

Econ 1335: Advanced Topics in Macroeconomics

Final Exam

Shanghai University of Finance and Economics - Fall 2015

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You have exactly **24** hours for this exam. The solutions are due **6pm, Tuesday December 8**. Please type your own answer, preferably using tex, and send it to my email address (audrb917@gmail.com) until 6pm. I will not consider submitted solutions that are written by hand or are late. **DO NOT DISCUSS WITH YOUR CLASSMATES.**

Problem 1: Permanent Income Hypothesis under Incomplete Information

Consider a representative household who solves the following problem:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha_0 + \alpha_1 c_t - \frac{\alpha_2}{2} c_t^2 \right) \quad (1)$$

subject to

$$c_t + a_{t+1} = y_t + (1+r)a_t$$

where $\beta, a_0, \alpha_0, \alpha_1, \alpha_2 > 0$ are given.

1. Derive the lifetime budget constraint.
2. Suppose that income process y_t follows the AR (1) process:

$$y_t = \rho y_{t-1} + \varepsilon_t \quad (2)$$

where $\rho \in (0, 1)$ and $\varepsilon_t \sim \mathbb{N}(0, \sigma^2)$.

Assuming that the agent is able to observe y_t at t . Hence, she know the history of the income profile $y^t = \{y_0, y_1, \dots, y_t\}$ but need to form an (rational) expectations about the future income process. Derive the optimal consumption plan.

3. Now assume instead that the agent cannot directly observe y_t at t . Instead, she observes a signal z_t ;

$$z_t = y_t + \frac{1}{\sqrt{\alpha_z}} \varepsilon_t^z \quad (3)$$

where $\varepsilon_t^z \perp \varepsilon_t \perp y_t$ and $\varepsilon_t^z \sim \mathbb{N}(0, 1)$.

Hence, she now has an incomplete information about the income process y_t , but she knows the realization of income upto $t - 1$. i.e. her information set is $\{y^{t-1}, z_t\}$. Derive the optimal consumption plan under the above information structure.

4. The signal z_t includes the information provided by the government. i.e. the precision of the signal α_z can be chosen by the government. Suppose that the government aims to maximize the ex-ante utility of the consumer, equation (1). What is the optimal choice of α_z by the government? (Hint: substitute your solution obtained from 3 to the objective function (1))

Exercise 2: Effect of Better Public Signal in Morris and Shin (1998) Economy

In the class, we derive the following equilibrium condition for the threshold $\theta^*(p)$:

$$\theta^*(p) = \Phi \left[\sqrt{\frac{\alpha_x}{\alpha_x + \alpha_p}} \Phi^{-1}(1 - c) - \frac{\alpha_p}{\alpha_x + \alpha_p} p \right] \quad (4)$$

where $p = \theta + \frac{1}{\sqrt{\alpha_p}} \varepsilon$ and Φ is the CDF of the standard normal distribution.

1. Analyze the effect of more precise public information on the threshold level, $\theta^*(p)$. For simplicity of the analysis, assume that ε is positive. Is better public signal able to lower the possibility of regime collapse? Go as far as you can and also provide your economic reasoning.

2. Information technology has developed over time. The development of technology can possibly affect the informativeness of private signal. In particular, we can interpret this situation as α_x becoming higher over time. Then what is the consequence of the better information technology on the threshold $\theta^*(p)$? Assume again $\varepsilon > 0$. Discuss if the better technology can be associated with more frequent financial crises in Morris and Shin economy. In describing your answer, go far as you can and provide your economic reasoning.