Variable Effort, Business Cycles, and Economic Welfare*

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Abstract

Recent literature has shown that productivity-driven business cycles are beneficial to consumers in the context of market clearing real business cycle model. In this paper, we argue that this notion of welfare-improving business cycles does not jointly satisfy (1) the balanced growth path property and (2) micro evidence on the Frisch labor supply elasticity. We then show that once "variable effort," a channel that plays an important role in the business cycle frequency but has been ignored by the previous literature on the welfare cost of business cycles, is introduced into the model, welfare-improving business cycles can be achieved in relatively plausible parameter regions.

JEL classification: E24, E30, E32

Keywords: Variable Effort, Real Business Cycle Model, Balanced Growth Path, Welfare Cost of Business Cycles

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1 Introduction

While it has been widely believed that business cycles are costly (Lucas (1987); and Barlevy (2004) for instance), two recent papers, Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014), have documented an interesting welfare property of the class of real business cycle (henceforth RBC) model with (1) variable production factors and (2) market clearing environment: Productivity-driven business cycles are beneficial to consumers under reasonable parameter values since consumers can exploit the economic fluctuations with endogenous changes in labor supply and capital accumulation.[1]

Consider the following restrictions on the utility function: (1) the balanced growth path property when preference exhibits separability between consumption and leisure ($\log c + v(l)$ with $v'(l) > 0$ and $v''(l) < 0$ (King, Plosser, and Rebelo (1988)))[2] and (2) Frisch labor supply elasticity[3] consistent with micro evidence (Chetty, Guren, Manoli, and Weber (2012)). Once these two restrictions are jointly considered, the fluctuations are costly. For instance, the canonical RBC model requires the Frisch elasticity given log utility on consumption be four, which is even greater than the "macro" Frisch elasticity (one (Chang and Kim (2006))).

We aim to provide a new perspective on this issue by introducing "variable effort" into an otherwise standard RBC model. The relevance of variable effort to business cycle models has been well recognized (Burnside, Eichenbaum, and Rebelo (1993); Bils and Cho (1994); and Lewis, Villa, and Wolters (2019)). In the sense that Bils and Chang (2003) analyze the welfare implication of sticky wages under a monetary shock, their work is related to this paper. However, our work deviates from theirs by analyzing the welfare cost of business cycles with the RBC model under a technology shock.

We find that introduction of effort substantially enlarges the parameter region in which business cycles are beneficial to consumers. If we take the Frisch labor supply elasticity and the effort elasticity to match the empirical evidence (Chang and Kim (2006) and Bils and Cho (1994)), the constant relative risk aversion (CRRA henceforth) parameter to obtain welfare-improving business cycles is about one, which ensures the balanced growth property. The required CRRA parameter in the standard RBC model, in contrast, is about 0.7, which is far below one. In Section 2 we inspect the mechanism behind

[1] Heiberger and Maußner (2020) show that Cho, Cooley, and Kim (2015)'s finding is not robust to an alternative perturbation solution with log-level specification of the exogenous shock. We instead use level specification of the exogenous shock, and hence their critique does not apply.

[2] While Lester, Pries, and Sims (2014) further consider a non-separable utility function, the model still requires a high Frisch elasticity to obtain welfare-improving business cycles (Table 2 of their paper).

[3] Throughout this paper, We use Frisch elasticity instead of Frisch labor supply elasticity for brevity.

this finding: procyclical effort, consistent with the data (Lewis and Dijcke (2019)), enhances the ability of the consumer to further exploit fluctuations. As a result, the parameter space to obtain welfare-improving business cycles can expand. Section 3 and 4 evaluate this finding in the dynamic setup. We further show that the effect of the effort channel on the welfare remains substantial in the extended model with investment adjustment cost. Hence, this paper contributes to the literature on the welfare cost of business cycles by shedding new light on a potentially important channel to be considered in future studies.

2 Inspecting the Mechanism

In this section, we analyze a static version of the model in Bils and Cho (1994) to study the underlying mechanism behind the less tight conditions on parameters needed to achieve welfare-improving business cycles.

The social planner solves the following problem:

$$V(C_t^*, H_t^*, \phi_t^*) = \max_{C_t, H_t, \phi_t} \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{H_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - H_t \frac{\phi_t^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} \tag{2.1}$$

subject to

$$C_t = Z_t H_t \phi_t$$

where C_t, H_t, ϕ_t, and Z_t are consumption, hours worked, effort per hours worked, and the exogenous technology shock, respectively. $\gamma > 0$ is the CRRA parameter, $\psi > 0$ is the Frisch labor supply elasticity, and $\tau > 0$ is the effort elasticity with respect to the wage rate.

Static structure of the model allows us to derive the first order conditions and the associated value function, $V(C_t^*, H_t^*, \phi_t^*)$, in the closed-form.[4] The environment becomes volatile with a mean-preserving spread of Z_t. For the more volatile environment to be preferred by the consumer to less volatile one, we need the value function to be a convex function of Z_t. i.e. $\frac{\partial^2 V(C_t^*, H_t^*, \phi_t^*)}{\partial Z_t^2} > 0$. The following proposition then summarizes our argument.

Proposition 1 (Generalized Lester, Pries, and Sims (2014))**.** *Consider the economy described above*

[4]For relevant derivations, see Appendix.

and assume that $\gamma \in (0, \frac{1}{2})$*. The consumer of this economy prefers the more volatile economy if and only if the Frisch labor supply elasticity is higher than a threshold level. Formally, the condition for welfare-improving business cycles is* $\psi > \psi^*$ *where* $\psi^* = \bar{\psi} - \frac{1-\gamma}{1-2\gamma}\frac{\tau}{1+\tau}$ *and* $\bar{\psi} = \frac{\gamma}{1-2\gamma}$*.*

The threshold level for welfare-improving business cycles without effort is denoted by $\bar{\psi}$, the level obtained by Lester, Pries, and Sims (2014). Hence, the above condition is achieved with a lower Frisch elasticity, ψ, given the same CRRA parameter whenever $\tau > 0$ since $\frac{1-\gamma}{1-2\gamma} > 0$ under our assumption. This is because the consumer has more production factors with which to exploit the business cycle.

Figure 2.1 visualizes the above proposition: We plot the indifference curves between the fluctuating- and the non-fluctuating economy: The area below (resp. above) the indifference curve is the region in which household prefers the more (resp. less) volatile economy. The line labeled "No effort" is the indifference curve of Lester, Pries, and Sims (2014); as the consumer is further allowed to adjust her effort ($\tau > 0$), the indifference curve shifts up.

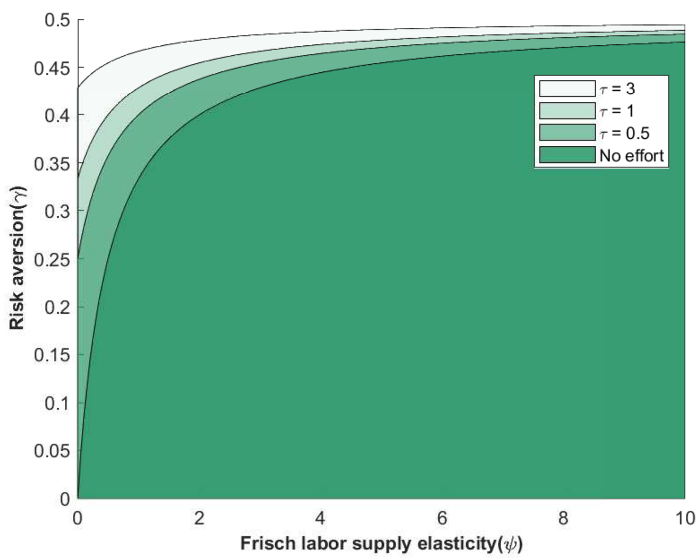

Figure 2.1: Indifference Curve in the Static Economy
Note: Shaded region is the area in which business cycles are welfare-improving.

3 The General RBC Model with Endogenous Effort

We now consider the dynamic model, a version of Bils and Cho (1994). The social planner now faces the following problem:

$$\max_{C_t,H_t,\phi_t,K_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - B_H \frac{H_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - B_\phi H_t \frac{\phi_t^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} \right] \tag{3.1}$$

subject to

$$C_t + K_{t+1} = Z_t K_t^{1-\alpha}(H_t\phi_t)^\alpha + (1-\delta)K_t \tag{3.2}$$

$$Z_{t+1} = (1-\rho) + \rho Z_t + \sigma\epsilon_t,\ \epsilon_t \sim \mathcal{N}(0,1) \tag{3.3}$$

where $\beta \in (0,1)$ is the discount factor, $\delta \in (0,1)$ and $\alpha \in (0,1)$ are the depreciation rate and the labor share, respectively. Following Lester, Pries, and Sims (2014), the productivity shock (Z_t) follows a stationary AR (1) process ($\rho \in (0,1)$). The shock specified in level ensures $\mathbb{E}(Z_t) = 1$.

We apply the perturbation method (Schmitt-Grohé and Uribe (2004)) to obtain the second-order approximation of the equilibrium conditions. We further define and compute the lifetime value of living in the steady-state economy (V^s) and that in the fluctuating economy ($V^f(\mathbf{C}_t, \mathbf{H}_t, \Phi_t)$) as follows:

$$V^s \equiv \frac{1}{1-\beta} U(C,H,\phi) \tag{3.4}$$

and

$$V^f(\mathbf{C}_t, \mathbf{H}_t, \Phi_t) \equiv U(C_t, H_t, \phi_t) + \beta \mathbb{E}_t V^f(\mathbf{C}_{t+1}, \mathbf{H}_{t+1}, \Phi_{t+1}) \tag{3.5}$$

where $\mathbf{X}_t \equiv \{X_\tau\}_{\tau=t}^{\infty}$ for any variable X_t and $\Phi_t \equiv \{\phi_\tau\}_{\tau=t}^{\infty}$.

Define λ as the compensation variation of consumption that the household in the fluctuating economy should receive at each period to satisfy the following relationship:

$$V^s = V^f((1+\lambda)\mathbf{C}_t, \mathbf{H}_t, \Phi_t) \tag{3.6}$$

If $\lambda = 0$, no cost is associated with living in the volatile economy. If $\lambda > 0$ (resp. $\lambda < 0$), however, the consumer becomes indifferent to the two economies when consumption flows are adjusted upward (resp. downward) by the factor λ, implying that business cycles are costly (resp. beneficial).[5]

As computational exercises, we choose parameter values in line with the literature: $\alpha = 0.66$, $\beta = 0.995$, and $\delta = 0.02$. $B_H > 0$ and $B_\phi > 0$ are chosen to ensure hours worked to be one-third at the steady-state. Parameters for shock process are normalized to be $\rho = 0.95$ and $\sigma = 0.01$.

4 Role of Effort: Evaluations

In this section, we evaluate our argument by varying key parameters.

4.1 Main Results Figure 4.1 plots the main result from our model: Each line indicates the indifference curve where consumers are indifferent between the steady-state economy and the volatile economy ($\lambda = 0$) and the area below (resp. above) the curve is the parameter region that supports welfare-improving (resp. welfare-detrimental) business cycles. The benchmark case is denoted by the line labeled "No effort": We plot the indifference curve in the absence of the effort, which is equivalent to Lester, Pries, and Sims (2014). Once we consider $\gamma = 1$ to achieve the balanced growth path and $\psi \in (0, 1)$ to be consistent with micro evidence, fluctuations are not preferred by the consumers. In particular, the minimum value of ψ is greater than four when $\gamma = 1$ for the fluctuations to be better-off, which is even higher than one, the macro Frisch elasticity (Chang and Kim (2006)). Hence, the previous finding that business cycles are welfare-improving does not seem to hold when we take the two empirical regularities into account.

As we allow endogenous effort with a positive τ, the region in which business cycles are welfare-improving expands and the area enlarges further as τ becomes higher. If $\psi = 1$, given $\tau = 1$ is required to match empirical regularities (Bils and Cho (1994)), the highest indifference curve yields the corresponding CRRA parameter to be about one, which supports the balanced growth path. If we instead take $\tau = 1/3$, the value estimated by Lewis, Villa, and Wolters (2019), the minimum value for ψ needed to obtain welfare-improving business cycles is about 2 when $\gamma = 1$, which is about half of the value we need without the effort. Hence, our finding implies that effort, which has been neglected by the previous literature, might play an important role in determining the welfare cost of business cycles.

[5] In the Appendix, we describe in detail how to compute λ.

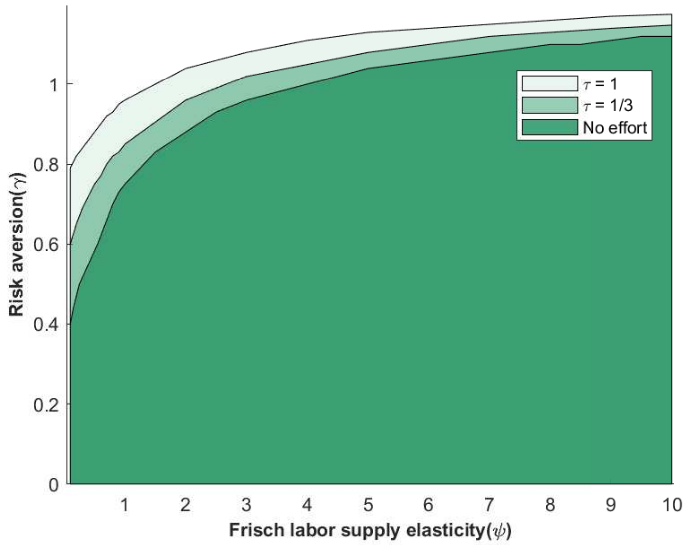

Figure 4.1: Parameter Restrictions for Welfare-Improving Business Cycles
Note: Shaded region is the area in which business cycles are welfare-improving.

4.2 Extension In this subsection, we discuss whether effort still has meaningful effects on welfare, when there exist an factor that lowers the consumer's ability to exploit the fluctuations (adjustment cost in investment). In particular, adjustment cost in investment takes the following form:

$$K_{t+1} = (1-\delta)K_t + \left(\frac{a_1}{1-\theta} \left(\frac{I_t}{K_t} \right)^{1-\theta} + a_2 \right) K_t \tag{4.1}$$

with $a_1 = \delta^\theta$ and $a_2 = -\frac{\delta\theta}{1-\theta}$. Following Boldrin, Christiano, and Fisher (2001) and Francis and Ramey (2005), we set $\theta = 4$.

We compute λ and plot the indifference curves in Figure 4.2: The effects of the effort channel on the welfare cost are still substantial; the indifference curves shift up as τ increases, which is consistent with the benchmark case. Thus, we can conclude that the role of effort in the RBC model in which economic fluctuations are productivity-driven is not minor. This implies that ignoring the unobserved factors might yield an upward bias in the welfare cost of business cycles.

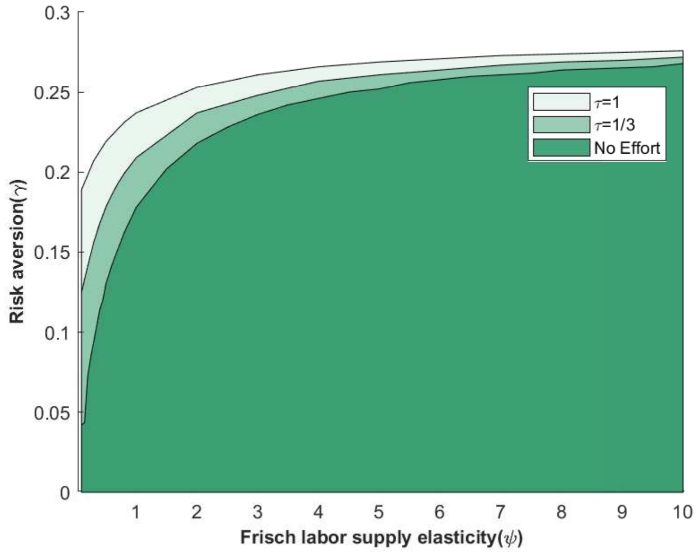

Figure 4.2: Role of Adjustment Cost in Investment
Note: Shaded region is the area in which business cycles are welfare-improving.

5 Concluding Remark

This paper introduces effort into the standard RBC model and evaluates the welfare consequences of such a change. We show that the parameter space required in order to obtain welfare-improving economic fluctuations substantially expands as the consumers are allowed to vary their effort, the unobserved factor. Our finding implies that previous findings on the welfare cost might be substantially biased upward as the unobserved factors, which are important in obtaining realistic features of business cycles, are not considered.

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Appendix. Derivations

A. Proof of Proposition 1 First-order conditions of the social planner problem are given as follows.

$$H_t^{\frac{1}{\psi}} + \frac{\phi_t^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} = C_t^{-\gamma} Z_t \phi_t \quad (.1)$$

$$H_t \phi_t^{\frac{1}{\tau}} = C_t^{-\gamma} Z_t H_t \quad (.2)$$

After some algebra, we can obtain closed-form solutions for aggregate variables as follows.

$$C_t = (\frac{1}{1+\tau})^{\frac{\psi}{1-\gamma}[1-\frac{\gamma(1+\frac{1}{\tau})(1+\psi)}{A}]} Z_t^{\frac{(1+\frac{1}{\tau})(1+\psi)}{A}} \quad (.3)$$

$$H_t = (\frac{1}{1+\tau})^{\psi[1-\frac{\gamma(1+\frac{1}{\tau})\psi}{A}]} Z_t^{\frac{(1+\frac{1}{\tau})(1-\gamma)\psi}{A}} \quad (.4)$$

$$\phi_t = (\frac{1}{1+\tau})^{-\frac{\gamma\psi}{A}} Z_t^{\frac{1-\gamma}{A}} \quad (.5)$$

where

$$A = (1+\frac{1}{\tau})\gamma\psi + (\frac{1}{\tau}+\gamma) \quad (.6)$$

Substituting the solutions into the objective function, we can obtain the value function as a function of Z_t:

$$V^*(Z_t) = \left(\frac{1}{1-\gamma} - \frac{1}{(1+\frac{1}{\psi})(1+\tau)} - \frac{\tau}{1+\tau}\right)(\frac{1}{1+\tau})^{\psi[1-\frac{\gamma(1+\frac{1}{\tau})(1+\psi)}{A}]} Z_t^{\frac{(1-\gamma)(1+\frac{1}{\tau})(1+\psi)}{A}} - \frac{1}{1-\gamma} \quad (.7)$$

The first and second derivatives of the value function are:

$$V^{*'}(Z_t) = (\frac{1}{1+\tau})^{\psi[1-\frac{\gamma(1+\frac{1}{\tau})(1+\psi)}{A}]} Z_t^{\frac{(1-\gamma)(1+\frac{1}{\tau})(1+\psi)}{A}-1} > 0 \quad (.8)$$

$$V^{*''}(Z_t) = (\frac{1}{1+\tau})^{\psi[1-\frac{\gamma(1+\frac{1}{\tau})(1+\psi)}{A}]} \left(\frac{(1-\gamma)(1+\frac{1}{\tau})(1+\frac{1}{\psi})}{(1+\frac{1}{\tau})\gamma + \frac{1}{\psi}(\frac{1}{\tau}+\gamma)} - 1\right) Z_t^{\frac{(1-\gamma)(1+\frac{1}{\tau})(1+\psi)}{A}-2} \quad (.9)$$

The convexity of the value function with respect to Z_t requires the second derivative to be greater than zero. One can obtain the following condition:

$$\frac{(1-\gamma)(1+\frac{1}{\tau})(1+\frac{1}{\psi})}{(1+\frac{1}{\tau})\gamma+\frac{1}{\psi}(\frac{1}{\tau}+\gamma)} \quad > \quad 1 \tag{.10}$$

Under $\gamma \in (0, \frac{1}{2})$, the consumer prefers the more volatility economy if and only if:

$$\psi \quad > \quad \frac{\gamma}{1-2\gamma} - \frac{1-\gamma}{1-2\gamma}\frac{\tau}{1+\tau} \tag{.11}$$

B. Computing the Compensation Variation of Consumption First, the lifetime value of living in the fluctuating economy is:

$$V^f(C_t, H_t, \Phi_t) \quad = \quad \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \left[\frac{C_{t+j}^{1-\gamma}-1}{1-\gamma} - B\frac{H_{t+j}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - BH_{t+j}\frac{\phi_{t+j}^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} \right] \tag{.12}$$

The value function can be divided into two parts:

$$V^f \quad = \quad V^{f,C} + V^{f,H} \tag{.13}$$

$$V^{f,C} \quad = \quad E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[\frac{C_{t+j}^{1-\gamma}-1}{1-\gamma} \right] \tag{.14}$$

$$V^{f,H} \quad = \quad -E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[B\frac{H_{t+j}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} + BH_{t+j}\frac{\phi_{t+j}^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} \right] \tag{.15}$$

The welfare cost of business cycles, measured by λ, is defined as follows:

$$V^s \quad = \quad V^f((1+\lambda)C_t, H_t, \Phi_t) \tag{.16}$$

$$= \quad E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[\frac{((1+\lambda)C_{t+j})^{1-\gamma}-1}{1-\gamma} - B\frac{H_{t+j}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - BH_{t+j}\frac{\phi_{t+j}^{1+\frac{1}{\tau}}}{1+\frac{1}{\tau}} \right] \tag{.17}$$

$$= \quad (1+\lambda)^{1-\gamma}\left(V^{f,C} + \frac{1}{(1-\gamma)(1-\beta)}\right) + V^{f,H} - \frac{1}{(1-\gamma)(1-\beta)} \tag{.18}$$

We can obtain the expression for λ:

$$\lambda \quad = \quad \left[\frac{V^s - V^{f,H} + \frac{1}{(1-\gamma)(1-\beta)}}{V^{f,C} + \frac{1}{(1-\gamma)(1-\beta)}} \right]^{\frac{1}{1-\gamma}} - 1 \tag{.19}$$