Financial Prices and Information Acquisition in Large Cournot Markets *

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Abstract

In the context of a large Cournot market with dispersedly informed firms, we show that while output decisions are strategic substitutes, private information acquisition decisions can be strategic complements. The reversal of incentives operates through the informational role played by the price of a financial asset whose payoff depends on firms' output decisions. Our results rely on a novel mechanism whereby, holding fixed the private information of financial traders, when firms become more privately informed the financial asset price becomes less informative.

Keywords: dispersed information, information acquisition, Cournot market, Grossman-Stiglitz

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1 Introduction

The question of how economic incentives translate into information acquisition incentives has been the subject of a rapidly expanding stream of literature in economics and finance.[1] Within the context of games with coordination motives, a benchmark result - as described by Hellwig and Veldkamp (2009) - establishes that in general, economic and information incentives exactly align, leading to the so-called "Knowing What Others Know" principle: information acquisition incentives are strategic substitutes (resp. complements) when economic incentives are strategic substitutes (resp. complements).

Subsequent studies have investigated whether this principle holds in more sophisticated informational environments. We contribute to this literature by developing a setting in which the informational role played by the price of a financial asset causes economic and information incentives to not align.

Our model consists of a large Cournot market where firms set output strategies and a financial market where traders take positions on a risky asset whose payoff is a function of firms' output. We show that while firms' output decisions are strategic substitutes, their private information investment decisions can be strategic complements. The pivotal mechanism behind this result is the inverse relationship between the precision of firms' private information and the precision of the publicly observable financial price. For a given precision of the private information of traders, when firms are privately more informed their output decisions become less predictable from the perspective of traders. The payoff of the asset is thus perceived as riskier, and traders who are risk-averse, take positions that are less sensitive to their private information. The final result is that less information is aggregated into the asset price.

Turning to information acquisition incentives, as all firms' private information is increased, we show that the marginal value of the individual firm's private information consists of two elements: a *strategic* effect and an *information* effect. The strategic effect takes into account that at the individual level, firms desire to set output differently from the aggregate. They thus value private information because it enables them to condition their strategies on information that is not common knowledge. Hence, whenever the share of information that is common knowledge declines, the marginal value of private information declines as well. In our main exercise, when private information precision is increased across all firms, the share of information that is common knowledge declines for two reasons: (i) because the precision of private information is exogenously increased and (ii) because the precision of the financial

[1] See Veldkamp (2011) for a textbook summary of the literature on information choice.

asset price declines by way of the reverse mechanism described above. The strategic effect thus works in the direction of substitutability: more private information across firms leads to a lower marginal value of private information. However, the decline in the asset price precision triggers an additional effect - the information effect - that works in the direction of complementarity. In the presence of a decline in overall information precision at the individual level, increasing back information precision, whether it is private or common knowledge, always has a positive marginal value. This principle is at work in our context: the drop in precision of the financial price increases the marginal value of the individual firm's private information. In our main proposition, we show that the information effect dominates the strategic effect under a wide range of parameter values. Hence, even though firms' output decisions are strategic substitutes, information acquisition decisions can be strategic complements.

Related Literature. In modeling the interaction of the financial market with the Cournot market, our paper draws from Angeletos and Werning (2006), with the important difference that we model financial traders as being distinct from players in the game, which are firms in our setting. The distinction plays a crucial role in delivering the inverse relationship between the precision of firms' private information and that of the financial price.

In addition to Hellwig and Veldkamp (2009), others have studied information acquisition in coordination settings similar to ours, such as Myatt and Wallace (2012) and Colombo, Femminis, and Pavan (forthcoming). In these papers, a crowding-out effect due to exogenous public information arises: in Myatt and Wallace (2012), when more agents focus on signals that are more public in nature, there is an incentive to discard private signals; in Colombo, Femminis, and Pavan (forthcoming), when public information is more precise agents' collection of private information is diminished. Our result on the inverse relationship between the precision of private information and the precision of the financial price is reminiscent of such crowding-out effects, but it is fundamentally different, as it concerns the notion of private information crowding out endogenous public information. Szkup and Trevino (2013) is perhaps the closest paper to ours in terms of motivation. In Szkup and Trevino (2013) information acquisition can exhibit substitutability even when the underlying game exhibits strategic complementarity. The key mechanism is that other players' better information sometimes decreases the individual cost of incurring a prediction error. Because of this externality, individual players might have an incentive to choose less precise information. Finally, several papers have studied the effects of endogenizing public information in games with coordination motives (e.g., Vives (2014) and Amador and Weill (2012)), but to the best of our knowledge, we are the first to endogenize public information through a financial market that trades claims on the outcome of a Cournot game.

The rest of the paper is organized as follows. Section 2 lays out the model of the Cournot market and the financial market. Section 3 derives the inverse relationship between firms' information precision and asset price precision. Section 4 studies information acquisition incentives. Section 5 concludes with a discussion of the empirical relevance of our results. Proofs can be found in Appendix A when not developed in the main text. A Supplementary Online Appendix contains several extensions of our results.

2 The Model

The model consists of two markets: a Cournot market and a financial market. In the Cournot market, a large number of firms set optimal production strategies, taking the inverse demand as given. In the financial market, a large number of traders decide how to allocate wealth between a risk-free asset and a risky asset. We are primarily interested in the information interaction that takes place between the two markets; we thus abstract from modeling their interaction through resource constraints.

2.1 Cournot Market In modeling the Cournot market, we follow Vives (2008) closely. There is a continuum of firms, indexed by $i \in [0,1]$, and of total measure 1, all producing a homogeneous output good. The individual firm faces the inverse demand curve

$$z = \theta - aQ, \tag{2.1}$$

where z is the unitary price of the firm's output, θ is a demand shock common across firms, Q is aggregate output and $a \geq 0$. Each firm faces a quadratic cost function $c(q) = \frac{1}{2}cq^2$, where $c > 0$ and q denotes the individual firm's output. The individual firm's profits are

$$\Pi(q, Q, \theta) \equiv zq - c(q) = (\theta - aQ)q - \frac{1}{2}cq^2. \tag{2.2}$$

At the time of choosing q, the individual firm does not know θ nor the aggregate production Q, but it observes two signals: a private signal $x_i = \theta + (\alpha_x)^{-\frac{1}{2}}\varepsilon_{x,i}$, with $\varepsilon_{x,i} \sim \mathcal{N}(0,1)$ - uncorrelated with θ and across firms - and $\alpha_x \geq 0$, and a public signal, $p = \theta + (\alpha_p)^{-\frac{1}{2}}\xi$, with $\xi \sim \mathcal{N}(0,1)$ and $\alpha_p \geq$

0. Firms are assumed to have an improper prior on θ over the real line.[2] We restrict our focus to symmetric linear equilibria. An equilibrium is a strategy $q : \mathbb{R}^2 \to \mathbb{R}$, linear in x_i and p, such that $q(x_i, p) = \arg\max_{q'} \mathbb{E}\big[\Pi(q', Q(\theta, p), \theta)|x_i, p\big]$, where $Q(\theta, p) = \int_x q(x, p) d\Psi(x|\theta, p)$, with $\Psi(x|\theta, p)$ denoting the conditional distribution of x given θ and p.[3] The fact that aggregate output Q is only a function of θ and p is a consequence of the i.i.d. assumption on the noise in private information.[4]

Under complete information, the equilibrium for the Cournot market is given by $q^*(\theta) = \rho\theta$, where $\rho \equiv \frac{1}{a+c}$. Under incomplete information, the individual strategy $q(x_i, p)$ is an equilibrium if and only if for all x_i and p, it satisfies[5]

$$q(x_i, p) = \mathbb{E}\big[(1-r) \cdot q^*(\theta) + r \cdot Q(\theta, p)|x_i, p\big], \tag{2.3}$$

where $r \equiv -a/c$. Equation (2.3) shows that individual and aggregate outputs are strategic substitutes in the Cournot market, with the degree of substitutability measured by r. We assume that $a < c$, which results in $r \in (-1, 0]$, restricting our attention to Cournot markets with weak substitutability.[6] Intuitively, when Q increases, the price schedule z shifts downward, and profit maximization is reached at a lower marginal cost, which corresponds to a lower q.

In deriving a closed form for (2.3), it is useful to introduce a notation that keeps the component of the individual firm's optimal strategy that depends on the firm's private information precision, $\alpha_{x,i}$, distinct from the component that depends on other firms' private precision, α_x, while maintaining $\alpha_{x,i} = \alpha_x$ to focus on symmetric equilibria. The equilibrium strategy of firm i can then be written as

$$q(x_i, p) = \rho\big[\psi\Lambda x_i + (1-\psi\Lambda)p\big], \tag{2.4}$$

[2]This assumption is made for analytical convenience so that the precision of the prior information drops from all of the signal extraction formulas. None of our results depend on this assumption, and for all purposes, one can imagine that the variance of the prior on θ is arbitrarily large but bounded.

[3]Note that $q \in \mathbb{R}$, so that negative quantities are a possibility. This is just a drawback of working with analytically convenient Gaussian distributions.

[4]We adopt the usual convention that the strong law of large numbers holds for a continuum of independent random variables with uniformly bounded variances; see the Technical Appendix in Vives (2008). Note that the result holds for a prior on θ with variance that is arbitrarily large but bounded.

[5]We follow the equilibrium characterization used by Angeletos and Pavan (2007).

[6]Relaxing this assumption would enrich the class of equilibria, which would add an additional dimension to our analysis that is beyond the scope of this paper.

where

$$\psi = \frac{\alpha_{x,i}}{\alpha_{x,i} + \alpha_p}, \qquad \text{and} \qquad \Lambda = \frac{\alpha_x + \alpha_p}{\alpha_x + \frac{\alpha_p}{1-r}}. \tag{2.5}$$

Compared to the complete information case, strategy (2.4) substitutes θ with a linear combination of the signals available, x_i and p. The weight that a standard signal extraction exercise would assign to the private signal, ψ, is distorted by the term Λ. For $r < 0$, one has $\Lambda > 1$, and the private signal x_i receives a disproportionately higher weight compared to the public signal p. Firm i, in trying to substitute away from other firms' strategies, ties her production decision to her private information while downplaying public information. Because all firms have the same strategy in a symmetric equilibrium, the distortion carries over in the aggregate so that from a signal extraction perspective, aggregate production is too sensitive to the fundamental θ and too insensitive to the public signal. Letting $\lambda \equiv \psi\Lambda$, when $\alpha_{x,i} = \alpha_x$ one has

$$\lambda = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}, \tag{2.6}$$

so that aggregate output can be expressed as

$$Q(\theta, p) = \rho\big[\lambda\theta + (1-\lambda)p\big]. \tag{2.7}$$

We adopt a Cournot market as the economic backbone of our analysis because it conveniently accommodates both incomplete information and strategic motives. The market can be interpreted as an industry in which the price of an individual firm's output depends on the uncertain state of the aggregate - or industry-wide - demand (θ) and on the unobserved output produced by other suppliers (Q). The profit-maximizing quantity for the individual firm then depends positively on the predicted state of the demand and negatively on the expected output across the industry, which is formalized in (2.3). The private signal x_i can be naturally interpreted as the information that a firm can privately gather about the underlying market conditions, while p represents information about those conditions that is publicly available, such as economic news or prices in financial markets. In our analysis, we model p as the price of a financial asset whose market structure we describe next.

2.2 Financial Market We model the financial market using the CARA-Gaussian framework of Grossman and Stiglitz (1980). The market consists of a large number of risk-averse traders indexed by $j \in [0, 1]$ with total measure 1, whose task is to allocate initial wealth w_0 into a risky asset that promises to pay a dividend d and a riskless asset with unitary return. Denoting the price of the risky asset by p, the utility of trader j is $V(w_j) = -e^{-\gamma w_j}$ with $w_j = w_0 + dk_j - pk_j$ and where k_j denotes trader j's demand for the risky asset.

We assume that the payoff d is a function of the aggregate output of the Cournot market, and for convenience, we set $d = Q$. Traders formulate predictions of the dividend conditional on the equilibrium price p and a private signal $y_j = \theta + (\alpha_y)^{-\frac{1}{2}}\varepsilon_{y,j}$, where $\alpha_y \geq 0$ is the precision of the signal and $\varepsilon_{y,j} \sim \mathcal{N}(0,1)$ is uncorrelated with θ and across traders. Traders' individual demands take the standard "mean-variance" form, $k(y_j, p) = \frac{\mathbb{E}(d|y_j,p)-p}{\gamma\mathbb{V}(d|y_j,p)}$. To prevent the price from perfectly revealing θ, we let the net asset supply be stochastic and equal $\mathbb{K}^s(\xi) = (\alpha_\xi)^{-\frac{1}{2}}\xi$, and we assume that traders do not observe ξ. Market clearing requires $\int_y k(y,p)d\Phi(y) = \mathbb{K}^s(\xi)$, where $\Phi(y)$ is the Gaussian cumulative density.

We focus on linear equilibrium prices of the form $p = \theta - (\alpha_p)^{-\frac{1}{2}}\xi$, so that solving for an equilibrium corresponds to solving for α_p. To simplify the algebra, we set $\rho = 1$ in (2.7),[7] and compute the conditional mean and variance for Q as

$$\mathbb{E}\big[Q(\theta,p)|y_j,p\big] = \lambda\frac{\alpha_y}{\alpha_y+\alpha_p}\big(y_j - p\big) + p, \qquad \text{and} \qquad \mathbb{V}\big[Q(\theta,p)|y_j,p\big] = \frac{\lambda^2}{\alpha_y+\alpha_p}. \tag{2.8}$$

Substituting these expressions into the asset demand and imposing market clearing, the fixed-point condition for α_p becomes

$$\alpha_p = \alpha_\xi\left(\frac{\alpha_y}{\gamma}\right)^2\left(\frac{\alpha_p}{(1-r)\alpha_x}+1\right)^2. \tag{2.9}$$

Equation (2.9) has two positive real solutions: a "high precision" one and a "low precision" one.[8] In addition, for $2\big(\frac{\alpha_y}{\gamma}\big)^2\alpha_\xi < (1-r)\alpha_x$, there is no real solution, and a linear equilibrium for the financial market does not exist.[9] While both levels of price precisions are legitimate equilibria, the use of the

[7] The restriction on ρ corresponds to setting $a + c = 1$. In the Supplementary Online Appendix, we present the analysis with unrestricted ρ and show that the above restriction is without a loss of generality.

[8] Multiplicity originates from the two-way feedback between the signal extraction problem of traders and firms: traders predict a dividend that itself is the outcome of a signal extraction problem, that of firms, which depends on the asset price and thus on traders' dividend predictions in the first place. Multiplicity has been obtained by Ganguli and Yang (2009) and Manzano and Vives (2011) in related settings.

[9] The price precision in equilibrium must be such that traders are willing to hold the risk associated with the asset payoff, which means that any factor that reduces the risk tolerance of traders works against existence. A higher γ directly reduces risk tolerance, a lower α_y increases perceived risk, and a smaller α_ξ increases the size of the net positions that traders have to hold. Finally, a higher $(1-r)\alpha_x$ makes firms rely more on their private information, which increases the perceived risk of the asset payoff. From a technical standpoint, non-existence is the consequence of having assumed that the noise in the private information of traders is uncorrelated, as demonstrated by Manzano and Vives (2011). In the Supplementary Online Appendix, we show that assuming a positive degree of correlation always ensures the existence of

stability criterion proposed by Manzano and Vives (2011) reveals that only the low precision solution is stable. In the following analysis, we restrict our attention to the stable solution.[10]

We now comment on two important features of the above financial market. (i) The assumption that the dividend of the asset is a function of output - an endogenous variable - as opposed to an exogenous fundamental introduces crucial feedback between the two markets of our model, whereby the financial price affects output and output affects the financial price. Arguably, a security with a payoff that is a function of some verifiable measure of performance such as sales or cash flows resembles real-world financial securities more than one whose payoff corresponds to some underlying industry- or economy-wide demand shock.[11] (ii) The financial price p is isomorphic to the price index of a market where a complete set of assets, representing claims on firms' individual outputs, are traded. In Appendix A.2 we show that such a price index is a noisy sufficient statistic for the fundamental θ under the assumption that the stochastic supplies of the individual assets contain an aggregate factor ξ. Focusing on a single asset with payoff equal to firms' aggregate output is, therefore, without a loss of generality.

Finally, we present a technical note on the timing of the two markets. The two-way feedback that we model implies that the financial price formation process and the output decision process happen simultaneously. We think of the interaction as unfolding in two stages. Working backwards, in stage 2 firms set their output strategies given the price p, and a Perfect Bayesian Equilibrium is determined taking the informational properties of p as given. In stage 1, the financial market determines the informational properties of p in a Rational Expectations Equilibrium in which traders act by anticipating the equilibrium of stage 2.

3 Firms' Private Information and Financial Price Precision

We are now in a position to analyze the relationship between the precision of firms' private information in the Cournot market and the precision of the financial asset price. The following proposition states a key result.

at least one equilibrium. In our analysis, we assume that the condition for the existence of two distinct real solutions to (2.9) always holds.

[10] See Appendix A.1 for a formal definition of stability and its application to the solutions of (2.9). Interestingly, the result of Proposition 1 below would be exactly reversed in the unstable equilibrium.

[11] Employing output as a payoff of the asset as opposed to the more textbook-like choice of using firms' profits is also convenient for analytical tractability, as it allows the closed-form derivation of the equilibrium price by avoiding running into quadratic functions of Gaussian distributions.

Proposition 1. *The precision of the financial asset price p is decreasing in the precision of the private information of firms in the Cournot market. Formally,*

$$\frac{\partial \alpha_p}{\partial \alpha_x} < 0. \tag{3.1}$$

The intuition for this result can be gained by looking at the individual traders' asset demand, which, for a trader with private signal y, is

$$k(y,p) = \frac{\alpha_y}{\gamma\lambda}\big(y - p\big). \tag{3.2}$$

Whether the trader takes a long or a short position on the asset depends on whether the private signal y is bigger or smaller than the asset price p. However, the magnitude of the position depends on the precision of a trader's private information, α_y, the degree of risk aversion, γ, and, crucially, the parameter λ. Recall from (2.7) that λ determines how strongly the aggregate output Q depends on the fundamental θ versus the public signal p. When λ is larger, the unobserved fundamental plays a bigger role in determining the asset dividend, which, in turn, makes the dividend become perceived as riskier by traders. In this sense, the role played by λ in (3.2) is analogue to the role played by the risk aversion coefficient γ. Because an increase in α_x corresponds to an increase in λ, it essentially operates as an increase in traders' risk aversion. As traders perceive the asset as riskier, they reduce their trading volumes, which makes the price less reactive to traders' private information, finally resulting in lower precision.[12] The intuition for our result is similar to the "residual risk" effect in Bond and Goldstein (forthcoming). In their setting, a policymaker can affect the dividend of the asset in a Grossman-Stiglitz market. As the policymaker conditions its intervention on more private information, traders perceive the dividend as riskier and reduce trading volumes, which makes the price less informative.[13]

The result of Proposition 1 is reminiscent of the crowding-out effects obtained by Myatt and Wallace (2012) and Colombo, Femminis, and Pavan (forthcoming) discussed in the Introduction. However, while in those studies more precise exogenous public information crowds out private information, ours is the first setting in which the more precise private information of firms crowds out the endogenous precision

[12]The result does not hinge upon traders being able to perfectly observe the asset price p. In Section B.3 of the Online Supplementary Appendix, we show that Proposition 1 holds also in an environment where traders submit orders without being able to condition on the asset price p, as in Kyle (1985). In that setting, the presence of a market-maker that sets the price conditional on her own private information about fundamentals makes traders perceive the asset as riskier when firms' output strategy is more sensitive to fundamentals.

[13]A similar mechanism, labeled the "uncertainty reduction effect," has been also used by Goldstein and Yang (forthcoming) to study strategic complementarities in trading on different types of information.

of public information - the financial price.[14]

It is important to stress that the source of private information that operates in the crowding-out of public information in Proposition 1 is from firms in the Cournot market. Increasing the precision of traders' private information α_y would in fact increase the precision of the financial price - a standard result in noisy rational expectations models. The endogeneity of the dividend is also important. If one were to assume an exogenous dividend such as $d = \theta$, the price precision would be $\alpha_p = \alpha_\xi \left(\frac{\alpha_y}{\gamma}\right)^2$, which is clearly invariant to α_x.

4 Information Acquisition

In this section, we relax the assumption of fixed private information precision, and we allow firms to vary the precision of the private signals they receive, α_x. We are interested in characterizing how the marginal value of the individual firm's private information is affected by the change in private information across all firms. The novelty of the analysis lies in the fact that the individual firm anticipates that the change in the information acquisition of the rest of the firms impacts the information precision of the financial price according to Proposition 1. This is the source of complementarity in information acquisition. Information choice happens at stage 0, before the financial market and the Cournot market operate. In stage 0, firms choose how much precision to acquire; subsequently, nature draws the vector of stochastic shocks. Stages 1 and 2, as described in Section 2, follow.[15]

4.1 Modeling Information Choice We assume that the precision of private information for an individual firm depends on the amount of information acquisition effort n that the firm exerts at stage 0. We capture this relationship by denoting the precision by $\alpha_x(n)$, where the function $\alpha_x(\cdot)$ is assumed to be non-negative, non-decreasing, weakly concave and differentiable.[16] The private signal received by

[14] In Rondina and Shim (2014), we show that a similar result holds when the dividend of the asset is a function of the outcome of a game of regime change, such as a currency attack game or a bank run game. We use the result to show that when the information precision of players in the game is made arbitrarily precise, the introduction of an endogenous public signal does not necessarily imply multiplicity of equilibria, a result that qualifies the criticism of Angeletos and Werning (2006) to the uniqueness result of Morris and Shin (1998).

[15] We focus on symmetric Perfect Bayesian Equilibria of the three-stage extensive form game and use backward induction to characterize the equilibrium strategies. Because each individual agent is infinitesimal, beliefs that are off the equilibrium path need not be specified, as any deviation from a single player cannot possibly be detected.

[16] Our analysis would not change if we allowed firms to choose the precision level rather than effort directly. We model the precision choice through effort purely for notational convenience to more easily keep track of which signal precisions are controlled by the individual firms and which are taken as given but still endogenous in equilibrium as they are a function of effort of all other firms.

firm i is now $x_i(n) = \theta + [\alpha_x(n)]^{-\frac{1}{2}}\varepsilon_{x,i}$. Firms choose n, taking into account how private information precision affects their expected payoff.[17] We denote expected profits at the beginning of stage 0 as $\mathbb{E}_0\left[\Pi^*(q, Q, \theta)\right]$, where Π^* is the payoff function conditional on equilibrium strategies being played at stages 1 and 2, while $\mathbb{E}_0$ denotes that expectations are taken with respect to the common prior before signals are realized.

For simplicity, we focus on symmetric equilibria in information effort choice, and we let $\mathbf{n}$ represent the common information acquisition effort across firms, which is taken as given by any individual firm. To recognize that effort $\mathbf{n}$ affects the precision of p, we denote the financial price precision by $\alpha_p(\mathbf{n})$. We can then embed the information choice of stage 0 in the characterization of the optimal strategy for the individual firm by writing the coefficients in (2.5) as $\psi = \psi(n, \mathbf{n}) \equiv \frac{\alpha_x(n)}{\alpha_x(n)+\alpha_p(\mathbf{n})}$ and $\Lambda = \Lambda(\mathbf{n}) \equiv \frac{\alpha_x(\mathbf{n})+\alpha_p(\mathbf{n})}{\alpha_x(\mathbf{n})+\frac{\alpha_p(\mathbf{n})}{1-r}}$.

Information sets at stage 0 are the same across all agents and consist of the structure of the game and the common improper prior over θ. Because the profit function is quadratic, the expected profits at stage 0 depend only on unconditional second moments. To economize our notation, let $\pi(n, \mathbf{n}) \equiv \mathbb{E}_0\left[\Pi^*(q, Q, \theta)\right]$; in Appendix A.4, we show that

$$\pi(n, \mathbf{n}) \propto \psi(n, \mathbf{n}) \frac{\Lambda(\mathbf{n})^2}{\alpha_p(\mathbf{n})} + \tilde{\pi}(\mathbf{n}), \tag{4.1}$$

where $\tilde{\pi}(\mathbf{n})$ does not depend on the choice variable n. The first term on the right-hand side of (4.1) corresponds to the sum of the unconditional second moments $\mathbb{E}_0[qQ]$ and $\mathbb{E}_0[q^2]$, net of the unconditional variance of θ. Intuitively, expected profits are increasing in $\psi(n, \mathbf{n})$ because as the precision of private information improves, the impact of the variance of the noise in q is reduced. Inspection of (4.1) also reveals that when the precision of the public information $\alpha_p(\mathbf{n})$ is varied, expected profits are affected through both ψ and Λ. This property plays a central role in our information acquisition results, to which we now turn.

4.2 Information Incentives in the Cournot Market As an intermediate step towards the main result of this section, it is useful to study the marginal value of private information acquisition

[17] Because our focus is on characterizing the marginal value of private information, we abstract from modeling the cost of information acquisition effort. As long as the marginal cost of effort does not depend on any of the equilibrium variables, any cost-related term would not affect our analysis of acquisition incentives.

effort n, which is

$$\pi_n \equiv \frac{\partial \pi(n,\mathbf{n})}{\partial n} \propto \frac{\partial \alpha_x(n)}{\partial n}\left(\frac{\Lambda(\mathbf{n})}{\alpha_x(n)+\alpha_p(\mathbf{n})}\right)^2. \tag{4.2}$$

The marginal value is always positive: everything else equal, when more information is available at the individual level, predictions are more accurate and second moments are lower. Note that π_n depends on two factors in addition to the exogenous change α_x: the distortion term $\Lambda(\mathbf{n})$ and a measure of the overall precision of the individual firm's information, $\left[\alpha_x(n)+\alpha_p(\mathbf{n})\right]$.

Complementarity or substitutability in information acquisition for firms can be evaluated by studying the sign of the cross-partial derivative

$$\pi_{n\mathbf{n}} \equiv \frac{\partial \pi_n}{\partial \mathbf{n}}. \tag{4.3}$$

When the sign of $\pi_{n\mathbf{n}}$ is positive, the marginal value of additional information at the individual firm level π_n is increasing in the precision of firms' private information at the aggregate level $\mathbf{n}$. In such a case, we say that a firm in the Cournot market exhibits complementarity in information acquisition with respect to other firms. When the sign is negative, the value of additional information at the individual level is decreasing with the precision of information at the aggregate level. In this case, we say that the firm exhibits substitutability in information acquisition.

Given our definition of $\Lambda(\mathbf{n})$, from (4.2), it is evident that π_n depends on $\mathbf{n}$ through $\alpha_x(\mathbf{n})$ and $\alpha_p(\mathbf{n})$. Both terms influence the distorting factor $\Lambda(\mathbf{n})$, but only the financial price precision matters for the overall precision of the individual information $\left[\alpha_x(n)+\alpha_p(\mathbf{n})\right]$. Differentiating π_n with respect to $\mathbf{n}$, one obtains

$$\pi_{n\mathbf{n}} \propto \Lambda_r \frac{\partial \alpha_x(\mathbf{n})}{\partial \mathbf{n}} + \left(-\Lambda_r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{n})} + \lambda_\alpha\right)\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}}, \tag{4.4}$$

where

$$\Lambda_r \equiv r\frac{(1-r)\alpha_p(\mathbf{n})}{(\alpha_p(\mathbf{n})+(1-r)\alpha_x(\mathbf{n}))^2}, \qquad \text{and} \qquad \lambda_\alpha \equiv -\frac{\Lambda(\mathbf{n})}{\alpha_x(n)+\alpha_p(\mathbf{n})}. \tag{4.5}$$

Recall that information precision and effort are positively related by assumption, which means that $\frac{\partial \alpha_x(\mathbf{n})}{\partial \mathbf{n}} > 0$. The application of Proposition 1 then immediately implies $\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} < 0$. In addition, for $r < 0$, one has $\Lambda_r < 0$, while λ_α is negative under any parametrization. We refer to Λ_r as the "strategic effect" coefficient, as its existence and magnitude depend on r, while we refer to λ_α as the "information effect" coefficient.

To interpret expression (4.4), it is useful to abstract for a moment from the change in the precision of the financial price and set $\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} = 0$. Note that this would correspond to assuming that the precision of p is exogenous. The following Lemma holds.

Lemma 1. *In the Cournot market with substitutability parameter r and with exogenous public signal p, the effort in private information acquisition at the individual firm level n and the aggregate effort $\mathbf{n}$ are always strategic substitutes.*

Because $r < 0$, one immediately has $\pi_{n\mathbf{n}} < 0$ from (4.4). Lemma 1 corresponds to the "Knowing What Others Know" principle of Hellwig and Veldkamp (2009). To understand the result, note that when p is exogenous, $\pi_{n\mathbf{n}} \propto \frac{\partial \Lambda(\mathbf{n})}{\partial \mathbf{n}}$. Recall that $\Lambda(\mathbf{n}) > 1$ measures the upwards distortion of the individual signal extraction coefficient ψ so that private information influences q more than public information. When $\mathbf{n}$ is increased, $\alpha_x(\mathbf{n})$ is increased and public information is further downplayed by all firms. For the individual firm, it is then optimal to reduce the degree of distortion by lowering $\Lambda(\mathbf{n})$, as public information has become less relevant. Lemma 1 is entirely driven by a strategic effect: private information is less valuable because it is less useful in differentiating one's action from the rest of the firms.

Now let p be endogenous. The resulting change in the precision of the asset price activates two additional effects. The first effect is a strategic effect similar to that of Lemma 1. As the precision of public information declines, all firms assign more relevance to their private information, which reduces the distortion $\Lambda(\mathbf{n})$, making private information less valuable. For $r < 0$, one has $-\Lambda_r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{n})} \frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} < 0$, so the "Knowing What Others Know" effect is reinforced.

There is a second effect - the information effect - that works in the opposite direction and is measured by $\lambda_\alpha \frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} > 0$. Recall that the financial price precision impacts $\left[\alpha_x(n) + \alpha_p(\mathbf{n})\right]$ in addition to $\Lambda(\mathbf{n})$ in the marginal value (4.2). The increase in the precision of private information across firms reduces the precision of the asset price, which in turn reduces the overall precision of information for the individual firm. Regardless of r, when less information is available, there is always a positive value in increasing the precision of information, which means that putting effort in increasing private information precision is desirable. The information effect thus works in the direction of complementarity.

While the existence of the strategic effect depends on r being different than zero, the information effect also operates when $r = 0$. This suggests that there are situations when r is not too large in absolute value, and the complementarity effect in information acquisition dominates the substitutability effect. We formalize this argument in a proposition.

Proposition 2. *In the Cournot market with substitutability parameter r and with public signal p generated by the financial market of Section 2.2, there exists a constant $r^* < 0$ such that for $\{r : r^* < r < 0\}$, the effort in private information acquisition at the individual firm level n and the aggregate effort $\mathbf{n}$ are strategic complements.*

To argue the existence of r^*, it is sufficient to consider the case of $r = 0$ so that $\pi_{n\mathbf{n}} > 0$ because $\Lambda_r = 0$. As r turns negative, by continuity of Λ_r, $\frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{n})}$ and $\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}}$ with respect to r, one can always find an open ball around $r = 0$ such that $\pi_{n\mathbf{n}} > 0$.

In the symmetric information acquisition equilibrium, where $n = \mathbf{n}$, the condition for r^* of Proposition 2 can be simplified to

$$r^* = -2\frac{\alpha_p(\mathbf{n})}{\alpha_x(\mathbf{n})}. \tag{4.6}$$

Note that $\alpha_p(\mathbf{n})$ is itself a function of r^*, which makes (4.6) non-linear in r^*. Everything else equal, when the precision of the financial price is higher, $|r^*|$ is bigger and the class of Cournot markets that feature complementarity in information acquisition is larger. Letting $\alpha_y = \alpha_\xi = 1$, $\gamma = 2$, and $\alpha_x(\mathbf{n}) = 1$, condition (4.6) returns $r^* = -.73$. In this case, firms in a Cournot market with substitutability parameter $r = -.5$ would display complementarity in private information acquisition. Consider now what happens if the risk aversion of financial traders is increased to $\gamma = 3$. Condition (4.6) informs us that the critical substitutability value would increase to $r^* = -.27$ in response. Firms in a Cournot market with $r = -.5$ would now display substitutability in information acquisition. The reason for the change in the direction of incentives lies, as expected, in the marginal value of increasing private information, $\lambda_\alpha \frac{\partial \alpha_p}{\partial \alpha_x}$. While the value remains positive, its magnitude becomes smaller as the marginal change in price precision drops from -0.15 to $-.01$. Intuitively, when traders are more risk-averse, the mechanism of Proposition 1 is weaker, which makes the overall information effect smaller, pushing the information incentives back towards the "Knowing what Others Know" benchmark of Lemma 1.

4.3 Traders' Information Acquisition

Up to this point, we have maintained the assumption that traders cannot invest in information acquisition. In this section, we relax the assumption and study the information incentives of firms when traders change their information acquisition effort.[18] Let m denote the individual effort of traders and $\mathbf{m}$ its aggregate counterpart, and let $\alpha_y(m)$ denote the precision of private information for a trader that exerts effort m.[19] We use $\alpha_p(\mathbf{m})$ to denote the dependence of the financial price on traders' information acquisition. We are now interested in studying the cross-partial $\pi_{n\mathbf{m}} \equiv \frac{\partial^2 \pi(n,\mathbf{m})}{\partial n\, \partial \mathbf{m}}$, where $\pi(n, \mathbf{m})$ denotes the expected profits for firms when α_y is substituted with $\alpha_y(\mathbf{m})$ in (4.1). It can be shown that

$$\pi_{n\mathbf{m}} \propto \left(-\Lambda_r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{m})} + \lambda_\alpha \right) \frac{\partial \alpha_p(\mathbf{m})}{\partial \mathbf{m}}, \tag{4.7}$$

where Λ_r and λ_α are as in (4.5). Adjusting the proof of Proposition 1 to the case of changes in traders' information precision α_y, one can show that $\frac{\partial \alpha_p(\mathbf{m})}{\partial \mathbf{m}} > 0$. Intuitively, when traders are more informed the dividend is less risky, which leads to larger trading volumes and more information aggregated in the asset price.

Setting $r = 0$ in (4.7) reveals that $\pi_{n\mathbf{m}} < 0$. In other words, the information effect here implies substitutability: as the public signal is more informative, the marginal value of firms' private information declines. When $r < 0$, the strategic effect Λ_r comes into play, working in the opposite direction. With more information that is common knowledge, it is valuable for firms to invest in private information to distance q from aggregate output. The question is then whether the strategic effect can be larger than the information effect, therefore turning substitutability into complementarity. Plugging Λ_r and λ_α into (4.7), complementarity results when

$$-r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{m}) + (1-r)\alpha_x(\mathbf{n})} - \frac{\alpha_p(\mathbf{m}) + \alpha_x(\mathbf{n})}{\alpha_p(\mathbf{m}) + \alpha_x(n)} > 0. \tag{4.8}$$

For this to be possible, one needs $\alpha_x(n)$ to be much bigger than $\alpha_x(\mathbf{n})$. Because in a symmetric equilibrium $\alpha_x(n) = \alpha_x(\mathbf{n})$, the above inequality would be unattainable for any $r \in (-1, 0]$. Our last proposition immediately follows.

Proposition 3. *In the Cournot market with public signal p generated by the financial market of Section 2.2 and symmetric equilibrium $n = \mathbf{n}$, the effort in private information acquisition at the individual firm*

[18] In the Online Supplementary Appendix, we report the analysis of the information incentives of traders when firms and traders change their information effort in the aggregate.

[19] The function $\alpha_y(\cdot)$ is assumed to have the same properties as $\alpha_x(\cdot)$, and the timing of the information choice is the same as the one for firms.

level n and the acquisition effort of traders $\mathbf{m}$ *are always strategic substitutes.*

The difference between Propositions 2 and 3 highlights the importance of recognizing how endogenous public information varies with private information. Throughout our analysis, we have assumed that traders and firms are distinct, which essentially corresponds to the notion of firms taking positions with negligible size - as a whole - in the financial market. Because more precise private information for firms or traders has opposite effects on the precision of the asset price, it is natural to ask what would happen if firms and traders overlapped. The answer to this question depends on the degree of overlapping. To see this, let μ denote the measure of firms that trade in the financial market as a share of the set of traders, and denote the precision of their private information by α_x. Correspondingly, the share of traders that are not firms is $1-\mu$, with private information precision denoted by α_y. The financial price precision therefore corresponds to

$$\alpha_p = \alpha_\xi \left(\frac{\mu\alpha_x + (1-\mu)\alpha_y}{\gamma}\right)^2 \left(\frac{\alpha_p}{(1-r)\alpha_x} + 1\right)^2. \tag{4.9}$$

If firms and traders exactly coincide so that $\mu = 1$, the result of Proposition 1 would reverse to $\frac{\partial \alpha_p}{\partial \alpha_x} > 0$, and the condition for complementarity in information acquisition would become unattainable in a symmetric equilibrium, as suggested by Proposition 3. The reason for this is found in the reversal of the information effect, $\lambda_\alpha \frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} < 0$: with a more informative financial price, private information precision is less valuable. The strategic effect is also reversed, but only for the portion that is related to the asset price, while for the portion due to the aggregate private information the effect is still negative. Clearly, the applicability of Proposition 2 depends on whether the value of μ is below or above some critical threshold level μ^* such that $\pi_{n\mathbf{n}} = 0$ for the particular context at hand. In many applications, it seems reasonable to think that the pool of traders is much larger than the pool of firms, so that while firms can trade in the financial market, they still represent only a small subset of market traders. At the same time, firms can sometime be large traders in specific financial markets in terms of volumes. In the end, whether μ is above or below the critical threshold μ^* remains a question of an empirical nature.

As a final remark, we note that Proposition 3 also captures the mechanism by which a change in the volatility of the stochastic supply $(\alpha_\xi)^{-\frac{1}{2}}$ can affect the information acquisition of firms. To see this, recall from Section 2.2 that an increase in α_ξ increases the precision of the financial price α_p. The marginal value of private information for an individual firm then takes the form (4.7), with $\frac{\partial \alpha_p}{\partial \alpha_\xi}$ replacing

$\frac{\partial \alpha_p(\mathbf{m})}{\partial \mathbf{m}}$. Applying the argument of Proposition 3 results in the value of effort in private information acquisition at the individual firm level n to be increasing in the volatility of the asset supply, ξ.

5 Conclusion

We conclude with a discussion on the empirical relevance of our results. Propositions 1 and 2 can be used to draw implications in terms of observed volatility in financial prices, such as stock prices. In our model, fluctuations in financial prices depend on two factors, θ and ξ. The former captures underlying economic fundamentals, such as real cash flows, while the latter captures conditions specific to financial markets, such as noise or liquidity trading. Proposition 1 indicates that the volatility unrelated to economic fundamentals, i.e. due to ξ, increases with the precision of private information held by firms traded in the market. The complementarity in information acquisition uncovered in Proposition 2 acts as an amplification mechanism that can turn small changes in the precision of firms' private information into larger changes in price volatility. Suppose that one can isolate an exogenous change in the cost of producing private information for firms - possibly as a consequence of changes in information technology or regulation, our results suggest two possible empirical implications. A first implication, testable with cross-sectional price data (and provided one can perform the challenging task of measuring the degree of substitutability at the individual firm level), is that stock prices of firms in Cournot markets with low substitutability should exhibit a substantially larger increase in volatility unrelated to economic fundamentals, compared to those of firms in markets with stronger substitutability. A second implication, testable with data at a more aggregate level, is that the volatility of financial markets should become less responsive to the volatility of indicators related to economic fundamentals, such as industrial production. In this respect, Engle, Ghysels, and Sohn (2013) document that the ratio of the standard deviations of daily U.S. stock returns and of industrial production growth increased from 1, for the period 1953-1984, to 3, for the period 1985-2004. Under the assumption that the cost of producing private information for firms declined in the latter period because of information technology improvements, the increase in the ratio is consistent with the mechanism of our model: more precise private information in output markets heightened the perceived riskiness of stock returns, which reduced the sensitivity of stock prices to industrial production fundamentals, and increased the relevance of fluctuations due to noise or liquidity trading. How much of the increase in the volatility ratio can be attributed to such a mechanism clearly requires the analysis of a more elaborated model than the one presented above. Nonetheless, our results identify an amplification channel due to strategic complementarity in information acquisition that bears the potential to make the mechanism quantitatively relevant. We leave such empirical analyses to future work.

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A Appendix A: Proofs

A.1 Stability Analysis The fixed point condition (2.9) has two distinct real solutions when $2\left(\frac{\alpha_y}{\gamma}\right)^2\alpha_\xi > (1-r)\alpha_x$. Both solutions are legitimate rational expectations equilibria, but they feature different stability properties around the equilibrium point. To be clear, if one considers the stability of the equilibria with respect to variations in the asset price, both equilibria are stable in the sense that the asset demand is always downward sloping at the equilibrium point in both cases. On these grounds, both solutions could be considered as acceptable from a theoretical point of view. This is, for instance, the notion of stability that is advocated by Ganguli and Yang (2009). We instead follow Manzano and Vives (2011) and argue that a different notion of stability needs to be applied for fixed-point conditions such as (2.9). Specifically, Manzano and Vives (2011) propose that if the equilibrium fixed-point condition can be written as $\alpha \equiv f(\alpha)$, stability should be defined as

Definition. *A fixed point α is stable if $|f'(\alpha)| < 1$.*

In our case, $f(\alpha) \equiv \alpha_\xi\left(\frac{\alpha_y}{\gamma}\right)^2\left(\frac{\alpha}{(1-r)\alpha_x}+1\right)^2$, which leads to the following result.

Proposition A1. *The high-precision solution to (2.9) is unstable; the low-precision solution is stable.*

Proof. Let $\delta \equiv \sqrt{\alpha_\xi}\alpha_y/\gamma$. The derivative $f'(\alpha_p)$ is

$$f'(\alpha_p) = 2\left(\frac{\delta}{(1-r)\alpha_x}\right)^2 \cdot \alpha_p + \frac{2\delta^2}{(1-r)\alpha_x}. \tag{A.1}$$

The solutions satisfying (2.9) are

$$\alpha_p = \frac{\left(1-\frac{2\delta^2}{(1-r)\alpha_x}\right) \pm \sqrt{1-4\delta^2\frac{1}{(1-r)\alpha_x}}}{2\left(\frac{\delta}{(1-r)\alpha_x}\right)^2}. \tag{A.2}$$

Substituting (A.2) into (A.1) yields

$$f'(\alpha_p) = 1 \pm \sqrt{1-4\delta^2\frac{1}{(1-r)\,\alpha_x}}. \tag{A.3}$$

The existence of two distinct equilibria requires $1-4\delta^2\frac{1}{(1-r)\alpha_x} > 0$, from which the result immediately follows. □

A.2 Single Financial Asset and Aggregate Price Index In this section, we show that the price of the financial asset we consider in Section 2.2 - whose payoff is aggregate output Q - is isomorphic to the price index of a financial market where a continuum of assets are traded, each with a payoff equal to the output of individual firms, q_i. Let the financial price of the asset with payoff q_i (henceforth "asset i") be denoted by p_i and define the aggregate index p as

$$p \equiv \int_0^1 p_i di. \tag{A.4}$$

In the financial markets a large number of risk-averse traders operates by conditioning their expectations on a trader-specific private signal $y_j = \theta + (\alpha_y)^{-\frac{1}{2}}\varepsilon_{y,j}$ and on the entire distribution of the financial prices $\{p_i, i \in [0,1]\}$. Because all of the assets are ex-ante identical, the optimal portfolio for any trader is to equally diversify across all assets. We therefore proceed by analyzing the demand of an individual trader for an arbitrary asset i. We assume that the stochastic supply of asset i consists of an aggregate factor, $(\alpha_\xi)^{-\frac{1}{2}}\xi$, with ξ distributed as $\mathcal{N}(0,1)$, and an asset-specific component $(\alpha_\eta)^{-\frac{1}{2}}\eta_i$, with η_i distributed as $\mathcal{N}(0,1)$, independent of the distribution of ξ. The aggregate factor in the individual asset supplies can naturally capture the noise trading that arises as a consequence of some aggregate liquidity shock that is unrelated to θ. For our purposes, an important consequence of the presence of the aggregate factor ξ is that the index p becomes a linear combination of the two aggregate shocks, θ and ξ, and thus fails to perfectly reveal the underlying fundamental θ.[20] We conjecture the functional form of the individual price as

$$p_i = b_\theta \theta + b_\xi \xi + b_\eta \eta_i. \tag{A.5}$$

Under our conjecture for p_i, the aggregate index takes the form $p = b_\theta\theta + b_\xi\xi$. Note that if our conjecture is verified, the price index p is a sufficient statistic for θ given the entire distribution of financial prices. This is because any of the p_i's contain the same noise term as p plus an idiosyncratic component. It follows that firms only employ the signal p in their optimal strategies. Let $\tilde{p} \equiv b_\theta^{-1} p$; the individual output q_i can then be written as $q_i = \lambda x_i + (1-\lambda)\tilde{p}$, where $\lambda = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}$ and $\alpha_p = (b_\theta/b_\xi)^2$. The conditional moments for a trader with signal y_j are

$$\mathbb{E}(q_i|y_j,\tilde{p}) = \lambda\left(\frac{\alpha_y}{\alpha_y+\alpha_p} y_j + \frac{\alpha_p}{\alpha_y+\alpha_p}\tilde{p}\right) + (1-\lambda)\tilde{p}, \quad \text{and} \quad \mathbb{V}(q_i|y_j,\tilde{p}) = \frac{\lambda^2}{\alpha_y+\alpha_p}. \tag{A.6}$$

The demand for asset i of trader j takes the usual mean-variance form $k_j^i = \frac{\mathbb{E}(q_i|y_j,\tilde{p}) - p_i}{\gamma \mathbb{V}(q_i|y_j,\tilde{p})}$. Substituting the expressions in (A.6) into k_j^i and aggregating over traders, one obtains the total demand for asset i as

$$k^i = \frac{\lambda\left(\frac{\alpha_y}{\alpha_y+\alpha_p}\right)\left(b_\theta\theta - p_i + b_\eta\eta_i\right) + p_i(1-b_\theta) - b_\eta\eta_i}{b_\theta \frac{\gamma\lambda^2}{\alpha_y+\alpha_p}}, \tag{A.7}$$

where we have used the relationship $\tilde{p} = (1/b_\theta)p_i - (b_i/b_\theta)\eta_i$ to substitute for $\tilde{p}$. Market clearing for asset i is then

$$k^i = (\alpha_\xi)^{-\frac{1}{2}}\xi + (\alpha_\eta)^{-\frac{1}{2}}\eta_i. \tag{A.8}$$

[20] Assuming more aggregate shocks than aggregate signals to avoid perfect revelation is a common practice in macroeconomic models of incomplete information. See, for instance, Angeletos and La'O (2009) and Amador and Weill (2010).

Substituting (A.7) into (A.8), one can solve for p_i and then match the coefficients with (A.5), which returns $b_\theta = 1$, $b_\xi = -\frac{\gamma\lambda}{\alpha_y}(\alpha_\xi)^{-\frac{1}{2}}$, and $b_\eta = -\frac{\gamma\lambda^2}{\alpha_p+\alpha_y}(\alpha_\eta)^{-\frac{1}{2}}$. It follows that $p = \tilde{p}$, and the aggregate price index satisfies

$$p = \theta - \frac{\gamma\lambda}{\alpha_y}(\alpha_\xi)^{-\frac{1}{2}}\xi. \tag{A.9}$$

The fixed point for α_p implied by (A.9) is indeed equivalent to (2.9). As a consequence, we can conveniently focus on a single financial asset with return Q, knowing that its price is equivalent to the price index of a market where a complete set of claims on firms' individual outputs are traded.

A.3 Proof of Proposition 1 Once again let $\delta \equiv \sqrt{\alpha_\xi}\alpha_y/\gamma$. Rearranging equation (A.2) the stable solution satisfies

$$2\left(\frac{\delta}{(1-r)\,\alpha_x}\right)^2\alpha_p - \left(1 - \frac{2\delta^2}{(1-r)\,\alpha_x}\right) = -\sqrt{1 - 4\delta^2\frac{1}{(1-r)\,\alpha_x}}. \tag{A.10}$$

The expression for $\partial\alpha_p/\partial\alpha_x$ is obtained by differentiating equation (2.9) with respect to α_x to get

$$\frac{\partial\alpha_p}{\partial\alpha_x} = -\frac{2\frac{\delta^2}{(1-r)}\left(\frac{\alpha_p}{(1-r)\alpha_x}+1\right)\frac{\alpha_p}{\alpha_x^2}}{\left(1-\frac{2\delta^2}{(1-r)\alpha_x}\right) - 2\left(\frac{\delta}{(1-r)\alpha_x}\right)^2\alpha_p}. \tag{A.11}$$

Substituting equation (A.10) into (A.11) yields

$$\frac{\partial\alpha_p}{\partial\alpha_x} = -\frac{\frac{\delta^2}{(1-r)}\left(\frac{\alpha_p}{(1-r)\alpha_x}+1\right)\frac{\alpha_p}{\alpha_x^2}}{\sqrt{1-4\delta^2\frac{1}{(1-r)\alpha_x}}} < 0, \tag{A.12}$$

which completes the proof.

A.4 Derivation of $\pi(n, \mathbf{n})$ Let $\Pi^*(q, Q, \theta)$ be the payoff function conditional on equilibrium strategies being played at stages 1 and 2. In the main text we defined $\pi(n, \mathbf{n}) \equiv \mathbb{E}_0\big[\Pi^*(q, Q, \theta)\big]$, where $\mathbb{E}_0$ denotes unconditional expectations taken before signals are realized. Recall that the profit function is specified as

$$\Pi(q, Q, \theta) = \theta q - aQq - \frac{1}{2}cq^2. \tag{A.13}$$

For convenience, we define $\lambda(n, \mathbf{n}) = \Lambda(\mathbf{n})\psi(n, \mathbf{n})$ so that individual output can be written as $q(x_i, p) = \lambda(n, \mathbf{n})x_i(n) + (1 - \lambda(n, \mathbf{n}))p(\mathbf{n})$. In a symmetric equilibrium $n = \mathbf{n}$, which implies $\lambda(\mathbf{n}, \mathbf{n}) = \lambda(\mathbf{n}) \equiv \frac{\alpha_x(\mathbf{n})}{\alpha_x(\mathbf{n})+\frac{\alpha_p(\mathbf{n})}{1-r}}$. Aggregate output can finally be written as $Q(\theta, p(\mathbf{n})) = \lambda(\mathbf{n})\theta + (1 - \lambda(\mathbf{n}))p(\mathbf{n})$. Using the specific form of the signals, the individual strategy for firm i in the Cournot market is

$$\begin{aligned} q(x_i(n), p(\mathbf{n})) &= \lambda(n, \mathbf{n})(\theta + \alpha_x(n)^{-1/2}\varepsilon_{x,i}) + (1 - \lambda(n, \mathbf{n}))(\theta - \alpha_p(\mathbf{n})^{-1/2}\xi) \\ &= \theta + \lambda(n, \mathbf{n})\alpha_x(n)^{-1/2}\varepsilon_{x,i} - (1 - \lambda(n, \mathbf{n}))\alpha_p(\mathbf{n})^{-1/2}\xi. \end{aligned}$$

Aggregating over firms

$$Q(\theta, p(\mathbf{n})) = \theta - (1 - \lambda(\mathbf{n}))\alpha_p(\mathbf{n})^{-1/2}\xi. \tag{A.14}$$

We can now derive the expressions for $\mathbb{E}_0(q^2)$, $\mathbb{E}_0(q\theta)$ and $\mathbb{E}_0(qQ)$. Using the equilibrium strategies one has

$$q^2 = \theta^2 + \lambda(n, \mathbf{n})^2\alpha_x(n)^{-1}(\varepsilon_{x,i})^2 + (1 - \lambda(n, \mathbf{n}))^2\alpha_p(\mathbf{n})^{-1}\xi^2 + \mathcal{C}_1(\theta, \varepsilon_{x,i}, \xi), \tag{A.15}$$

where $\mathcal{C}_1(\theta, \varepsilon_{x,i}, \xi)$ summarizes all of the relevant covariance terms that are zero under $\mathbb{E}_0$. Similarly,

$$q\theta = \theta^2 + \mathcal{C}_2(\theta, \varepsilon_{x,i}, \xi), \tag{A.16}$$

$$qQ = \theta^2 + (1 - \lambda(n, \mathbf{n}))(1 - \lambda(\mathbf{n}))\alpha_p(\mathbf{n})^{-1}\xi^2 + \mathcal{C}_3(\theta, \varepsilon_{x,i}, \xi). \tag{A.17}$$

In applying $\mathbb{E}_0$ to these expressions, recall that $\mathbb{E}_0(\xi^2) = \mathbb{E}_0((\varepsilon_{x,i})^2) = 1$. Under the improper prior assumption one would also have that $\mathbb{E}_0(\theta^2)$ is not well defined. As discussed in the paper, we imagine the unconditional variance of the prior on θ as very large, but bounded, and we use the improper prior assumption to simplify the notation for the signal extraction problem. Substituting the above expressions into the expected profits, one has

$$\mathbb{E}_0\big[\Pi^*(q, Q, \theta)\big] = -\frac{1}{2}c\left[\frac{[\lambda(n, \mathbf{n})]^2}{\alpha_x(n)} + \frac{(1 - \lambda(n, \mathbf{n}))^2}{\alpha_p(\mathbf{n})}\right] - a(1 - \lambda(n, \mathbf{n}))\frac{1 - \lambda(\mathbf{n})}{\alpha_p(\mathbf{n})} + \bar{\pi}, \tag{A.18}$$

where $\bar{\pi} \equiv \mathbb{E}_0(\theta^2)\big[1 - a - \frac{1}{2}c\big]$. Recall that $\lambda(n, \mathbf{n}) = \Lambda(\mathbf{n})\psi(n, \mathbf{n})$, then

$$\frac{[\lambda(n, \mathbf{n})]^2}{\alpha_x(n)} + \frac{(1 - \lambda(n, \mathbf{n}))^2}{\alpha_p(\mathbf{n})} = \frac{1}{\alpha_p(\mathbf{n})}\Big(\Lambda(\mathbf{n})\psi(n, \mathbf{n})(\Lambda(\mathbf{n}) - 2) + 1\Big) = -\frac{\lambda(n, \mathbf{n})}{\alpha_p(\mathbf{n})}\big(1 + r(1 - \lambda(\mathbf{n}))\big) + \frac{1}{\alpha_p(\mathbf{n})}.$$

Given our definition of $r \equiv -\frac{a}{c}$, it immediately follows that

$$\pi(n, \mathbf{n}) \propto \frac{1}{2}\frac{\lambda(n, \mathbf{n})}{\alpha_p(\mathbf{n})}\big[1 + r(1 - \lambda(\mathbf{n}))\big] + r(1 - \lambda(n, \mathbf{n}))\frac{1 - \lambda(\mathbf{n})}{\alpha_p(\mathbf{n})} + \bar{\pi}(\mathbf{n}) = \psi(n, \mathbf{n})\frac{\Lambda(\mathbf{n})^2}{\alpha_p(\mathbf{n})} + \tilde{\pi}(\mathbf{n}), \tag{A.19}$$

where $\bar{\pi}(\mathbf{n}) \equiv \frac{\bar{\pi}}{c} - \frac{1}{2\alpha_p(\mathbf{n})}$ and $\tilde{\pi}(\mathbf{n}) \equiv \bar{\pi}(\mathbf{n}) + \frac{r(1-\lambda(\mathbf{n}))}{\alpha_p(\mathbf{n})}$.

B Supplementary Online Appendix

B.1 Asset Market Equilibrium with Unrestricted ρ. In the main text, we set $\rho = 1$ for convenience. Here, we show that the normalization is essentially without a loss of generality. Using (2.7) one can show that

$$\begin{aligned}\mathbb{E}\big[Q(\theta,p)|y_j,p\big] &= \rho\lambda\mathbb{E}[\theta|y,p] + \rho(1-\lambda)p \\ &= \rho\lambda\frac{\alpha_y}{\alpha_p+\alpha_y}y_j + \rho\bigg(1-\lambda\frac{\alpha_y}{\alpha_p+\alpha_y}\bigg)p,\end{aligned}$$

and $\mathbb{V}\big[Q(\theta,p)|y_j,p\big] = \rho^2\frac{\lambda^2}{\alpha_p+\alpha_y}$. The individual asset demand is then

$$k(y_j,p) = \frac{\rho\lambda\frac{\alpha_y}{\alpha_p+\alpha_y}(y_j-p) + (\rho-1)p}{\gamma\rho^2\frac{\lambda^2}{\alpha_p+\alpha_y}}. \tag{B.1}$$

Imposing market clearing and solving for p gives

$$p = \frac{\rho\lambda\alpha_y}{\rho\lambda\alpha_y + (1-\rho)(\alpha_p+\alpha_y)}\theta - \frac{\gamma\rho^2\lambda^2\big(\alpha_\xi\big)^{-\frac{1}{2}}}{\rho\lambda\alpha_y + (1-\rho)(\alpha_p+\alpha_y)}\xi. \tag{B.2}$$

Substituting for $\lambda = \frac{\alpha_x}{\alpha_x+\frac{\alpha_p}{1-r}}$ and rearranging the fixed-point condition, α_p can be written as

$$\alpha_p = \alpha_\xi\bigg(\frac{\alpha_y}{\gamma}\bigg)^2\bigg(1+\frac{\alpha_p}{(1-r)\alpha_x}\bigg)^2\bigg(\frac{1}{\rho}+\frac{1-\rho}{\rho}\frac{\alpha_p}{\alpha_y}\bigg(1+\frac{\alpha_y+\alpha_p}{(1-r)\alpha_x}\bigg)\bigg)^2. \tag{B.3}$$

When $\rho = 1$, the fixed-point condition coincides with the one used in the main text. The question is then how relevant the restriction on ρ is for the result of Proposition 1. To address the issue, let $f(\alpha_p) \equiv \alpha_\xi\big(\frac{\alpha_y}{\gamma}\big)^2\big(1+\frac{\alpha_p}{(1-r)\alpha_x}\big)^2$ and $h(\alpha_p) \equiv \big(\frac{1}{\rho}+\frac{1-\rho}{\rho}\frac{\alpha_p}{\alpha_y}\big(1+\frac{\alpha_y+\alpha_p}{(1-r)\alpha_x}\big)\big)^2$, then

$$\frac{\partial\alpha_p}{\partial\alpha_x} = -\frac{\alpha_p}{\alpha_x}\frac{f'(\alpha_p)h(\alpha_p) + \frac{1-\rho}{\rho}f(\alpha_p)2\sqrt{h(\alpha_p)}\frac{\alpha_p+\alpha_y}{(1-r)\alpha_y\alpha_x}}{1-\big(f'(\alpha_p)h(\alpha_p)+f(\alpha_p)h'(\alpha_p)\big)}. \tag{B.4}$$

Requiring α_p to be a stable solution results in $|f'(\alpha_p)h(\alpha_p) + f(\alpha_p)h'(\alpha_p)| < 1$, which means that the sign of (B.4) depends only on the sign of the numerator. Upon inspection of the terms in the numerator, one realizes that in order for $\frac{\partial\alpha_p}{\partial\alpha_x} > 0$, which would contradict the result of Proposition 1, a necessary but not sufficient condition is $\rho > 1$, which means that for $\rho \in (0,1]$ Proposition 1 always holds. Assuming $a + c > 1$ would be enough to ensure this. In addition, because $\frac{1-\rho}{\rho} < 0$ when $\rho > 1$, it is also necessary that $\sqrt{h(\alpha_p)} > 0$, which places an upper bound on ρ of the form

$$\rho < 1 + \frac{(1-r)\alpha_x}{\alpha_p\big(\alpha_p+\alpha_y+(1-r)\alpha_x\big)}. \tag{B.5}$$

It is then easy to show that under many numerical parameterizations of our model, Proposition 1 would still hold for any value of $\rho > 1$ as well.

B.2 Existence of Linear Equilibrium of Financial Market In this section, we show that if one allows the private signals of traders to be correlated, the existence of at least one equilibrium for the financial market of Section 2.2 is always guaranteed. Let the private signal of trader j be specified as $y_j = \theta + (\alpha_y)^{-1/2} \varepsilon_{y,j}$, where $\varepsilon_{y,j} \sim \mathcal{N}(0,1)$ and $cov(\varepsilon_{y,j}, \varepsilon_{y,l}) = \beta$ for $j \neq l$, $\beta \in [0,1]$, and where l is any arbitrary trader. Given θ, the average private signal is

$$\mathbb{E}[y_j|\theta] = \theta + \tilde{\varepsilon}, \tag{B.6}$$

where abusing the notation $\tilde{\varepsilon} \equiv (\alpha_y)^{-1/2} \int_j \varepsilon_{y,j} dj$ and $\tilde{\varepsilon} \sim \mathcal{N}(0, \beta/\alpha_y)$. The conjectured linear price is $p = \theta + \tilde{\varepsilon} - \omega\xi$, where ξ is the stochastic supply and ω is the precision we want to solve for. Note that the precision of p is

$$\alpha_p = \left(\frac{\beta}{\alpha_y} + \omega^2\right)^{-1}. \tag{B.7}$$

Recall that the individual asset demand is $k(y_j, p) = \frac{\alpha_y}{\gamma\lambda}(y_j - p)$ where $\lambda = \frac{(1-r)\alpha_x}{(1-r)\alpha_x + \alpha_p}$. Market clearing implies

$$\mathbb{E}[k(y_j,p)|\theta,p] = \frac{\alpha_y}{\gamma\lambda}(\theta + \tilde{\varepsilon} - p) = (\alpha_\xi)^{-\frac{1}{2}}\xi. \tag{B.8}$$

Solving for p one obtains $p = \theta + \tilde{\varepsilon} - \frac{\lambda\gamma}{\alpha_y}(\alpha_\xi)^{-\frac{1}{2}}\xi$, so that $\omega \equiv \frac{\lambda\gamma}{\sqrt{\alpha_\xi}\alpha_y}$. Using (B.7) and substituting for λ, the fixed-point condition for α_p is

$$\alpha_p = f(\alpha_p) \equiv \frac{\alpha_y^2\left(\frac{\alpha_p}{(1-r)\alpha_x} + 1\right)^2}{\beta\alpha_y\left(\frac{\alpha_p}{(1-r)\alpha_x} + 1\right)^2 + \frac{\gamma^2}{\alpha_\xi}}. \tag{B.9}$$

This expression is a third-order polynomial in α_p so that there are at most three equilibria. A solution to the fixed point always exists. To see this, notice that $f(0) > 0$ and $\lim_{\alpha_p\to\infty}[f(\alpha_p) - \alpha_p] = \lim_{\alpha_p\to\infty}\left[\frac{\alpha_y}{\beta} - \infty\right] < 0$. Because $f(\alpha_p)$ is continuous, the function $f(\alpha_p)$ must cross the 45-degree line at least once. Note that for $\beta = 0$, the argument for existence would not hold.

B.3 Financial Price Precision under Kyle(1985)'s "Market Order" Structure In this section, we investigate to what extent the result of Proposition 1 relies on traders perfectly observing the financial price. We do this by replacing the assumption of competitive equilibrium and adopting instead the "market order" approach developed by Kyle (1985). We show that the result of Proposition 1 is robust to traders not observing the financial price.

The environment for firms in the Cournot market is identical to the one adopted in the main text. Aggregate output is specified as

$$Q(\theta, \tilde{p}) = \lambda\theta + (1-\lambda)\tilde{p}, \tag{B.10}$$

where $\lambda = \frac{(1-r)\alpha_x}{(1-r)\alpha_x + \alpha_p}$, and α_p is the precision of $\tilde{p}$, which is a linear transformation of the financial price p that preserves information (see more on this below). The financial market consists of a continuum of risk-averse traders indexed by $j \in [0,1]$ that face the problem of allocating their wealth into a risk-free asset or a risky asset.[21] The risky asset has a price p and promises a payoff of Q. An arbitrary trader j cannot condition on the price of the asset, but she submits a market order k_j set to maximize her expected utility conditional on their private signal $y_j = \theta + (\alpha_y)^{-\frac{1}{2}}\varepsilon_{y,j}$. The expected utility of traders takes the usual exponential form, which means that the order submitted is specified as

$$k(y_j) = \frac{\mathbb{E}(Q - p|y_j)}{\gamma\mathbb{V}(Q - p|y_j)}. \tag{B.11}$$

The total market order submitted by traders is then given by $k = \int_0^1 k(y_j)dj$. In addition to risk-averse traders, there are noisy traders in the market that submit a total order of $-(\alpha_\xi)^{-\frac{1}{2}}\xi$. We denote the total market order of all traders by $z = k - (\alpha_\xi)^{-\frac{1}{2}}\xi$.

The market maker collects the order and determines the price as the best prediction of the asset payoff Q conditional on the total market order z. Namely, it is assumed that

$$p = \mathbb{E}[Q(\theta, \tilde{p})|z]. \tag{B.12}$$

Given this price, the market maker takes a position so that the financial market clears. We assume that the market maker has a prior on θ that is not degenerate and it is given by a Gaussian distribution with a mean zero and a standard deviation $(\alpha_\theta)^{-\frac{1}{2}} > 0$.[22] We proceed by conjecturing that the total market order takes the linear form

$$z = \beta\theta - (\alpha_\xi)^{-\frac{1}{2}}\xi, \tag{B.13}$$

where β is to be determined in equilibrium. Under this conjecture, we define $\tilde{z} = \beta^{-1}z$ so that $\mathbb{E}[Q(\theta, \tilde{p})|z] = \mathbb{E}[Q(\theta, \tilde{p})|\tilde{z}]$. The expression for the equilibrium price becomes

$$p = \lambda\mathbb{E}(\theta|\tilde{z}) + (1-\lambda)\mathbb{E}(\tilde{p}|\tilde{z}). \tag{B.14}$$

Given our conjecture on z, we can also conjecture that the price takes the linear form $p = h^{-1}\theta - h^{-1}(\alpha_p)^{-\frac{1}{2}}\xi$,

[21] In the original model by Kyle (1985), traders are risk-neutral. Here, we maintain the risk-aversion assumption as we are interested in evaluating the robustness of our result only to the financial price formation process, not to the preferences of traders.

[22] This assumption is necessary in order to keep the equilibrium price well defined. In Kyle (1985), an equivalent assumption is made so that the equilibrium is well defined only when the prior of the market maker on the asset fundamentals is non-degenerate (in Kyle (1985)'s notation this corresponds to $\Sigma_0 < \infty$).

for some real coefficients h and α_p. This suggests the transformation $\tilde{p} = hp = \theta - (\alpha_p)^{-\frac{1}{2}}\xi$. Note that the information contained in p and in $\tilde{p}$ about θ is equivalent, and therefore, $\tilde{p}$ can be used as a signal rather than p, which is what is done in equation (B.10). Because firms know the financial price p at the time they choose their output, the market maker can set $\mathbb{E}(\tilde{p}|\tilde{z}) = \tilde{p}$. Substituting $\tilde{p}$ with hp in (B.14) and rearranging one obtains

$$p = \frac{\lambda}{1-(1-\lambda)h}\mathbb{E}(\theta|\tilde{z}). \tag{B.15}$$

The fundamental prediction of the market maker is

$$\mathbb{E}(\theta|\tilde{z}) = \frac{\beta^2\alpha_\xi}{\alpha_\theta + \beta^2\alpha_\xi}\tilde{z} = \frac{\beta^2\alpha_\xi}{\alpha_\theta + \beta^2\alpha_\xi}\Big(\theta - \beta^{-1}(\alpha_\xi)^{-\frac{1}{2}}\xi\Big). \tag{B.16}$$

Substituting (B.16) into (B.15) and matching the coefficients in our conjecture for p, it is immediate that $\alpha_p = \alpha_\xi\beta^2$. The last step in solving for an equilibrium is then to obtain a fixed-point condition for α_p. Towards that goal, we notice that

$$Q(\theta,\tilde{p}) - p = \lambda\theta - \big(1-(1-\lambda)h\big)p = \lambda\Big(1 - \frac{\alpha_p}{\alpha_\theta+\alpha_p}\Big)\theta + \lambda\frac{\alpha_p}{\alpha_\theta+\alpha_p}(\alpha_p)^{-\frac{1}{2}}\xi, \tag{B.17}$$

where we have substituted β^2 for α_p/α_ξ. It then follows that

$$\mathbb{E}(Q-p|y_j) = \lambda\Big(1 - \frac{\alpha_p}{\alpha_\theta+\alpha_p}\Big)y_j, \tag{B.18}$$

and

$$\mathbb{V}(Q-p|y_j) = \lambda^2\Big(1 - \frac{\alpha_p}{\alpha_\theta+\alpha_p}\Big)^2\frac{1}{\alpha_y} + \lambda^2\Big(\frac{\alpha_p}{\alpha_\theta+\alpha_p}\Big)^2\frac{1}{\alpha_p}. \tag{B.19}$$

Substituting into (B.11) and aggregating over traders to obtain k, one finally has

$$z = \frac{\frac{\alpha_\theta}{\alpha_\theta+\alpha_p}}{\gamma\lambda\Big[\Big(\frac{\alpha_\theta}{\alpha_\theta+\alpha_p}\Big)^2\frac{1}{\alpha_y} + \Big(\frac{\alpha_p}{\alpha_\theta+\alpha_p}\Big)^2\frac{1}{\alpha_p}\Big]}\theta - (\alpha_\xi)^{\frac{1}{2}}\xi. \tag{B.20}$$

Given our conjecture for z and rearranging terms, we get the following fixed-point condition for α_p,

$$\alpha_p = f(\alpha_p) \equiv \alpha_\xi\Big(\frac{\alpha_y}{\gamma}\Big)^2\Big(\frac{\alpha_p}{(1-r)\alpha_x}+1\Big)^2\Big(\frac{1}{\frac{\alpha_\theta}{\alpha_\theta+\alpha_p}+\frac{\alpha_p}{\alpha_\theta+\alpha_p}\frac{\alpha_y}{\alpha_\theta}}\Big)^2. \tag{B.21}$$

The equation has multiple real solutions, when they exist. We focus here on the solution with the lowest precision, which is stable according to our criterion. We begin by noting that $f(0) > 0$, and because f is continuous for $\alpha_p > 0$ this means that at the solution with the lowest precision, f intersects the 45-degree line from above. Next we note that the only difference between (B.21) and the fixed point condition (2.9) is the third term in brackets on the right-hand side. From closer inspection, it is immediate to see that the precision of firms' private information α_x does not enter such term. It follows that when α_x increases, f shifts downward and it crosses the 45-degree line at a lower α_p. We summarize this result in the following proposition.

Proposition B1. *Suppose that the equilibrium financial price p is the lowest precision equilibrium determined according to the variant of the market order model of Kyle (1985) described above. Then, the precision α_p is decreasing in the precision of the private information of firms in the Cournot market. Formally,*

$$\frac{\partial \alpha_p}{\partial \alpha_x} < 0. \tag{B.22}$$

The intuition for this result is still related to the perception of risk of traders when α_x changes, as in Proposition 1. However, the role of the market maker is also essential. While p is not observed by traders, it is perfectly observed by firms. Traders internalize this indirectly through the market maker pricing formula. This is best visible from expression (B.17). Note that for $\alpha_\theta = 0$ the net return of purchasing the asset would just be a function of the stochastic supply ξ. This is because Q and p would be equally sensitive to the fundamental θ, and the effect of θ would completely cancel. In this case, because traders have no information on ξ, their positions will invariably be equal to zero, and an equilibrium would not exist as no price could possibly force traders to absorb the stochastic supply. When $\alpha_\theta > 0$, the pricing function of the market maker is such that the signal represented by the market order receives a weight strictly less than 1, which results in Q being more sensitive to θ when compared to p. As a consequence, the payoff of the asset becomes proportional to $\lambda\theta$, which means that λ matters for traders' positions, exactly as it does in the competitive equilibrium setting. Put it differently, the private information of market makers, represented by α_θ, exposes traders to the risk about θ, which allows them to use their private information to take positions on the asset, and in so doing makes such positions sensitive to the perceived risk which co-varies with λ.

In equilibrium, when α_x is higher, λ is higher and as equation (B.11) shows, the total market order is less sensitive to the fundamental θ. Because the equilibrium price is set by the market maker so to equal the best prediction of θ conditional on the total market order, it follows that conditional on holding fixed α_θ, the equilibrium price is as informative about θ as the market order z, which, as we just argued, is less informative when α_x increases.

B.4 Information Incentives for Traders.

In this section, we study the private information acquisition incentives for traders. As in the main text, we denote the precision of private information as $\alpha_y(m)$, where m is individual effort, while we denote the effort level across all traders by $\mathbf{m}$. The expected utility for an individual trader at stage 0 conditional on the equilibrium strategies being played at stages 1 and 2 can be written as[23]

$$v(m,\mathbf{n},\mathbf{m}) \equiv \mathbb{E}_0\left[V^*\left(w_0+(d-p)k_j\right)\right] = -\exp[-\gamma w_0]\left(\frac{\alpha_p(\mathbf{n},\mathbf{m})}{\alpha_p(\mathbf{n},\mathbf{m})+\alpha_y(m)}\right)^{\frac{1}{2}}, \tag{B.23}$$

where V^* denotes that utility is evaluated for equilibrium strategies, similarly to Π^*. The notation $\alpha_p(\mathbf{n},\mathbf{m})$ is meant to capture the notion that both the information acquisition effort across firms $\mathbf{n}$ and across traders $\mathbf{m}$ affect the precision of the public signal p. The unconditional expected utility for traders is inversely related to the signal extraction coefficient $\frac{\alpha_p}{\alpha_p+\alpha_y}$: when the equilibrium price is more informative, the expected utility is lower because private information is less useful in predicting the risky dividends. To study the information incentives of traders, we want to study the cross partials, $v_{m\mathbf{m}} = \frac{\partial v(m,\mathbf{n},\mathbf{m})}{\partial m \partial \mathbf{m}}$, and $v_{m\mathbf{n}} = \frac{\partial v(m,\mathbf{n},\mathbf{m})}{\partial m \partial \mathbf{n}}$. From (B.23) one can show that

$$\operatorname{sign}\big(v_{m\mathbf{m}}\big) = \operatorname{sign}\left[\frac{\partial \alpha_p(\mathbf{n},\mathbf{m})}{\partial \mathbf{m}}\big(\alpha_y(m) - 2\alpha_p(\mathbf{n},\mathbf{m})\big)\right], \tag{B.24}$$

and the expression for $\operatorname{sign}\big(v_{m\mathbf{n}}\big)$ can be obtained by substituting $\frac{\partial \alpha_p(\mathbf{n},\mathbf{m})}{\partial \mathbf{m}}$ with $\frac{\partial \alpha_p(\mathbf{n},\mathbf{m})}{\partial \mathbf{n}}$ in (B.24). The following proposition immediately follows.

Proposition B2. *In the financial market of Section 2.2, the effort in private information acquisition at the individual trader level m and the aggregate effort of traders $\mathbf{m}$ (resp. of firms $\mathbf{n}$) are strategic substitutes (resp. complements) if and only if $2\alpha_p(\mathbf{n},\mathbf{m}) > \alpha_y(m)$.*

Application of Proposition 1 shows that $\frac{\partial \alpha_p(\mathbf{n},\mathbf{m})}{\partial \mathbf{m}} > 0$ and $\frac{\partial \alpha_p(\mathbf{n},\mathbf{m})}{\partial \mathbf{n}} < 0$. When $\mathbf{m}$ increases, p becomes more precise and private information is less valuable at the margin when the precision of the price is already twice that of private information. When $\mathbf{n}$ increases, on the other hand, p becomes less precise, and private information is more valuable at the margin when not larger than twice the precision of the price. In a symmetric equilibrium $m = \mathbf{m}$, a sufficient condition for the private information acquisition of traders to be always substitute with respect to $\mathbf{m}$ and always complement with respect to $\mathbf{n}$ is

$$\frac{1}{2}\alpha_\xi \alpha_y(\mathbf{m}) > \gamma^2, \tag{B.25}$$

which can hold under a large class of parameters' values provided that the existence of α_p is maintained.

B.5 Information Incentives when $r > 0$. In the main paper, we have focused our attention on a Cournot market where economic actions are substitutes. It is possible to extend our analysis to contexts in which economic actions are complements. Here, we present a version of our Proposition 2 for the case of $r > 0$. In the setting of the paper, let us replace the Cournot market with a coordination economy where there are a continuum of agents $i \in [0,1]$; the individual action of agent i is denoted by a_i. Let $\Psi(a)$ denote the cumulative distribution

[23] Details of the derivation are available from the authors upon request.

function for individual actions across the population; the average action is $A \equiv \int a d\Psi(a)$. Let $\theta \in \mathbb{R}$ represent an exogenous payoff relevant state of the world. We assume that player's i payoff function is

$$U\big(a, A, \theta\big) = -\frac{1}{2}\bigg(a - (1-r)\theta - rA\bigg)^2. \tag{B.26}$$

where $r > 0$, so that the individual action a and the aggregate action A are strategic complements. The information structure and the modeling of information acquisition are identical to those assumed in the main paper, with the only nominal difference that firms are now referred to as agents. In this context, the following result holds.

Proposition B3. *In the coordination economy with $r > 0$ and with public signal p generated by the asset market of Section 2.2 with $d = A$, the effort in private information acquisition at the individual agent level n and the aggregate effort $\mathbf{n}$ are always strategic complements.*

Recall that

$$\pi_{n\mathbf{n}} \propto \Lambda_r \frac{\partial \alpha_x(\mathbf{n})}{\partial \mathbf{n}} + \bigg(-\Lambda_r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{n})} + \lambda_\alpha\bigg)\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}}. \tag{B.27}$$

Regardless of r, $\lambda_\alpha < 0$ and Proposition 1 still ensures that $\frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} < 0$. For $r > 0$, however, $\Lambda_r > 0$, which means that all of the terms in the above expression take a positive value. Recall also that the term $\Lambda(\mathbf{n})$ represents the adjustment that individual agents apply to their actions to take into account that other agents overreact to the common noise that is present in the public signal. In the presence of complementarity the adjustment is downward, so that $\Lambda(\mathbf{n}) < 1$. When aggregate private information acquisition across agents $\mathbf{n}$ is increased, all agents devote more importance to their private signals and less to their public signal. In the presence of strategic complementarity, individual agents recognize this and adjust $\Lambda(\mathbf{n})$ upwards, in order to take into account the reduced correlation across actions from the common noise of the public signal. The effect just described is represented by $\Lambda_r \frac{\partial \alpha_x(\mathbf{n})}{\partial \mathbf{n}} > 0$. As a consequence, when $\mathbf{n}$ is increased, the value of private information is enhanced for the individual agent. This effect is reinforced by the simultaneous reduction of the precision of public information, which corresponds to $-\Lambda_r \frac{\alpha_x(\mathbf{n})}{\alpha_p(\mathbf{n})} \frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} > 0$. Finally, to the extent that public information is declining, private information is always more valuable in contrasting the overall information reduction, which is the effect captured by $\lambda_\alpha \frac{\partial \alpha_p(\mathbf{n})}{\partial \mathbf{n}} > 0$.