of Crises,” *Working Paper.*

Sims, C. A. (2003): “Implications of Rational Inattention,” *Journal of Monetary Economics*, 50, 665–690.

A Appendix. Additional Results

We first analyze the extent to which the probability of the regime collapse depends on some of the key parameters as in Metz (2002). Recall that $A(\theta^*) = \theta^*$ and hence θ^* denotes the size of an aggregate attack at the equilibrium. Hence, the probability of the regime collapse becomes higher when the threshold level for θ, θ^*, becomes higher. The following proposition summarizes the comparative statics on θ^* that holds regardless of the underlying parameter values.

Proposition A1 (Comparative Statics on θ^*: Effect of c and p)**.** *The regime collapses more easily when c or p becomes lower. Formally, $\frac{\partial\theta^*}{\partial c} < 0$ and $\frac{\partial\theta^*}{\partial p} < 0$.*

Proof. Equilibrium condition (13) implicitly determines θ^* and hence we can differentiate with respect to each parameters:

$$\frac{\partial\theta^*}{\partial c} = -\frac{(1-\lambda)\sqrt{\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}}\phi(\beta^i)\frac{1}{\phi(\Phi^{-1}(c))} + \lambda\sqrt{\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}}\phi(\beta^i)\frac{1}{\phi(\Phi^{-1}(c))}}{1-(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{\alpha_x^e}}\phi(\beta^e)} < 0 \tag{23}$$

$$\frac{\partial\theta^*}{\partial p} = -\frac{(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) + \lambda\frac{\alpha_p}{\sqrt{\alpha_x^e}}\phi(\beta^e)}{1-(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{\alpha_x^e}}\phi(\beta^e)} < 0 \tag{24}$$

where $\beta^i \equiv \frac{\alpha_p}{\sqrt{\alpha_x^i}}(\theta^* - p) - \sqrt{\frac{\alpha_x^i+\alpha_p}{\alpha_x^i}}\Phi^{-1}(c)$ and $\beta^e \equiv \frac{\alpha_p}{\sqrt{\alpha_x^e}}(\theta^* - p) - \sqrt{\frac{\alpha_x^e+\alpha_p}{\alpha_x^e}}\Phi^{-1}(c)$.

□

This is an intuitive result and generalizes Metz (2002) (Proposition 1 and 2): When it is not that costly to attack the regime, it would increase the chance of attacking the regime for all players, resulting in high θ^*. When the public signal is realized to be low, every player would suspect that the regime is more vulnerable to attack, and hence the size of the aggregate attack enlarges.

On the contrary, some of the key parameters do not have global effects. Rather, they do depend on the status of the economy whether it is a "low-cost low-p" environment or "high-cost high-p" environment. The next proposition presents the finding on the effect of changes in precision of public information (α_p) on the probability of regime collapse.

Proposition A2 (Comparative Statics on θ^*: Effect of α_p)**.** *Let $c \in [0, \frac{1}{2})$ (resp. $c \in [\frac{1}{2}, 1]$) and $p < \theta^*$ (resp. $p > \theta^*$) and there exists a unique threshold equilibrium as in Proposition 1. More precise public information increases (resp. lowers) the probability of the regime collapse. Formally, $\frac{\partial \theta^*}{\partial \alpha_p} > 0$ (resp. $\frac{\partial \theta^*}{\partial \alpha_p} < 0$).*

Proof. Again, we differentiate the equilibrium condition (13) with respect to α_p:

$$\frac{\partial \theta^*}{\partial \alpha_p} = \frac{(1-\lambda)\frac{1}{\sqrt{\alpha_x^i}}\left(\theta^* - p - \frac{\Phi^{-1}(c)}{2\sqrt{\alpha_x^i+\alpha_p}}\right)\phi(\beta^i) + \lambda\frac{1}{\sqrt{(1+\mu)\alpha_x^i}}\left(\theta^* - p - \frac{\Phi^{-1}(c)}{2\sqrt{(1+\mu)\alpha_x^i+\alpha_p}}\right)\phi(\beta^e)}{1-(1-\lambda)\frac{\alpha_p}{\sqrt{\alpha_x^i}}\phi(\beta^i) - \lambda\frac{\alpha_p}{\sqrt{\alpha_x^e}}\phi(\beta^e)} \tag{25}$$

It is easy to verify that the above expression is positive when $p \leq \theta^*$ and $c \in [0, \frac{1}{2})$ and is negative when the opposite holds.

□

Again, this finding is a generalization of Metz (2002) (Proposition 4): Whether more precise public information leads to greater probability of the regime collapse or not crucially depends on (1) cost of attacking the regime and (2) the realized public signal.