Frequency-Specific Effects of Macroprudential Policies∗

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Abstract

Are macroprudential policies effective tools for stabilization and for welfare improvement? The answer to this question crucially depends on the frequency and the sector that we consider. Using a financial-sector augmented New Keynesian DSGE model, we find that a set of macroprudential policies that are commonly used both in theory and in practice have different frequency-specific effects on the economy: when a countercyclical capital requirement is implemented, loan volatility is reduced while inflation rate volatility is amplified at all frequencies. In contrast, when the Taylor rule is extended to respond to loan growth, loan is stabilized only at the relatively high frequencies while output volatility increases at all frequencies. Lastly, at equilibrium, a loan-to-value (LTV) ratio regulation does not seem to have much impact on our model economy. More aggressive policies worsen such problems. Hence, our findings unveil design limits of well known macroprudential policies; they can be effective at the targeted sectors and frequencies while having significant perverse effects on other sectors and frequencies. We further analyze spectral welfare costs and show that welfare analysis without considering frequency-specific effects might be misleading.

JEL classification: E32, E44, E58

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1 INTRODUCTION

The worldwide recession from the Global Financial Crisis was different from other recessions in the history since the Great Depression, because it was initiated from the collapse of financial market. A number of researches have investigated the cause of the financial disruption. Among them, Borio (2012) and Drehmann, Borio, and Tsatsaronis (2012) propose an idea of financial cycles which drive the movements of financial markets, independent of the traditional business cycle. They define financial cycles as “the self-reinforcing interactions between perceptions of value and risk, risk-taking and financing constraints which translate into financial booms and busts.” Not only financial cycles are shown to have much lower frequencies (8 to 32 years per cycle) than usual business cycles have (1.5 years to 8 years per cycle), the infrequent downturns of financial cycles, compared to business cycles, can initiate financial crises. They warn the possible divergence between business and financial cycles and emphasize the role of the monetary policy to take a balanced approach when stabilizing both cycles with one policy tool. For example, they suggest the monetary authority should be ready to tighten whenever financial imbalances show signs of building up, even if inflation appears to be under control in the near term.

Meanwhile, other researchers (Bernanke (2013), Yellen (2014)) point out the limitation of the monetary policy tool on targeting multiple policy objectives. They rather propose to use additional policy tools, for instance macroprudential policies. The macroprudential policies are designed to strengthen the resilience of the financial system against potential economic downturns and to actively limit the build-up of financial risks (BIS (2010)). Hence, the policy objective of macroprudential policies is different from that of the monetary policy that is designed to stabilize output and inflation.

This debate brings our attention to the proper method for policy evaluation. The traditional approach limits its focus to the policy effects only at the business cycle frequency. This approach, however, may not be valid for macroprudential policies considering financial cycles move at lower frequencies than the business cycle frequency. In other words, we should consider the power shifts between low frequency and high frequency resulting from a policy. Thus, this paper suggests two new approaches to this literature; First, we analyze “frequency-specific” effects of macroprudential policies, following Brock, Durlauf, and Rondina (2008). That is, we compute the variances of the key macro variables at different frequencies and compare them with ones under different macroprudential policies in order to detect frequency-specific effects. On the other hand, we also conduct a “spectral welfare analysis”,

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1The notion that financial market matters for economic cycles was already pointed out by earlier literature such as Wicksell (1936). We appreciate an anonymous referee for pointing this out.

2See BIS (2014) for further discussion.
following Otrok (2001), by comparing the frequency-specific welfare under different policies.

In doing so, we develop a New Keynesian DSGE model with a financial sector by extending models of Iacoviello (2014) and Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Agents are subject to borrowing constraints so that the shocks are amplified and propagated through balance sheet channels. Following Iacoviello (2014), we introduce two classes of shocks: non-financial and financial shocks. Non-financial shocks are common shocks in the New Keynesian literature, including aggregate TFP shock, investment-specific technology shock, aggregate demand shock, and monetary policy shock. Financial shocks include default shocks that transfer wealth from savers to borrowers in the case of default, loan-to-value (henceforth LTV) shocks that change lenders’ subjective perception of the riskiness of collateral, and housing demand shocks.

We choose three kinds of macroprudential policy tools that are widely used in related literature and also in practice. The first policy is an extended Taylor rule that aims to stabilize not only inflation and GDP gap, but also loan growth. The second policy is the counter-cyclical capital requirement from the Basel III regulatory framework that requires banks to accumulate additional capital buffers in good times for the possible losses in bad times. The third policy is a time-varying LTV regulation that tightens the cap on LTV ratio when the growth rate of housing price is high.

Our strategy to evaluate and compare the performances of different policies is described as follows. We first generate sets of simulated time series for each policy regime using our DSGE model. Following Otrok (2001), we then compute the spectral densities of key variables that provide variances at each frequency. By comparing frequency-specific variances across policy regimes, we can evaluate which policy is more effective in lowering variances at a given frequency. On the other hands, we compute the spectral welfare using band-pass filtered (Baxter and King (1999)) time series. By comparing frequency-specific welfare gains across policy regimes, we can evaluate which policy generates higher welfare gains.

Our quantitative experiments provide several notable findings. First, the extended Taylor rule has several perverse effects on the volatility mostly at lower frequencies. Output and inflation fluctuations are amplified across almost all frequencies, especially more at the low frequencies, while loan fluctuations are amplified only at low frequencies. That is, the extended Taylor rule stabilizes loan fluctuations only at high

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3Iacoviello (2014) highlights the importance of financial shocks which approximately accounted for two-thirds of the output drops during the Great Recession

4Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015) calls this type of shocks as risk perception shocks

5In this paper, we classify the extended monetary policy as a macroprudential policy for the purpose of distinguishing from typical monetary policy, although it is rather considered as an another version of monetary policy in practice.

6One might ask if other filtering methods can be used instead of the approximate (but optimal) filter suggested by Baxter and King (1999): As Estrella (2007) showed in his paper, the filtering method we choose performs well when the (generated) data is stationary, which is the case in our paper. Hence choice of Baxter-King filter is innocuous for our purpose.
frequencies. Even worse, welfare deteriorates across almost all frequencies. Second, the countercyclical capital requirement successfully achieves its policy objective by stabilizing loan fluctuations across all frequencies. However, it also incurs sizable costs of amplifying both output and inflation fluctuations across all frequencies. Interestingly, the adverse effects are more severe at high frequencies, which is in contrast to the case of the extended Taylor rule. In other words, this policy has a clear trade-off; it reduces the risk of financial crisis in the long-run, but at the cost of short-run variability.\footnote{We thank an anonymous referee for point this out.} Lastly, the time-varying LTV regulation does not make significant changes in frequency-specific variances and generates negligible welfare gains, compared to the benchmark economy. All findings above highlights the importance of frequency-specific macroprudential policy evaluation.

Our paper has two important contributions to the existing literature. First, this paper is the first attempt to analyze frequency-specific effects of macroprudential policies and to measure their frequency-specific welfare gains. While the frequency-specific effects of traditional monetary policy have been studied (Brock, Durlauf, and Rondina (2008) for instance), those of macroprudential policy has not yet been analyzed in this regard. Hence, our work is the first step to fill this gap in the literature. Second, this paper derives policy implications of widely-used macroprudential policies based on frequency-specific analysis, which is another new findings in this literature. In particular, we find a clear trade-off between the stabilization of real and financial markets under macroprudential policies.

The remainder of this paper is organized as follows. After taking a literature review in Section 2, we introduce the notion of frequency-specific effects of policy in Section 3. Our main DSGE model is then introduced in Section 4 with parameterization and preliminary analysis in Section 5. Key findings from our model are presented in Section 6 and Section 7. In Section 8, we conclude the paper.

\section{Literature Review}

Our paper is related to the literature that analyzes the role and the effectiveness of macroprudential policies. The conventional method to evaluate the performances of policies adopted by the previous literature is the welfare-cost approach; they compute the value of lifetime utility under different policy regimes and compare them using the compensational variation in terms of consumption.\footnote{See Footnote 3 of Schmitt-Grohé and Uribe (2006) for details of the approach.} One stream of literature focuses on measuring the welfare cost of policies. \textit{Van Den Heuvel (2008)} measures the welfare cost of bank capital requirements and shows that the regulations produce 0.1\% to 1\% loss in consumption in the U.S economy. \textit{Nguyen (2014)} applies a general equilibrium model to the dynamic banking sector to show that
the increase in bank capital requirements to the optimal level can produce welfare gains greater than 1% of lifetime consumption. Another stream of the literature tries to answer in which situations macroprudential policies are effective. Benes and Kumhof (2015) shows countercyclical bank capital requirements can create a precautionary motive to banks when the creditworthiness (or riskiness) of borrowers depreciates. Bailliu, Meh, and Zhang (2015) compare different sets of macroprudential regimes and find that welfare gains are largest when macroprudential policies react to financial shocks rather than productivity shocks. Lastly, a group of literature search for the optimal coordination between monetary and macroprudential policies. Quint and Rabanal (2014), and Suh (2012) find the optimal simple rule for monetary and macroprudential policies in the Euro Area and the U.S, respectively. Collard, Dellas, Diba, and Loisel (2014), and Angeloni and Faia (2013) support the view that implementing macroprudential policies along with monetary policies is important due to risk-taking behaviors by banks. On the other hand, Kiley and Sim (2014) find that an optimal monetary policy without macroprudential policies is sufficient to ensure efficiency even under the financial shock. Woodford (2012) suggests a modified inflation targeting framework to take account of financial stability concerns alongside traditional stabilization objectives. Our approach is unique since we focus on the frequency-specific effects of implementing macroprudential policies while the studies mentioned above do not consider the possible frequency-specific effects.

This paper is also related to the literature that applies the notion of design limits approach to macroeconomics. Brock, Durlauf, and Rondina (2008) and Brock, Durlauf, and Rondina (2013) show that unless the central bank implements a policy that is optimally designed to stabilize the economy at every frequency, the monetary policy can have unexpected negative effects at certain frequencies. The main difference between our paper and their series of papers is that we study the frequency-specific effects of macroprudential policies using a medium-scale New Keynesian model with financial frictions while they consider the small-scale New Keynesian model to derive the optimal monetary policy; to our best knowledge, our paper is the first to consider the possible design limits of macroprudential policies.

Literature that adopts alternative domain of data, especially frequency domain, to study macroeconomic properties of model and data is also closely related to our work. For instance, there is a large literature that applies a Wavelet analysis into the macro analysis; see Yogo (2008), Crowley and Hallett (2014), and Crowley and Hallett (2015) as examples. While the Wavelet analysis has an advantage over our approach using the band-pass filter (Baxter and King (1999)) in the sense that it can further capture time-varying volatility observed in the data (Yogo (2008)), we believe that the approach we take is appropriate for our purpose since we are not interested in the time-varying property of macro variables but instead try to unveil properties of the model-simulated data.
Our analysis is further related to the literature on spectral welfare analysis. To our best knowledge, Otrok (2001) is the only paper in this regard. While the methodology is identical, our paper is distinctive from his paper as he considered a simple partial equilibrium model of consumers while we consider a full-blown general equilibrium model.

3 Frequency-Specific Effects: a Brief Description

In this section, we introduce the primary concepts and steps taken in our main quantitative exercises. Suppose that we have a covariance-stationary macro variable \( \{Y_t\}_{t=-\infty}^{\infty} \), which is defined in the time domain. This variable oscillates over time so that it can be described as the weighted sum of periodic functions of the form cosine and sine functions. Then the spectral density function of the time series \( Y_t \), \( s_Y(\omega) \), can be described as follows.

\[
s_Y(\omega) = \frac{1}{2\pi} \left[ \sum_{k=-\infty}^{\infty} \lambda_k \exp(-i\omega k) \right] \tag{3.1}
\]

where \( \omega \in [0, \pi] \) is the frequency, \( \lambda_k \) is the \( k \)-th autocovariance of \( Y_t \), and \( i = \sqrt{-1} \). Then using De Moivere’s theorem, symmetry of autocovariance, and properties of cosine and sine functions, we can obtain the spectral density of the following form:

\[
s_Y(\omega) = \frac{1}{2\pi} \left[ \lambda_0 + 2 \sum_{k=1}^{\infty} \lambda_k \cos(\omega k) \right] \tag{3.2}
\]

The spectral density function provides the information on the extent to which a specific frequency contributes to the variance of the series. One good property of the spectral density is that the sum of all spectral density is equal to the variance of the variable. Formally,

\[
\mathbb{V}(Y_t|R_i) = \int_{-\pi}^{\pi} S_{Y_t|R_i}(\omega) d\omega \tag{3.3}
\]

where \( R_i \) is the policy regime \( i \) under which the time series \( Y_t \) is simulated and \( \mathbb{V} \) denotes variance. Hence, we can interpret spectral density at each frequency as the variance at each frequency.

In general, the main objective of a particular policy is to stabilize its target variables, such as output, inflation, or credit (or loans). In this sense, the spectral density can be used to evaluate policy effects.

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9We closely follow Chapter 6 of Hamilton (1994) for notation and formula.

10One might additionally consider cross-spectra to obtain further insights on policy implications, which is abstract from our analysis.
particularly the heterogeneous effects on the variance over different frequencies. It is important since a policy designed to stabilize variables at some specific frequencies, for instance a macroprudential policy designed to stabilize variables at low frequencies, may have an adverse effect at different frequencies. This property is known as the “design limits” of a policy. For instance, Brock, Durlauf, and Rondina (2008) considers an example that shows a policy that is supposed to minimize the overall variance of variables can increase the variance of the series at the high frequencies. The finding that the welfare gains (or losses) associated with different policies can vary across frequencies (Otrok (2001)) further raises the importance of our approach when it comes to studying the frequency-specific effects of macroprudential policies.

In this paper, we compute two frequency-specific effects of macroprudential policies; (1) frequency-specific variance and (2) frequency-specific welfare. The steps for the spectral variance analysis are as follows.

1. Under different policy regimes, simulate a model economy with all exogenous shocks.\footnote{We would like to point out that all exogenous shocks are included in our quantitative exercise; one might argue that only one exogenous shock should be introduced when simulating the model economy, productivity shock for instance, so that we can examine the effectiveness of a policy when the specific shock is particularly considered. We decide to introduce all exogenous shocks in the simulation by the following reasons. First, as is well-known and will be shown later, macroprudential policies do not work in the economy in which financial shocks are excluded. Hence, analysis with non-financial shocks only will be not useful for our purpose. Second, it is not clear which financial shock to be included in the simulation since contribution of financial shocks to the fluctuations of the aggregate economy is not clearly studied. For instance, historical variance decomposition of the model in Iacoviello (2014) (Figure 3 in his paper) shows that importance of a particular shock is different across variables. In addition, the macroprudential policy in the real world is not set to respond to a single shock; the policy works whenever the financial market fluctuates. Hence, it seems to be more natural to consider all exogenous shocks in the simulation.}

2. Compute the spectral density of the simulated series, take the average across simulations, and compare the density functions obtained under different policy regimes.

In particular, we will consider the effectiveness of macroprudential policies in comparison with the benchmark economy without any such policies. If the policy is effective at some frequencies, the spectral density will become lower than that from the benchmark economy. If it has an adverse effect, the spectral density will be higher than that from the benchmark economy. Therefore, if a policy is effective in stabilizing the economy as a whole, the spectral density of key macro variables, such as output, loan, consumption, and inflation rate, will be lower than the corresponding spectral density from the benchmark economy, which will be studied in details in Section 6.2.

The steps for the spectral welfare analysis are as follows,

1. Under different policy regimes, simulate a model economy with all exogenous shocks.
2. Apply the band pass filter by (Baxter and King (1999)) to the simulated series from Step 1 and obtain the filtered series.\textsuperscript{12}

3. Compute the spectral utility for each frequency band \(i\), and compute the average values of life-time utilities under different policy regimes.

Again, we compare the life-time utility of macroprudential policy regime with that of the benchmark economy, and compute the welfare gain (or loss). Details of the analysis will be discussed in Section 7.

4 The Model

The model introduced in this section is not particularly new by itself, but includes the key features of models with financial frictions. In particular, our model builds upon the model of Iacoviello (2014) (see Section 3); we incorporate New-Keynesian features to the original setup by Iacoviello (2014), similar to Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Our strategy to keep the model consistent with the previous literature is in order to minimize the model-specific factors that can possibly affects equilibrium behaviors.

The model economy consists of patient households, impatient households, entrepreneurs, retail banks, investment banks, retailers, and monetary authorities. Two financial intermediaries have different roles in the economy; retail banks lend funds to both impatient households and investment banks while they draw deposits from patient households. Investment banks, however, obtain fund only from retail banks and lend to entrepreneurs. In order to obtain the hump-shape behavior of macro variables, we have the habit formation and various adjustment costs introduced.

4.1 Households

Patient and impatient household are distributed on a unit interval. Patient households have a higher discount factor than impatient households, namely \(\beta_s > \beta_b > 0\). Hence, in equilibrium only patient households save deposits, while impatient households borrow.

4.1.1 Patient Households

The representative patient households (saver), denoted as \(s\), solve the following expected lifetime utility maximization problem by choosing optimal consumption \(C_s^t\), hours worked \(N_s^t\), housing \(H_s^t\), capital holding \(K_s^t\) and saving in the bank \(d_s^t\), taking prices as given:

\textsuperscript{12}A fruitful extension of our work would be considering time-specific and frequency-specific domains together as in Crowley and Hallett (2015). For instance, if one tries to extend our analysis to take “the Great Moderation” into account, consideration of the Wavelet analysis can be beneficial.

\textsuperscript{13}A model in Iacoviello (2005) also shares similar features.
\[
\max \ E_0 \sum_{t=0}^{\infty} \beta_s^t \left[ \varepsilon_t^c \ln \left( C_t^s - hC_{t-1}^s \right) + \varepsilon_t^c \varepsilon_t^h \nu_s^h \ln H_t^s - \nu_n^s \left( \frac{N_t^s}{1 + \phi} \right)^{1+\phi} \right]
\]

(4.1)

where \( \beta_s \in (0, 1) \) is a discount factor of patient households, \( h \in [0, 1] \) is a parameter that governs habit formation, \( \phi > 0 \) is the inverse Frisch elasticity, and \( \nu_s^h > 0 \) (resp. \( \nu_n^s > 0 \)) is a relative utility parameter from housing (resp. working). \( \varepsilon_t^c \) is an exogenous shock to preference for consumption and housing jointly, and \( \varepsilon_t^h \) is an exogenous shock to housing preference, one of the financial shocks in our model economy.

Budget constraints for the patient households are as follows.

\[
C_t^s + \frac{K_t^s}{\varepsilon_t^s} + p_t^H \left( H_t^s - H_{t-1}^s \right) + d_t + AC_{\phi,t} + AC_{K^s,t} = w_t^s N_t^s + r_t^K d_{t-1} + \left( r_t^K + 1 - \delta \right) K_{t-1}^s
\]

(4.2)

where the price of consumption goods is normalized to 1 \( (P_t \equiv 1) \), \( p_t^H \) is the real price of housing, \( r_t^K \) is a gross real interest rate from the deposit. Households rent capital to entrepreneurs at the rental rate \( r_t^K \), and receive the real wage \( w_t^s \) for labor supply. We define \( AC_{x,t} \), a convex real external adjustment cost for any variable \( x_t \), as follows:

\[
AC_{x,t} = \frac{\iota_x}{2} \left( x_t - x_{t-1} \right)^2
\]

(4.3)

where \( \iota_x \geq 0 \) is an adjustment cost parameter and \( x \) is the steady state level for \( x_t \).

4.1.2 Impatient Households

Similar to patient households, the representative impatient households (borrowers), denoted as \( b \), also choose the optimal level of consumption, \( C_t^b \), hours worked \( N_t^b \), and housing stock \( H_t^b \). As the discount factor of impatient households \( \beta_b \) is smaller than that of patient households, they prefer spending to saving and borrow from the banking sector to fund their spending. However, due to the financial friction, they cannot borrow as much as they want and lenders (retail banks) ask for collateral to secure loans. Since the only asset of impatient households is housing stock, the level of new bank loans depends on the discounted value of the house they own.

The problem of impatient households can be written as follows:

\[
\max \ E_0 \sum_{t=0}^{\infty} \beta_b^t \left[ \varepsilon_t^c \ln \left( C_t^b - hC_{t-1}^b \right) + \varepsilon_t^c \varepsilon_t^h \nu_s^h \ln H_t^b - \nu_n^b \left( \frac{N_t^b}{1 + \phi} \right)^{1+\phi} \right]
\]

(4.4)

Iacoviello (2014) interprets it as an aggregate spending shock.
subject to

\[ C^b_t + p_t^H [H_t^b - H_{t-1}^b] + r_t^b l_{t-1}^b + AC_{p_t} = u_t^b N_t^b + l_t^b + \varepsilon_t^b \]  \hspace{1cm} (4.5)

\[ l_t^b \leq \rho_b l_{t-1}^b + (1 - \rho_b) \left[ \gamma_t^{Hb} \frac{p_t^H}{p_{t+1}^H} H_t^b - \varepsilon_t^b \right] \]  \hspace{1cm} (4.6)

where \( l_t^b \) denotes bank loans, paying a gross interest rate \( r_t^b \), and \( u_t^b \) is the real wage rate. \( \varepsilon_t^b \geq 0 \) is a default shock for impatient households, which is another financial shock in our model; this can be interpreted as a wealth redistribution shock between borrowers and lenders since this shock increases the net wealth of impatient household (borrower) while it lowers the net wealth of retail banks (lenders).\(^{15}\)

Contrary to Iacoviello (2014), we assume that the default shock also negatively affects the borrowing constraint of the impatient households in order to capture the idea that the default on existing loans can limit the level of new loans.

Equation (4.6) is the borrowing constraint of impatient households, where \( \rho_b \in [0, 1] \) allows for the slow adjustment of bank loans over time.\(^{16}\) Borrowers cannot borrow more than the fraction of \( \gamma_t^{Hb} \) of the expected value of their housing stock. Here we assume that this constraint is imposed by government policies, so called LTV regulation, \( \gamma_t^{Hb} \), which is composed of two parts as follows:

\[ \gamma_t^{Hb} = \gamma_0^{Hb} \varepsilon_t^{lb} - \gamma_1^{Hb} \left( \frac{p_t^H}{p_{t+1}^H} - 1 \right) \]  \hspace{1cm} (4.7)

where the first term is a constant LTV regulation and the other term is a time-varying regulation. \( \gamma_0^{Hb} \) in the first term is the constant maximum LTV ratio cap, imposed by the policy, while \( \varepsilon_t^{lb} \) captures lenders’ subjective perceptions of the riskiness of the housing stock. We call this shock a risk perception shock (or LTV shock). The time-varying LTV regulation is one of the popular macroprudential tools to stabilize housing prices. If \( \gamma_1^{Hb} > 0 \), the LTV cap becomes tighter (lower) as housing prices increase. That is, it becomes more difficult for impatient households to borrow from banks with the collateral (housing) she/he holds.

### 4.2 Entrepreneurs

A continuum of entrepreneurs, denoted as \( e \), produces intermediate goods \( X_t^e \) and sell at a price of \( p_t^X \) in a competitive market. They hire workers and combine them with housing stock \( H_{t-1}^e \) and capital (both produced by themselves, \( K_t^e \), and rent from patient households, \( K_{t-1}^s \)).

The Cobb-Douglas production technology can be written as:

\(^{15}\)See Iacoviello (2014) for more discussions.
\(^{16}\)For the parameters to govern slow adjustment of loans, see Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015).
\[ X_t = \varepsilon_t^e \left( (K_{t-1}^e)^{1-\omega^k} (K_{t-1}^s)^{1-\omega^s} \right)^\alpha \left( H_{t-1}^e \right)^\nu \left( (N_t^s)^{\omega^s} (N_t^b)^{1-\omega^s} \right)^{(1-\alpha-\nu)} \]  

(4.8)

where \( \varepsilon_t^e \) is a neutral productivity shock and \((1 - \omega^k)\) and \(\omega^s\) are shares of patient households’ capital and labor, respectively.

Similarly to impatient households, entrepreneurs also face a borrowing constraint when making financing decisions. They solve the following problem:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \log \left( C_t^e - hC_{t-1}^e \right) 
\]

(4.9)

where \( \beta^e < \beta^s \) is assumed and \( C_t^e \) is the consumption of the entrepreneur. They are subject to the following constraints:

\[
C_t^e + \frac{K_t^e}{\varepsilon_t^e} + p_t^H[H_t^e - H_{t-1}^e] + w_t^sN_t^s + w_t^bN_t^b + r_t^eK_{t-1}^e + r_t^e l_{t-1}^e + AC^e_{K^e,t} + AC^e_{L^e,t} = p_t^H X_t + \frac{1 - \delta}{\varepsilon_t^e} K_{t-1}^e + l_t^e + \varepsilon_t^e \]

(4.10)

\[
l_t^e \leq \rho_e l_{t-1}^e + (1 - \rho_e) \left( \frac{p_{t+1}^H}{p_t^H} \gamma_{t+1}^H \frac{H_t^e}{\varepsilon_t^{e,t} - 1} \gamma_t^K K_t^e - \gamma_t^N (w_t^sN_t^s + w_t^bN_t^b) - \varepsilon_t^e \right) \]

(4.11)

Equation (4.10) is the budget constraint of the representative entrepreneur where \( r_t^e \) is a gross real interest rate on entrepreneur loans \( l_t^e \). Similarly to equation (4.6), \( \varepsilon_t^e \) is a default shock to entrepreneurs, which captures losses on banks and gains from entrepreneurs. Equation (4.11) is the borrowing constraint for entrepreneurs. Contrary to impatient households, entrepreneurs can use both housing and capital stocks as collateral when borrowing from banks. \( \gamma_t^H \) and \( \gamma_t^K \) are the ratio of housing and capital they can pledge, respectively. \( \gamma_t^H \) shares the same implication with the LTV regulation on impatient households’ housing stock.

\[
\gamma_t^H = \gamma_0^H \varepsilon_t^e - \gamma_1^H \left( \frac{p_t^H}{p_0^H} - 1 \right) \]

(4.12)

However, the amount of loan capacity decreases due to the working capital assumption. Similarly to Iacoviello (2014), Aoki, Benigno, and Kiyotaki (2009), and Neumeyer and Perri (2005), entrepreneurs
are assumed to pay for some portion of wage bills in advance, i.e. \( \gamma^N_{t+1} \in (0,1] \). We assume \( \gamma^K_{t+1} = \gamma^K_0 \varepsilon^K_t \) and \( \gamma^N_{t+1} = \gamma^N_0 \varepsilon^N_t \), where \( \varepsilon^K_t \) is a risk perception shock which is applied to housing stock, capital and wage at the same time.

### 4.3 Retail Banks

Retail banks, denoted as \( r \), collect deposits from patient households and lend to impatient households \( b_t \) and investment banks \( i_t \). As we assume \( \beta_r < \beta_s \), retail banks prefer debt to equity.\(^\text{17}\)

To prevent banks from high leverage, regulators impose a cap on banks’ capital ratio relative to the total asset. It is called as the minimum capital requirement.

The utility maximizing problem of retail banks is:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^r_t \log \left( C^r_t - hC^r_{t-1} \right) \tag{4.13}
\]

subject to the following constraints:

\[
C^r_t + l^b_t + l^i_t + r^d_t d_{t-1} + AC_{d^r,t} + AC_{l^{b,i},t} + AC_{\rho^r,t} = d_t + r^d t^b_{t-1} + r^d t^i_{t-1} - \varepsilon^b_t - \varepsilon^i_t \tag{4.14}
\]

\[
l^b_t + l^i_t - d_t - \varepsilon^b_t - \varepsilon^i_t \geq \rho_r (l^b_{t-1} + l^i_{t-1} - d_{t-1} - \varepsilon^b_{t-1} - \varepsilon^i_{t-1}) + (1 - \rho_r) \left[ \eta^b t^b_t + \eta^i t^i_t - \varepsilon^b_t - \varepsilon^i_t \right] \tag{4.15}
\]

where \( r^d_t \) is a gross real interest rate on loans to investment banks. \( \varepsilon^b_t \) and \( \varepsilon^i_t \) in the budget constraint (4.14) are defaults shocks on loans to households and investment banks, which lower the level of bank equity. Equation (4.15) is the bank capital requirement regulation constraint. If we assume \( \rho_r = 0 \) and \( \eta^b_t = \eta^i_t \) for simplicity, it can be rewritten as

\[
\frac{\text{(equity)}}{\text{(total assets)}} = \frac{l^b_t + l^i_t - d_t - \varepsilon^b_t - \varepsilon^i_t}{l^b_t + l^i_t} \geq \eta^b_t
\]

which means retail banks should retain a certain level of equity, proportional to assets.

Similarly to the LTV regulation, the capital requirement regulation also consists of two parts as follows:

\[
\eta^j_t = \eta^j_0 + \eta^j_1 \left( \frac{t^b_t/Y_t}{t/Y} - 1 \right) \quad \text{where } j \in \{b,i\} \tag{4.16}
\]

\(^\text{17}\)The preference of debt over equity can also be introduced by tax treatment on debt, equity dilution cost, or liquidity premium on deposits.
where the first term is a constant capital requirement regulation and the next term is a time-varying regulation. \( \eta^j_0 \) in the first term is the constant minimum capital requirement. The time-varying capital requirement regulation is called a counter-cyclical capital requirement regulation. It requires banks to hold more equity when loans expand much faster than output. That is, the policy is counter-cyclically tightened when the credit expands. Assuming \( \eta^j_t > 0 \), \( \eta^j_t \) is positively related to the deviation of loan to GDP ratio from the steady state value. In the extreme case when \( \eta^j_t = 0 \), the particular asset \( j \) is considered to be riskless.

4.4 Investment Banks  Investment banks, denoted as \( i \), obtain funds from the retail banks and lend to entrepreneurs. Investment banks are also subject to capital requirement regulation. The utility maximization problem of the representative investment bank is given by

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \log \left( C^i_t - hC^i_{t-1} \right)
\]

subject to the following constraints:

\[
C^i_t + l^e_t + r^i_t l^i_{t-1} + AC^i_{et} + AC^i_{et} = l^e_t + r^i_t l^i_{t-1} + \varepsilon^i_t - \varepsilon^e_t
\]

\[
l^i_t \leq \rho^i l^i_{t-1} + (1 - \rho^i) [(1 - \eta^i_t) l^e_t + \varepsilon^i_t - \varepsilon^e_t]
\]

Budget constraint (4.18) and capital requirement constraint (4.19) (written in a form of borrowing constraint\(^{18}\)) are similar to other agents’ constraints. We will skip the definition of \( \eta^e_t \), which is exactly same with \( \eta^i_t \) and \( \eta^b_t \).

4.5 Retailers  Monopolistic competitive retailers purchase goods from the entrepreneurs in a competitive market and differentiate them into intermediate goods, as in the typical New Keynesian literature. The technology is linear: \( Y_i(z) = X^c_t - F^e(z) \) where \( F(z) \) are fixed costs to make the steady-state profit of the retailer zero. Then retailers sell intermediate goods, \( Y_i(z) \), to the final goods-producing firm at a price of \( P(z) \). Final output \( Y_t \) is given by

\[
Y_t = \int_0^1 Y_i(z) \frac{z^{\varepsilon-1}}{\varepsilon} \, dz
\]

where \( \varepsilon > 1 \).

\(^{18}\)It is equivalent to \( \frac{\text{equity}}{\text{total assets}} = \frac{l^e_t - l^i_t + \varepsilon^i_t - \varepsilon^e_t}{l^i_t} \geq \rho^i_t \), if \( \rho^i = 0 \)
The cost minimization problem of the final goods-producing firm yields the inverse demand function

\[ Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \]  \hspace{1cm} (4.21)

and the aggregate price index \( P_t = \left( \int_0^1 P_t(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}}. \)

Each retailer chooses optimal price \( P_t(z); \) following Calvo (1983), the retailer can adjust the price with probability \( 1 - \theta. \) If the retailer is not able to adjust its price, \( P_t(z) = P_{t-1}(z). \) Each retailer maximizes its market value:

\[
\max_{P_t(z)} E_0 \sum_{t=0}^{\infty} \beta_t^s \lambda_t^s \left[ P_t(z)Y_t(z) - P_t^X X_t^t \right] \]  \hspace{1cm} (4.22)

subject to the equation (4.21). The optimal price level for firm \( z \) in period \( t \) is:

\[
P_t^*(z) = \frac{\varepsilon}{\varepsilon - 1} \left( \frac{\varepsilon}{\varepsilon - 1} \right) \sum_{j=0}^\infty \left( \theta \beta_s \right)^j \lambda_t^s \frac{P_{t+j}^X}{Y_{t+j}} \]  \hspace{1cm} (4.23)

We assume a symmetric equilibrium case where \( P_t^* = P_t^*(z), \forall z, \) thus the aggregate price level evolves according to \( P_t = \left[ (1 - \theta)(P_t^*)^{1-\varepsilon} + \theta (P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \)

4.6 Monetary Authority We assume that the monetary authority conducts a monetary policy following the extended Taylor rule, which incorporates the loan (or credit) as an additional determinant of the policy rate \( R_{t+1}^i = r_t^i E_t \pi_{t+1}, \) the nominal inter-bank interest rate:

\[
\frac{R_t^i}{R_t^c} = \left( \frac{R_{t-1}^i}{R_t^c} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y} \right)^{\gamma_Y} \left( \frac{l_t}{T} \right)^{\gamma_L} \right]^{1-\rho_R} \]  \hspace{1cm} (4.24)

where \( \pi_t \equiv \frac{P_t}{P_{t-1}} \) is a gross inflation rate, \( l_t = l_t^b + l_t^c + l_t^e \) is a total loan in the economy. Variables without time subscript indicates steady-state levels. \( \rho_R \) is a smoothing parameter of policy rate and \( \gamma_\pi, \gamma_Y, \) and \( \gamma_L \) are feed-back parameters of corresponding variables. It becomes a standard Taylor rule if we set \( \gamma_L = 0. \)

4.7 Housing Market We assume that housing supply is exogenously given as \( \bar{H}. \) Then the housing market clearing condition is given by
\[
\bar{H} = H^c_t + H^b_t + H^e_t 
\] (4.25)

In what follows, we normalize \( \bar{H} \) as one, without loss of generality.

### 4.8 Exogenous Shocks

We have four non-financial shocks (Spending Shock \( \varepsilon^c_t \)), Investment-specific technology shock \( \varepsilon^k_t \), TFP shock \( \varepsilon^z_t \) and monetary policy shock \( \varepsilon^R_t \)) and six financial shocks (housing demanding shock \( \varepsilon^h_t \)), three default shocks \( \varepsilon^b_t \), \( \varepsilon^e_t \), and \( \varepsilon^i_t \)), two risk perception shocks \( \varepsilon^{lb}_t \) and \( \varepsilon^{le}_t \)) hence 10 exogenous shocks as total. For \( x \in \{c, k, z, R, h, lb, le\} \), the exogenous shock process \( \varepsilon^x_t \) is assumed to follow an AR (1) process:

\[
\log \varepsilon^x_t = \rho^x \log \varepsilon^x_{t-1} + u^x_t 
\] (4.26)

where \( u^x_t \) is the i.i.d. shock that is normally distributed with mean 0 and variance \( \sigma_x \). Default shocks \( x \in \{b, e, i\} \), are defined as level instead of log level.

## 5 Calibration and Preliminary Analysis

### 5.1 Parameterization

We use the estimated values in Iacoviello (2014) and conventional values for calibration. For instance, we set the patient households discount factor at 0.9925 to target 3% annual risk-free interest rate. As in Iacoviello (2014), our value for capital depreciation is higher than the typical number in the literature, 0.025, because housing is the additional factor of production which does not depreciate. Following the standard NK-DSGE literature, the elasticity of substitution for intermediate varieties, \( \varepsilon \), is calibrated as 11 to target the steady state mark-up at 10%. The coefficients in the Taylor rule are also usual numbers to ensure the determinate equilibrium of the model. Table 5.1 shows our benchmark calibration for the parameters and Table 5.2 represents the parameterization for exogenous shocks used in our model.\(^{19}\)

### 5.2 Basic Results: Impulse Response Functions

Throughout our analysis, we will compare four model economies with different types of macroprudential policies. The benchmark economy (Model 1) is set to have the 70% LTV regulation \((\gamma^Hb = 0.7)\), the 8% constant minimum capital requirement, and the monetary policy neutral to loan changes. Other economies have different macroprudential policies as follows:

\(^{19}\)In Appendix A, we report the steady-state values of key macro variables under the benchmark calibration.
Table 5.1: Benchmark Calibration (Benchmark Economy)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>0.9925</td>
<td>Discount factor, patient household</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>0.94</td>
<td>Discount factor, impatient household</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>0.94</td>
<td>Discount factor, entrepreneur</td>
</tr>
<tr>
<td>$\beta_i, \beta_l$</td>
<td>0.945</td>
<td>Discount factor, banks</td>
</tr>
<tr>
<td>$\nu_h, \nu_b$</td>
<td>0.075</td>
<td>Housing preference parameter</td>
</tr>
<tr>
<td>$\nu_s, \nu_b$</td>
<td>2</td>
<td>Labor preference parameter</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>Inverse Frisch elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.035</td>
<td>Rate of capital depreciation</td>
</tr>
<tr>
<td>$h$</td>
<td>0.8</td>
<td>Habit formation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
<td>Total capital share in production</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.67</td>
<td>Wage share of patient household</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>0.64</td>
<td>Capital share of patient household</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.04</td>
<td>Housing share in production</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>11</td>
<td>Elasticity of substitution for intermediate varieties</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.78</td>
<td>Calvo Parameter</td>
</tr>
<tr>
<td>$\iota_{K_s}$</td>
<td>1.73</td>
<td>Capital adjustment cost, household</td>
</tr>
<tr>
<td>$\iota_{K_e}$</td>
<td>0.59</td>
<td>Capital adjustment cost, entrepreneur</td>
</tr>
<tr>
<td>$\iota_{d_s}$</td>
<td>0.10</td>
<td>Deposit adjustment cost, household</td>
</tr>
<tr>
<td>$\iota_{d_r}$</td>
<td>0.14</td>
<td>Deposit adjustment cost, bank</td>
</tr>
<tr>
<td>$\iota_{h_b}$</td>
<td>0.37</td>
<td>Household loan adjustment cost, household</td>
</tr>
<tr>
<td>$\iota_{h_{rb}}$</td>
<td>0.47</td>
<td>Household loan adjustment cost, retail bank</td>
</tr>
<tr>
<td>$\iota_{e}$</td>
<td>0.07</td>
<td>Entrepreneur loan adjustment cost, entrepreneur</td>
</tr>
<tr>
<td>$\iota_{l_{rb}}$</td>
<td>0.06</td>
<td>Entrepreneur loan adjustment cost, investment bank</td>
</tr>
<tr>
<td>$\iota_{l_{rv}}$</td>
<td>0.47</td>
<td>Interbank loan adjustment cost, retail bank</td>
</tr>
<tr>
<td>$\iota_{l_{iv}}$</td>
<td>0.05</td>
<td>Interbank loan adjustment cost, investment bank</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>0.70</td>
<td>Speed of deleveraging, impatient household</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>0.65</td>
<td>Speed of deleveraging, entrepreneur</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.24</td>
<td>Speed of loan adjustment, retail bank</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.70</td>
<td>Speed of loan adjustment, investment bank</td>
</tr>
<tr>
<td>$\gamma_{K_s}$</td>
<td>0.7</td>
<td>LTV ratio on housing</td>
</tr>
<tr>
<td>$\gamma_{K_e}$</td>
<td>0.9</td>
<td>LTV ratio on entrepreneur capital</td>
</tr>
<tr>
<td>$\gamma_{h_b}$</td>
<td>0.08</td>
<td>Minimum capital requirement, households loan</td>
</tr>
<tr>
<td>$\gamma_{h_{rb}}$</td>
<td>0.08</td>
<td>Minimum capital requirement, entrepreneur loan</td>
</tr>
<tr>
<td>$\gamma_{h_{iv}}$</td>
<td>0.08</td>
<td>Minimum capital requirement, interbank loan</td>
</tr>
<tr>
<td>$\rho^R$</td>
<td>0.75</td>
<td>Interest rate inertia, monetary policy</td>
</tr>
<tr>
<td>$\gamma^\pi$</td>
<td>1.5</td>
<td>Inflation targeting parameter, monetary policy</td>
</tr>
<tr>
<td>$\gamma^Y$</td>
<td>0.125</td>
<td>Output targeting parameter, monetary policy</td>
</tr>
<tr>
<td>$\gamma^L$</td>
<td>0</td>
<td>Financial targeting parameter, monetary policy</td>
</tr>
</tbody>
</table>

- Model 1: No macroprudential policy (benchmark economy)
- Model 2: Extended Taylor rule ($\gamma^L = 0.125$)
Table 5.2: Benchmark Calibration: Exogenous Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^C$</td>
<td>0.994</td>
<td>Autocorr. of spending shock</td>
</tr>
<tr>
<td>$\rho^K$</td>
<td>0.916</td>
<td>Autocorr. of investment-specific technology shock</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.839</td>
<td>Autocorr. of TFP shock</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>0.932</td>
<td>Autocorr. of housing demand shock</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.969</td>
<td>Autocorr. of default shock (impatient HH)</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>0.992</td>
<td>Autocorr. of default shock (entrepreneur)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.916</td>
<td>Autocorr. of default shock (investment bank)</td>
</tr>
<tr>
<td>$\rho_{lb}$</td>
<td>0.839</td>
<td>Autocorr. of LTV shock (impatient HH)</td>
</tr>
<tr>
<td>$\rho_{le}$</td>
<td>0.873</td>
<td>Autocorr. of LTV shock (entrepreneur)</td>
</tr>
</tbody>
</table>

| $\sigma^C$ | 0.025 | s.d of spending shock |
| $\sigma^K$ | 0.025 | s.d of investment-specific technology shock |
| $\sigma_z$ | 0.007 | s.d. of TFP shock |
| $\sigma_H$ | 0.0348 | s.d. of housing demand shock |
| $\sigma_b$ | 0.0013 | s.d. of default shock (impatient HH) |
| $\sigma_e$ | 0.0011 | s.d. of default shock (entrepreneur) |
| $\sigma_i$ | 0.0011 | s.d. of default shock (investment bank) |
| $\sigma_{lb}$ | 0.0115 | s.d. of Risk perception(LTV) shock (impatient HH) |
| $\sigma_{le}$ | 0.0204 | s.d. of Risk perception(LTV) shock (entrepreneur) |

- Model 3: Counter-cyclical capital requirement ($\eta^j_1 = 0.25$ for $j = b, i, e$)
- Model 4: Time-varying LTV regulation on housing ($\gamma^{Hb}_1 = \gamma^{Eb}_1 = 0.3$)

The parameters are chosen in the following sense. In Model 2, the response of the central bank to loans is the same as to output. In Model 3, capital requirement increases by 0.25% in response to 1% increase in loans. In Model 4, LTV regulation decreases by 3% in response to 10% increase in housing prices.\(^{20}\)

Before we present our main results, we first show impulse response functions of our model economies to selected exogenous shocks (TFP and default shock to entrepreneurs) to check if the models behave well consistently with the usual economic intuition and different policies result in the different impulse responses to the variables.\(^{21}\) As will be turned out later, the intuition discussed below is useful to understand our main findings.

Figure 5.1 is the collection of impulse response functions to one-time-one-unit shock to the aggregate productivity. As usually argued in the literature, different sets of macroprudential policies do not have

\(^{20}\)In Online Appendix C.2, we provide additional results on robustness of our findings by changing policy parameters.

\(^{21}\)When alternative exogenous shocks are considered, economic intuitions one can obtain are not different from what we study in the main text. See Appendix B that reports results for other shocks, including housing preference shock and monetary policy shock.
much impact on the response of macro variables when the real shock hits the economy. Shapes are consistent with the usual intuition; key variables all increase due to high productivity in this economy.

Figure 5.1: Impulse Response Functions: Productivity Shock
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement, and Model 4 is the economy with time-varying LTV on impatient household.

Figure 5.2 is the collection of impulse response functions to one-time-one-unit shock to default shock to entrepreneurs. Since this is a negative redistribution shock to banks, they will lower loans, which triggers economic downturns. Overall, different macroprudential policies are still ineffective to lower the responsiveness of the economy to the shock. One noticeable observation is that inflation significantly increases over time when the extended Taylor rule is implemented (Model 2): this comes from the fact that interest rates further change from the changes in loan size. Interest rate(s) decrease more in this case so that the incentive to consume increases. However, the incentive of consumers for consumption smoothing, which is enhanced by the habit formation, requires less changes in interest rates. As a result, the inflation rate is required to increase in equilibrium so that interest rates do not change much by the Taylor rule. Greater response of consumption than under other policies naturally follows. Lastly the greater response of consumption and output in Model 3 versus Model 1 comes from the successfully controlled loan market; more loan in equilibrium implies that more deposit is required by financial intermediary so that patient households consumption instead decreases.

Therefore, quick preview of the effectiveness of different policies with impulse response functions shows that in most cases macroprudential policies do not achieve their goals aiming to lower the effects of exogenous shocks. If any, it is mostly observed from the policy requiring banks to accumulate counter-
Figure 5.2: Impulse Response Functions: Default Shock to Entrepreneur
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement, and Model 4 is the economy with time-varying LTV on impatient household.

It turns out that findings discussed above do not significantly change when an aggressive macroprudential policy is considered. While the aggressive policy achieves its goal to stabilize the financial market more effectively than the weak policy does, negative impacts on the real sector are also amplified; see Appendix B.1.

6 Frequency-Specific Effects on Variance

6.1 Volatility at Business Cycle Frequency

Before we proceed to analyze the frequency-specific effectiveness of macroprudential policies, as a benchmark to our main analysis, we first report the standard deviation of the key variables at the business cycle frequency. In particular, we simulate the model economy 1,000 times with each simulation setting the total period at 1,024. We then filter each of the series with \( \kappa = 200 \) by applying the band-pass filter (Baxter and King (1999)) to obtain the series of frequency between 2 years/cycle and 8 years/cycle where \( \kappa \) is the number of leads/lags used in the approximation of the filtering. We then compute the standard deviation of each series and take the average of standard deviations from each simulation. Table 6.1 summarizes the results; as we are interested in the performance of the macroprudential policies relative to the benchmark economy without such policies, we compute the standard deviation of key variables relative to that obtained in the benchmark economy.

Several observations are noteworthy. First, the macroprudential policy that tightens the LTV ratio in response to housing prices (Model 4) does not seem to change the business cycle properties of key macro variables. Second, the extended Taylor rule (Model 2) much more amplifies the variance of consumption
Table 6.1: Relative Standard Deviation of Key Variables

<table>
<thead>
<tr>
<th></th>
<th>std(Y)</th>
<th>std(C)</th>
<th>std(π)</th>
<th>std(L)</th>
<th>std(L/Y)</th>
<th>std(p_H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2</td>
<td>1.01</td>
<td>1.17</td>
<td>1.15</td>
<td>1.03</td>
<td>1.01</td>
<td>1.13</td>
</tr>
<tr>
<td>Model 3</td>
<td>1.00</td>
<td>1.05</td>
<td>1.14</td>
<td>0.87</td>
<td>0.93</td>
<td>1.07</td>
</tr>
<tr>
<td>Model 4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Model 5</td>
<td>1.03</td>
<td>1.08</td>
<td>1.23</td>
<td>0.77</td>
<td>0.88</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Note: All values are ratios relative to the benchmark economy. Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement, Model 4 is the economy with tight LTV on housing. Model 5 is the economy with aggressive countercyclical capital requirement (the policy parameter (η^j_1) is twice of that in Model 3).

than other policies when compared to the benchmark economy, while it does not achieve its original goal of stabilizing the financial market. Even worse, the volatility of inflation rate also increases dramatically. Hence, from the traditional perspective on the role of central bank, which emphasizes output and inflation stabilities, it is not recommended for the central bank to directly take loan into account. Third, stabilizing the financial market at the business cycle frequency is the most successful in the economies with countercyclical capital requirement policies (Model 3 (weak policy) and Model 5 (aggressive policy)), while the output fluctuations are somewhat similar to the benchmark economy. Finally, the aggressive macroprudential policy (Model 5) is more effective in the financial market stabilization than the weak policy (Model 3) at the cost of amplifying the fluctuation in the real sector.

6.2 Frequency-Specific Effects of Different Policies  We now turn to our main analysis, which analyzes the frequency-specific effects of macroprudential policies. In so doing, we take the steps described in Section 3. Since our main purpose here is to evaluate the performance of different policies versus the benchmark economy without any macroprudential policy, we compute the spectral density of each variable of the specific policy regime and compare the results to those of the benchmark model. In each figure, the vertical axis denotes the spectral density relative to model 1 at each frequency and the horizontal axis denotes frequencies from low to high frequency.

Firstly, when the extended Taylor rule is implemented in the economy, output fluctuations are amplified at every frequency while the negative effect is greater at the relatively low frequency. In contrast, the countercyclical capital requirement is also overall ineffective in stabilizing output compared to the benchmark economy, but the negative effect is much smaller than the economy with the extended Taylor rule. In addition, the output fluctuations are more amplified at the relatively high frequency. Therefore, this policy, even though it could be effective in stabilizing the financial market as shown later, clearly has a slight negative effect on the real sector, which is one of the design limits of the macroprudential policy.
This finding is in line with our previous discussions on impulse response functions; lower loan volatility would also decrease deposit volatility, which would amplify the consumption fluctuation and then the output fluctuation. Consumption volatility exhibits similar patterns; it is dramatically exacerbated under the extended Taylor rule. Almost at every frequency, the relative magnitude of the consumption volatility is about thirty percent higher than that of the benchmark economy. This is because interest rates directly respond to loan so that the Euler equation affects consumption more than economies under different policy regimes. In contrast, the LTV policy (Model 4) has a relatively smaller effect both on consumption and output fluctuations; output volatility slightly increases in the long-run hence this policy also has an adverse effect on the real sector while consumption volatility barely changes at every frequency.

![Figure 6.1a: Output](image1)

**Figure 6.1: Frequency-Specific Effects on Real Sector**

Note: Each line indicates relative volatility of the variable obtained under the particular policy regime when compared that obtained under benchmark economy (model 1).

How about loan fluctuations, which is the main objective of macroprudential policies? Figure 6.2a shows the result; importantly, the extended Taylor rule satisfies the main goal only at the relatively high frequency (higher than 8 years/cycle). It rather amplifies the financial market fluctuations at the relatively low frequency. In the sense that the financial cycle exhibits a much lower frequency (8 to 32 years/cycle), this implies that the extended Taylor rule does not achieve its goal to stabilize the financial cycles. Together with our observations from Figure 6.1a, this further implies that the extended Taylor rule affects output and loan fluctuations in the opposite direction to the original objective of the macroprudential policy. On contrary, the countercyclical capital requirement is very effective in stabilizing loan fluctuations regardless of the frequency. However the output fluctuations are amplified at every frequency under this policy. This implies that the macroprudential policies that effectively stabilize the financial market pay the cost to destabilize the goods market. In the mean time, the policy
to tighten LTV ratio on housing (Model 4) is not that effective in the financial market when compared to the countercyclical capital requirement. This means that while the adverse effect of the policy to tighten LTV ratio on the fluctuations in the real sector is smaller than other policy options, it comes at the cost of less effectiveness on the loan market, which is the main concern of the macroprudential policy. The observation from Figure 6.3a is consistent with the above discussions. Figure 6.2b provides an additional evidence that the performance of the extended Taylor rule in reducing the variability of the housing market is the worst among all policy options that we consider.

We also point out that tightening LTV ratio seems to be overall ineffective in equilibrium\(^{22}\) since it is not the policy that reacts to the changes in loan; other policies, in contrast, react to the changes in loan (potentially loan-to-output ratio). Rather, LTV policy responds to housing prices, but its propagation is relatively weak compared to the effects of the overall loan on the decision of the banks. The changes in volatilities are smaller when compared to other policies under an aggressive LTV policy (see Figure C.1 in Online Appendix C.2). In contrast, the countercyclical capital requirement is effective since it really reacts to the fluctuations in loan to GDP ratio. It is effective in lowering loan fluctuations at every frequency since banks accumulate enough capital during boom times in preparation for possible losses during recessions, which results in less changes in the overall loan level.

Lastly, we also consider the effects of macroprudential policies on inflation rate (Figure 6.3b). First, we can observe that the negative effect of the extended Taylor rule (Model 2) on inflation rate volatility is

\(^{22}\) Another way to interpret our finding is that the size of positive effect and negative effect from the policy are of the similar magnitude hence they cancel out each other at the equilibrium.
Figure 6.3: Frequency-Specific Effects on Financial Sector and Price

Note: Each line indicates relative volatility of the variable obtained under the particular policy regime when compared that obtained under benchmark economy (model 1).

observed for the frequency lower than about 3 years/cycle. The high frequency that cannot be observed from the business cycle analysis can actually lower the volatility of inflation rate while the variance of inflation rate becomes much greater as we consider lower frequency. Therefore the negative effect on the inflation rate volatility is maximized at the lowest frequency under the extended Taylor rule. The intuition for this result is in line with our discussions on the impulse response functions in Section 5.2; as the interest rate directly responds to loan in this economy, the inflation rate should adjust in order for consumption smoothing. As a result, the inflation rate volatility becomes higher in this economy. The countercyclical capital requirement (Model 3) on the fluctuations of inflation rate is further amplified at every frequency. Hence, our result provides an important lesson for the central bank; the currently well-known macroprudential policies can ruin the goal of the central bank that aims to stabilize the inflation rate.

In summary, our exercise implies that the currently well-known macroprudential policies can be effective in stabilizing the financial market, especially when the countercyclical capital requirement is implemented. However, there is a substantial cost to achieve its effectiveness in the financial sector; it can amplify the fluctuations in the real sector or in inflation rate. Especially the performance of the extended Taylor rule is worse than that of countercyclical capital requirement in many dimensions. Contrary to these policies, equilibrium effects of tightening LTV ratio on housing on the aggregate economy is small when compared to other policies. Hence, the design limits of macroprudential policies in our

23If Hodrick-Prescott filter is used to obtain the filtered data, this problem becomes exaggerated since it does not filter out movement at the very high frequency.
model is two folds. First, the financial sector stabilization is associated with the real sector (or inflation rate) destabilization. Second, the effectiveness of policy is different across frequencies; for instance, when the extended Taylor rule is implemented, the inflation rate volatility increases as the frequency becomes lower while the opposite is observed in the financial market.

Next, we study if the effects of the aggressive macroprudential policy can be different from the less aggressive macroprudential policy. As in the previous section, we only consider the countercyclical capital requirement since it is the most effective policy in terms of the loan market stabilization. We again compute the relative spectral densities of the economy with the weak macroprudential policy ($\eta^1_j = 0.25$ for $j = b, i, e$, thick blue line) and with the aggressive macroprudential policy ($\eta^1_j = 0.5$ for $j = b, i, e$, circled green line), and plot the spectral densities in Figure 6.4.

![Figure 6.4: Frequency-Specific Effects: Weak vs. Aggressive Policy](image_url)

Note: Relative volatility compared to model 1 at each frequency. ‘Weak policy’ is the economy with countercyclical capital requirement with $\eta^1_j = 0.5$, and ‘Aggressive policy’ is the economy with countercyclical capital requirement with $\eta^1_j = 1$ for $j = b, i, e$.

Two conclusions can be drawn from the figures. First, the aggressive policy is more effective in stabilizing the financial market than the weak one as expected. This achievement, however, leads to amplified fluctuations in the real sector compared to the weak policy. Particularly, the output, consumption, and inflation rate volatility at each frequency are much more exacerbated under the aggressive policy.

---

\[24\] In the sense that we vary the key policy parameter, this exercise can be interpreted as a robustness check. In Online Appendix C.2, we present more robustness check results.
worse, the negative effects on the real sector increase as frequencies become higher. If monetary authorities care about the aggregate fluctuations at the business cycle frequency, which is usually defined as the fluctuations between about 2 years/cycle and 8 years/cycle, the aggressive macroprudential policy is not recommended; it increases output volatility compared to the benchmark economy approximately by 5% point, consumption volatility approximately by 20% point, and inflation rate volatility by more than 50% point at the business cycle frequency.

7 Frequency-Specific Effects on Welfare

One might raise a concern that computing variance may not directly translate into measuring welfare.\footnote{For instance, more volatile economy can be welfare-improving (Cho, Cooley, and Kim (2015))} This section deals with such concerns by conducting a spectral welfare analysis as in Otrok (2001). In doing so, we consider a representative (average) consumer whose utility function takes the following form:

\[
V^r = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \varepsilon_t^c \ln \left( C_t^r - hC_{t-1}^r \right) - \nu_n \frac{(N_t^r)^{1+\phi}}{1 + \phi} \right]
\]

where \( r \) denotes a policy regime \( r \), \( C_t^r \) denotes aggregate consumption, and \( N_t^r \) denotes aggregate hours worked. Since we have four types of agents (household, entrepreneur, and banks) that maximize their own utility functions, we aggregate consumption of different agents in order to obtain \( C_t^r \) so that it represents aggregate consumption of this economy. \( N_t^r \), hours worked under regime \( r \), is also obtained by aggregating hours worked by patient household and that by impatient household.\footnote{Similar strategy is taken by Suh (2012). Aggregate housing demand is not included in the above utility function as the aggregate housing supply is assumed to be fixed as one so that its value does not change across the policy regime.} Parameters for the spectral welfare exercise are taken from Table 5.1; especially for \( \beta \), we choose the average value of \( \beta \) between patient and impatient household, which does not change our conclusion reported below.

Our objective is to evaluate performances of different macroprudential policies compared to the benchmark economy without any macroprudential policy, hence we compare the value, \( V^r \), the life-time value associated with Model \( r > 1 \) with the value associated with Model 1 (benchmark economy), \( V^1 \), by considering the following \( \lambda \) adjusted value function:

\[
V^{r, \lambda} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \varepsilon_t^c \ln (1 + \lambda) \left( C_t^r - hC_{t-1}^r \right) - \nu_n \frac{(N_t^r)^{1+\phi}}{1 + \phi} \right]
\]

where \( \lambda \) measures the welfare gain (or loss) in terms of consumption variation.
For \( r > 1 \), \( \lambda \) solves \( \mathbb{V}^{1,\lambda} = \mathbb{V}^r \) so that it satisfies the following formula\(^{27}\)

\[
\lambda \approx (1 - \beta)(\mathbb{V}^r - \mathbb{V}^1)
\]  

(7.3)

We note that \( \lambda \) is positive (resp. negative) when the macroprudential policy is welfare-improving (resp. welfare detrimental). In particular, as we are interested in the frequency-specific welfare gains, we compute \( \lambda_i \) for each frequency \( i \) by computing the spectral utility functions. The steps to compute the welfare formula (7.3) is described in Section 3.

7.1 Average Welfare Gains

We first compute the welfare gain by directly using the simulated series, not the filtered series; hence, the welfare gain is not frequency-specific but is consistent with the previous literature. We call this measure average welfare gain (loss). Results are reported in Table 7.1:

<table>
<thead>
<tr>
<th>Welfare Gains (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended Taylor rule</td>
</tr>
<tr>
<td>Countercyclical capital requirement</td>
</tr>
<tr>
<td>Time-varying LTV regulation on housing</td>
</tr>
<tr>
<td>Aggressive countercyclical capital requirement</td>
</tr>
</tbody>
</table>

Note: We multiply 100 to \( \lambda \).

Several points are noteworthy here. As can be inferred from our previous analysis, the Extended Taylor rule is, in general, welfare-detrimental. While the size is much smaller, LTV policy is also welfare-detrimental. On contrary, countercyclical capital requirement is welfare-improving and the size of the welfare gain is comparable to that obtained by Suh (2012). The last row shows the welfare gain from more aggressive countercyclical capital requirement, which confirms the above finding that this policy is welfare-improving. As a whole, Table 7.1 shows that, consistently with the findings reported in the previous section, the countercyclical capital requirement is the best among the macroprudential policy tools considered in our paper. In addition, negative welfare gains from two macroprudential policies (LTV policy and extended Taylor rule) imply that destabilizing effect of the policies on real sectors dominates the positive effect, including loan market stabilization, even at the business cycle frequency.\(^{28}\)

\(^{27}\)We ignore \( \mathbb{E}_0(\varepsilon_t) \) as it is constant across different regimes and is a constant \( \left( \mathbb{E}_0(\varepsilon_t) = \exp \left( \frac{1}{2} \sigma^2 \right) > 0 \right) \).

\(^{28}\)We appreciate an anonymous referee’s comment on this observation.
7.2 Frequency-Specific Welfare Gains  This section presents our main results on frequency-specific welfare gains. Figure 7.1a shows the welfare gain (loss) of each policy at different frequencies and Figure 7.1b compares the welfare gains (loss) of weak and aggressive countercyclical capital requirement. We first point out that unlike Otrok (2001), the average welfare gain obtained in the previous section is not additively decomposed into the spectral welfare gains reported in this section. In Otrok (2001), the welfare loss from the economic fluctuations can be summarized by the consumption volatility as denoted by Lucas (1987). As a result, the sum of the spectral welfare cost, which is also a (linear) function of frequency-specific variance, which is an additive decomposition of the overall variance, is equal to the average welfare cost. This is not the case in our experiment; the value function is not a linear function of consumption volatility as the model is second-order approximated at the equilibrium (Schmitt-Grohé and Uribe (2004)).

![Figure 7.1a: Benchmark Economy](image1)

![Figure 7.1b: Effects of Aggressive Policy](image2)

Figure 7.1: Spectral Welfare Analysis

Note: Each line indicates spectral welfare gain obtained under the particular policy regime when compared to welfare obtained under the benchmark economy (model 1).

Key observations can be summarized as follows. First, the LTV regulation barely affects the welfare gain or loss, consistent to our finding in the previous analysis. Second, the welfare loss from the extended Taylor rule is not concentrated at a specific frequency band; rather, it is broadly observed across all frequency bands. Interestingly, there is a welfare gain at the relatively low frequency. At the business cycle frequency (2-8 years), however, there is a welfare loss. Lastly, similar to the Extended Taylor rule, the welfare loss from the countercyclical capital requirement is dispersed along the frequency. Except at the frequency of 7-10 years/cycle, there is a welfare loss from the policy. Moreover, we can observe from Figure 7.1b that aggressive countercyclical capital requirement (dotted green line) widens the welfare loss at every frequency when compared to the less aggressive policy (solid blue line). This is also a finding consistent with Figure 6.4; more aggressive policy makes the real sector more volatile so that welfare loss
from the policy increases at every frequency.

Our findings through this section raise a cautionary note on the welfare analysis with macroprudential policies; even a policy seems to be effective, such as increasing the average welfare (Table 7.1), it may not hold with the spectral welfare analysis. In other words, the effectiveness of macroprudential policies on the aggregate economy should be carefully examined as their effects are frequency-specific.

8 Conclusion

This paper evaluates the performances of widely-used macroprudential policies with the financial-sector augmented New Keynesian DSGE model. Our results are somewhat disappointing. The extended Taylor rule is not recommended since its perverse effects on the real sector are observed across almost all frequencies. On the other hand, the counter-cyclical capital requirement achieves its policy objective by stabilizing the financial market but at the cost of destabilizing the real sector. The time-varying LTV regulation responding to the growth rate of housing price does not significantly alter the equilibrium properties of the model and hence the (possible) welfare gain is small.

Our findings suggest important policy implications: First, macroprudential policies are not successful to stabilize both the real and financial sectors. More importantly, macroprudential policies conflict with the price-stability objective of monetary policy, since they amplify the volatility of inflation across all frequencies. Second, our findings suggest that policy-makers should design macroprudential policies after careful consideration of their adverse effects. Both frequency-specific variance and spectral welfare approaches emphasize the trade-offs between positive effects on financial sector and negative effects on real sector. Lastly, our findings also support the argument that the monetary policy is not recommended to respond actively to the loan growth, because macroprudential policies are proven to be more effective compared to the monetary policy.

Our work can be extended in many directions: First, our findings can be empirically verified with sufficiently long time-series data. Second, it would be an important extension to find an “optimal” policy-mix between monetary and macroprudential policies. Moreover, considering frequency-specific effects of macroprudential policies, finding an optimal combination of policies across all frequencies could be another interesting extension. We leave these extensions as a future work since they are beyond the scope of our work.
REFERENCES


WICKSELL, K. (1936): Interest & Prices. Sentry Press. 1


A Appendix. Key Steady-State Values

Table A.1: Steady-State Values for Variables of Interest

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption-GDP Ratio</td>
<td>0.77</td>
</tr>
<tr>
<td>Investment-GDP Ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>Aggregate Loan-GDP Ratio</td>
<td>3.63</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>5.25%</td>
</tr>
</tbody>
</table>

Note: Interest rate spread is the difference between the annualized deposit rate and the annualized business loan rate.

B Appendix. Impulse Response Functions to Other Shocks

In this section, we present figures of impulse response functions on other exogenous shocks. In particular, we consider three additional shocks: (1) default shock to entrepreneurs, (2) housing preference shock, and (3) monetary policy shock.

Figure 5.2 plots impulse response functions to the positive shock to the default of entrepreneurs; as this is a negative redistribution shock to banks, they will lower loans, which triggers economic downturns. While some macroprudential policies (countercyclical capital requirements in particular) seem to be effective in lowering loan fluctuations, most other policies are not effective. Figure B.1 presents impulse response functions to the shock to the housing preference of households. Given fixed housing supply, the increase in housing demand means soaring housing prices, which grows the wealth of average agents in this economy. Hence, the economy experiences boom. Note again, macroprudential policies are not effective in terms of lowering the responsiveness of the economy to the exogenous shocks. Lastly, Figure B.2 shows the impulse responses to the positive monetary policy shock. Higher policy rates usually dampen the economy; all variables exhibit patterns commonly observed in the recession, and different macroprudential policies do not show different patterns.

B.1 Effectiveness of More Aggressive Policy

In this section, we evaluate the performance of more aggressive macroprudential policies. In particular, we consider the macroprudential policies on countercyclical capital requirements since it seems to be more effective than other policies in stabilizing loan fluctuations. Figure B.3 to B.6 show the impulse response functions to the productivity shock, default shock to entrepreneurs, housing preference shock, and monetary policy shock, respectively. The thick blue line represents the responses from the benchmark economy, the dotted green line represents
Figure B.1: Impulse Response Functions: Housing Preference Shock  
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement, and Model 4 is the economy with time-varying LTV on impatient household.

Figure B.2: Impulse Response Functions: Monetary Policy Shock  
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement, and Model 4 is the economy with time-varying LTV on impatient household.

the economy with the weak macroprudential policy ($\eta_j^1 = 0.25$ for $j = b, i, e$), and the circled red line represents the economy with the aggressive macroprudential policy ($\eta_j^1 = 0.5$ for $j = b, i, e$ hence the coefficient is twice from the weak policy economy).

We first note that in terms of lowering loan responses, the aggressive macroprudential policy is mostly effective. In other words, the aggressive policy achieves its goal to stabilize the financial market more effectively than the weak policy does. However, the negative impacts on the real sector are also amplified (see Figure B.4 to B.6); this comes from the fact that smaller response in loan results in smaller response
Figure B.3: Impulse Response Functions: Productivity Shock

Note: ‘Benchmark’ denotes the economy with $\eta_j^1 = 0$, ‘Weak policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.25$, and ‘Aggressive policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.5$ for $j = b, i, e$. in deposit, which thus increases the responsiveness of consumption and output as discussed above.

Figure B.4: Impulse Response Functions: Default Shock to Entrepreneur

Note: ‘Benchmark’ denotes the economy with $\eta_j^1 = 0$, ‘Weak policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.25$, and ‘Aggressive policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.5$ for $j = b, i, e$. in deposit, which thus increases the responsiveness of consumption and output as discussed above.
Figure B.5: Impulse Response Functions: Housing Preference Shock
Note: ‘Benchmark’ denotes the economy with $\eta_j^1 = 0$, ‘Weak policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.25$, and ‘Aggressive policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.5$ for $j = b, i, e$.

Figure B.6: Impulse Response Functions: Monetary Policy Shock
Note: ‘Benchmark’ denotes the economy with $\eta_j^1 = 0$, ‘Weak policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.25$, and ‘Aggressive policy’ is the economy with countercyclical capital requirement with $\eta_j^1 = 0.5$ for $j = b, i, e$.

C Supplementary Online Appendix: Not for Publication

C.1 Equilibrium Conditions

C.1.1 Patient Household $\lambda_s^t$ the Lagrangian multiplier attached to the budget constraint.
\[ C_t^s = \frac{\varepsilon_i^t}{C_t^s - hC_{t-1}^s} - h\beta_t E_t \left( \frac{\varepsilon_{t+1}}{C_{t+1}^s - hC_t^s} \right) \] (C.1)

\[ K_t^s = \frac{1}{\varepsilon_t^k} + \frac{\partial AC_{K_t^s}}{\partial K_t^s} = \beta_s E_t \left( \frac{\lambda_t^{s+1}}{\lambda_t^s} \left( r_{t+1}^K + 1 - \delta \right) \right) \] (C.2)

\[ H_t^H = \frac{\varepsilon_i^t \lambda_t^s}{H_t^s \lambda_t^s} + \beta_s E_t \left( \frac{\lambda_t^{s+1}}{\lambda_t^s} p_{t+1}^H \right) \] (C.3)

\[ N_t^s = \frac{\nu_s(N_t^s)^\phi}{\lambda_t^s} \] (C.4)

\[ d_t^* = 1 + \frac{\partial AC_{d^*t}}{\partial d_t} = \beta_s E_t \left( \frac{\lambda_t^{s+1}}{\lambda_t^s} r_{t+1}^d \right) \] (C.5)

\[ \lambda_t^s = \frac{K_t^s}{\varepsilon_t^k} + p_{t+1}^H [H_t^s - H_{t-1}^s] + d_t + AC_{d^*t} + AC_{K_t^s} = w_t^s N_t^s + r_{t+1}^d d_t + (r_{t+1}^K + 1 - \delta) K_{t-1}^s \] (C.6)

**C.1.2 Impatient Household** \( \lambda_t^b \) (resp. \( \mu_t^b \)) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

\[ C_t^b = \frac{\varepsilon_i^t}{C_t^b - hC_{t-1}^b} - h\beta_t E_t \left( \frac{\varepsilon_{t+1}}{C_{t+1}^b - hC_t^b} \right) \] (C.7)

\[ H_t^H = \frac{\varepsilon_i^t \lambda_t^b}{H_t^b \lambda_t^b} + \beta_t E_t \left( \frac{\lambda_t^{b+1}}{\lambda_t^b} p_{t+1}^H \right) + \mu_t^b (1 - \rho_b) \gamma_b E_t \left( \frac{p_{t+1}^H}{r_{t+1}^b} \right) \] (C.8)

\[ N_t^b = \frac{\nu_b(N_t^b)^\phi}{\lambda_t^b} \] (C.9)

\[ \lambda_t^b = \frac{1}{\varepsilon_t^b} + \frac{\partial AC_{\lambda_t^b}}{\partial \lambda_t^b} = \mu_t^b + \beta_t E_t \left( \frac{\lambda_t^{b+1}}{\lambda_t^b} \left( r_{t+1}^b - \rho_b \mu_t^b \right) \right) \] (C.10)

\[ C_t^b + p_{t+1}^H [H_t^b - H_{t-1}^b] + r_{t+1}^bh_t + AC_{\mu_t^b} = w_t^b N_t^b + r_{t+1}^b d_t + (r_{t+1}^K + 1 - \delta) K_{t-1}^b \] (C.11)

\[ \mu_t^b = \rho_b d_{t-1} + (1 - \rho_b) \left( H_{t-1}^b \mu_{t-1}^b \right) \] (C.12)

**C.1.3 Entrepreneur** \( \lambda_t^e \) (resp. \( \mu_t^e \)) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).
\[
\lambda_t^e = \frac{1}{C_t^e - hC_{t-1}^e} - \beta_e \varepsilon_t - \beta_e \varepsilon_t \frac{h}{C_{t+1}^e - hC_t^e} \tag{C.13}
\]

\[
p_t^H = \beta_e \varepsilon_t \left[ \frac{\lambda_{t+1}^e}{\lambda_t^e} p_t^H (1 + r_{t+1}^H) \right] + \mu_t^e (1 - \rho_e) \gamma_t^H p_{t+1}^H r_{t+1}^H \tag{C.14}
\]

\[
\frac{1}{\varepsilon_t^e} + \frac{\partial AC_{K^e,t}}{\partial K_t^e} = \beta_e \varepsilon_t \left[ \frac{\lambda_{t+1}^e}{\lambda_t^e} (1 + r_{t+1}^K - \delta) \right] + \mu_t^e (1 - \rho_e) \gamma_t^K \tag{C.15}
\]

\[
(N_t^b)^e = (1 - \rho_e) \gamma_t^N p_t^X X_t \tag{C.16}
\]

\[
(N_t^b)^e = (1 - \rho_e) \gamma_t^N (1 - \omega^n) p_t^X X_t \tag{C.17}
\]

\[
r_t^K = \alpha(1 - \omega_k) p_t^X X_t / K_{t-1}^e \tag{C.18}
\]

\[
l_t^e = 1 - \frac{\partial AC_{c,t}}{\partial l_t^e} = \mu_t^e + \beta_e \varepsilon_t \left[ \frac{\lambda_{t+1}^e}{\lambda_t^e} (r_{t-1}^e - \rho_c \mu_{t+1}^e) \right] \tag{C.19}
\]

\[
\frac{1}{\varepsilon_t^e} + \frac{\partial AC_{K^e,t}}{\partial K_t^e} + \frac{p_t^H [H_t^e - H_{t-1}^e]}{\varepsilon_t^e} + w_t^s N_t^s + w_t^b N_t^b + r_t^K K_{t-1}^e + r_t^e l_{t-1}^e + AC_{K^e,t} + AC_{c,t} \tag{C.20}
\]

\[
l_t^e = \rho_c l_{t-1}^e + (1 - \rho_c) \left( \gamma_t^e p_t^H H_{t-1}^{e*} \frac{r_{t+1}^e}{r_t^f} + \gamma_t^K K_{t-1}^e - \gamma_t^N (w_t^s N_t^s + w_t^b N_t^b) - \varepsilon_t^e \right) \tag{C.21}
\]

where \( X_t = \varepsilon_t^e (K_{t-1}^e)^{1-\omega_K} (K_{t-1}^s)^{1-\omega_K} (H_{t-1}^{e*}) N_t^s (1 - \omega_k)^{1-\omega_k} N_t^b (1 - \omega_k)^{1-\omega_k} \). From the firm’s production function, \( p_t^H r_t^H = \nu p_t^X X_t / H_{t-1}^{e*} \)

**C.1.4 Retail Banks** \( \lambda_t^e \) (resp. \( \mu_t^e \)) the Lagrangian multiplier attached to the budget constraint (resp. capital requirement constraint).
\[ C^r_i \]
\[ d_t \]
\[ l^b_t \]
\[ l^i_t \]
\[ \lambda^r_t \]
\[ \mu^r_t \]
\[ \lambda^l_t \]
\[ \mu^l_t \]

C.1.5 **Investment Banks** \( \lambda^l_t \) (resp. \( \mu^l_t \)) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

\[ C^l_t \]
\[ l^c_t \]
\[ l^b_t \]
\[ l^i_t \]
\[ \lambda^c_t \]
\[ \mu^c_t \]

C.1.6 **Retailers**

\[
\max_{P^t(z)} \sum_{j=0}^{\infty} (\theta_{i+1})^j \lambda^r_{i+1} \left[ P^r_t(z) - P^X_{i+1} \right] Y_{i+1}(z) \\
\text{subject to} \\
Y_{i+1}(z) = \left( \frac{P^r_t(z)}{P^r_{i+1}} \right)^{-\varepsilon} Y_{i+1} 
\]
We can rewrite the problem as:

\[
\max_{P_t^*(z)} \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j} \left[ (P_t^*(z))^{1-\varepsilon} - P_{t+j}^e (P_t^*(z))^{-\varepsilon} \right]
\]

\[
[P_t^*(z)] = \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j} \left[ (1 - \varepsilon) P_t^*(z) + \varepsilon P_{t+j}^X \right] = 0
\]

\[
P_t^* (z) = \frac{\varepsilon \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j} P_{t+j}^X}{\varepsilon - 1 \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j}} \equiv \frac{\varepsilon}{\varepsilon - 1} P_{t,j}^f \tag{C.34}
\]

\[
F_{1,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j} P_{t+j}^X
= \lambda_t^s P_t^e Y_t + \theta \beta_s \mathbb{E}_t F_{1,t+1} \tag{C.35}
\]

\[
F_{2,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_t^s P_{t+j}^e Y_{t+j}
= \lambda_t^s P_t^e Y_t + \theta \beta_s \mathbb{E}_t F_{2,t+1} \tag{C.36}
\]

or equivalently, with \( P_t^* = P_t^*(z) \),

\[
P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{f_{1,t}}{f_{2,t}} P_t \tag{C.37}
\]

\[
f_{1,t} = \lambda_t^s Y_t \mu_t^e + \theta \beta_s \mathbb{E}_t f_{1,t+1} \pi_{t+1}^t \tag{C.38}
\]

\[
f_{2,t} = \lambda_t^s Y_t + \theta \beta_s \mathbb{E}_t f_{2,t+1} \pi_{t+1}^t \tag{C.39}
\]

where \( f_{1,t} = F_{1,t} / P_t^{e+1} \) and \( f_{2,t} = F_{2,t} / P_t^e \).

Therefore, substituting equation (C.37) to the following equation yields
\[ \pi_t = \left(1 - \theta\right) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} + \theta \left[ \frac{\varepsilon f_{1,t}}{\varepsilon - 1 f_{2,t} - \varepsilon} \right]^{1-\varepsilon} \]

hence the equations (C.38), (C.39), and (C.40) implicitly determines \( f_{1,t}, f_{2,t}, \) and \( \pi_t. \)

**C.1.7 Market Clearing Conditions**  
We have

\[ 1 \equiv \bar{H} = H_t^s + H_t^b + H_t^e \]  
\[ Y_t = C_t + K_t + (1 - \delta)K_{t-1} \]  
\[ l_t = l_t^s + l_t^b + l_t^e \]

where \( C_t = C_t^s + C_t^b + C_t^r + C_t^e + C_t^t, K_t = K_t^s + K_t^r, \) and \( Y_t = X_t - \frac{Y_t}{1-\varepsilon}. \) Finally, we have a monetary policy rule (4.24).

**C.2 Robustness Checks**  
In this subsection, we present several robustness check of our finding by altering key policy parameters.

![Figure C.1: Robustness Check: LTV Policy](image)

Note: Relative volatility compared to model 1 at each frequency. Solid blue line represents the benchmark economy with \( \gamma_1^{Hb} = \gamma_1^{Eb} = 0.3 \) and dotted red line represents the alternative economy with \( \gamma_1^{Hb} = \gamma_1^{Eb} = 0.6. \)
Figure C.2: Robustness Check: Extended Taylor Rule
Note: Relative volatility compared to model 1 at each frequency. Solid blue line represents the benchmark economy with $\gamma_l = 0.125$ and dotted red line represents the alternative economy with $\gamma_l = 0.15$.

C.3 Welfare Analysis for Households In this subsection, we present the spectral welfare cost for each household, patient household and impatient household. While we use the same equation, equation (7.3), to obtain the spectral welfare cost, the lifetime utility is now measured at each household level. In other words, we use equation (4.1) for patient household’s welfare and equation (4.4) for impatient household’s welfare. Results are plotted in Figure C.3 and C.4:
Figure C.4: Spectral Welfare Analysis: Impatient Household