

101562: Intermediate Macroeconomics

Problem Set 5

Shanghai University of Finance and Economics - Fall 2014

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The solutions are due *Monday December 8* before 5pm. Please submit your solution to TA. Enjoy!

Exercise A (Two-Period Model: Importance of Considering Present Value)

Suppose that you are employed. Your current income is $e > 0$ and future income is $e' > 0$. Your employer comes to you and asks the following change in your income schedule: reducing future income by x units but increasing current income by x units. The interest rate is given as $r > 0$ and there is no tax in this economy.

1. Would you accept this offer? What would happen to your consumption plan if you accept the offer? Use a diagram in your answer.
2. Now consider the opposite option: reducing current income by x units but increasing future income by x units. Would you accept this offer? Use a diagram in your answer.
3. Say you are the employer and you would like to lower current income by x units because you do not have enough money. What would be the minimum changes in the future income that the employee would accept the change?

Exercise B (Two-Period Model: Asset Pricing)

Consider a two-period model. A consumer is given current income, $e > 0$, and future income, $e' > 0$. She would like to maximize her lifetime utility by solving the following problem:

$$\max_{c_1, c_2, b, s} u(c_1, c_2)$$

subject to

$$(1) \quad c_1 + qs + b = e$$

$$(2) \quad c_2 = e' + (1 + r)b + (q' + d)s$$

where b is the bond purchased by the consumer whose net return is r and s is the stock purchased by the consumer. q is the price of the stock in the initial period. In the next period, the consumer receives dividend, d , from the stock and then sells the stock at price q' at the stock market.

1. Derive the four first order conditions by using the Lagrangian approach.
2. Combine the first order conditions for b and s . Then you can obtain the equation for pricing the stock. Provide the economic reasoning behind the equation.

3. Suppose that the equation you derive in question 2 does not hold because r is higher than the equilibrium level. i.e. No-Arbitrage condition does not hold. What will happen in the asset markets?

4. Suppose that there is an announcement that the dividend of the stock will become higher. Discuss the changes in q after the announcement when r is fixed using the equation you derive in question 2.

Exercise C (Two-Period Model: Ricardian Equivalence with proportional income tax)

Consider an economy with a representative consumer who lives for two periods. Her current and future income are all $e > 0$. She would like to maximize her lifetime utility subject to the budget constraint. Formally,

$$\max_{c_1, c_2, s} u(c_1, c_2) = u(c_1) + \beta u(c_2)$$

subject to

$$(1) c_1 + s = (1 - t)e$$

$$(2) c_2 = (1 - t')e + (1 + r)s$$

where $0 < \beta < 1$ and $0 < t, t' < 1$. Hence, t and t' are tax rates; the government imposes proportional income tax.

Suppose that the government spending in each period is assumed to be the same as g .

1. What is the government present-value budget constraint?
2. The government imposes a new tax policy to decrease t to $t - 0.1$ (tax rate decreases by 10%, $t - 0.1$ is assumed to be still positive) but to increase t' . Using the government present-value budget constraint, compute the required changes in the future tax rate.
3. What happens to the consumer's consumption plan when the tax policy changes as in question 2? Discuss the Ricardian equivalence in the presence of proportional tax. (Hint: Use the consumer's life-time budget constraint)

Exercise D (Two-Period Problem with Capital)

Suppose that a firm's profit maximization problem is given as follows.

$$\max_{I_t, K_{t+1}, h_t, h_{t+1}} \pi_t + \frac{\pi_{t+1}}{1+r} = (z_t f(h_t, K_t) - w_t h_t - I_t) + \frac{1}{1+r} [z_{t+1} f(h_{t+1}, K_{t+1}) + (1 - \delta) K_{t+1} - w_{t+1} h_{t+1}]$$

subject to

$$K_{t+1} = I_t + (1 - \delta) K_t$$

where f is continuous, differentiable, concave, and increasing in h and K and $0 < \delta < 1$ is the depreciation rate of capital. $\frac{1}{1+r}$ is the discount factor for the firm. We assume that the remaining capital in the next period can be sold in the goods market at the same price with the good produced by the firm. Hence, the next period revenue for this firm is the sum of produced good ($z_{t+1} f(h_{t+1}, K_{t+1})$) and remaining capital ($(1 - \delta) K_{t+1}$).

Here, the firm owns the capital ($K_t > 0$ is assumed to be given) and decides demands for labor, (future) capital and how much to invest in capital that will be used in the next period production.

1. Use the Lagrangian approach to derive the first order conditions.
2. Combine the two first order conditions with respect to I_t and K_{t+1} to derive the demand for investment. Draw the diagram where r is in the vertical axis and I_t is in the horizontal axis. (note: I_t and K_{t+1} are positively related)

3. Using the resource constraint of the economy, $Y = C + I + G$, derive the (current) output demand curve in the two-period model. Draw the output demand curve and discuss the differences in the slope of the curve with the model without the capital.
4. What is the relationship between the demand for current labor (h_t) and interest rate (r)? Discuss its implications on the output supply curve and draw it in the diagram.
5. What happens in this economy when the government increases G_t ?