

# 101562: Intermediate Macroeconomics

## Problem Set 2

Shanghai University of Finance and Economics - Fall 2014

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The solutions are due *Wednesday October 8* at the beginning of class. The second review session is scheduled to be held also on *Wednesday October 8* (Please check the website later). Enjoy!

### Exercise A (T/F)

Please label each statement true (T) or false (F) and briefly explain.

1. If the production function is Cobb-Douglas and is constant returns to scale, the sum of income share of each factor is equal to 1.

2. Since there is no long-run effect of changing the saving rate, increasing the saving rate would not have any important effects on the economy.

3. Suppose that the current level of capital stock is below the golden rule level. If the policymaker of the economy values the current consumer much more than the future consumer (say the election is soon), she will not conduct a policy to increase the saving rate.

4. Consider the two economies that are identical except the initial levels of capital stock. Then the Solow model predicts that the growth rates of output at the steady-state are different between the two economies.

### Exercise B (Simple Mathematics)

Please prove the following statements that are already introduced in the class.

1.  $\ln\left(\frac{X_{t+1}}{X_t}\right) \approx \frac{X_{t+1}-X_t}{X_t}$  when  $X_t \approx X_{t+1}$ . (Hint: 1st order Taylor approximation around  $X_t$ )

2. If  $Z_t = \frac{Y_t}{X_t}$ ,  $\gamma_Z = \gamma_Y - \gamma_X$  where  $\gamma_A$  is the growth rate of variable  $A$ .

### Exercise C (Firm's Problem with Cobb-Douglas Production Function)

Consider a firm's profit maximization problem:

$$\max_{K,L} \pi = F(K, L) - wL - rK$$

where  $F$  is continuous, differentiable, concave, and increasing. Two production factors are inelastically supplied at  $\bar{L}$  and  $\bar{K}$ , respectively, and all markets are assumed to be perfectly competitive.

1. Suppose that the production function is given as follows.

$$F(K, L) = AK^\alpha L^\beta$$

where  $\alpha, \beta \in (0, 1)$  and  $A > 0$  denotes the productivity.

Derive the first order conditions of the firm. What is the relationship between  $r$  and  $K$ ?

2. In the equilibrium, supply is equal to demand. Draw the diagram of capital market where  $r$  is on the vertical axis and  $K$  is on the horizontal axis. Where is the equilibrium?

3. Now suppose that there is an increase in  $A$ . i.e. the productivity of this economy increases. What happens to equilibrium capital ( $K$ ) and interest rate ( $r$ )? Draw the diagram of the capital market to indicate the changes in equilibrium and explain your findings.

4. Derive the profit of the firm at the equilibrium. Economist X argues that we should assume  $\alpha + \beta = 1$  for this firm's problem to have a competitive equilibrium. Do you agree with her argument? Please provide your reasoning to support your opinion about Economist X. (Hint: What happens if either  $\alpha + \beta < 1$  or  $\alpha + \beta > 1$ ?)

### Exercise D (Basic Solow Model)

Consider the basic Solow model whose production function is given as follows.

$$f(k_t) = k_t^\alpha$$

where  $\alpha \in (0, 1)$ . Here  $L$ , the size of population, is assumed to be one.

1. Derive the steady-state level of capital stock ( $k^*$ ). What is the effect of changes in  $\delta$  on the steady-state level of capital? Explain with the diagram we used in the class and provide your reasoning.

2. Derive the golden rule level of capital ( $k^g$ ). What is the golden rule of saving rate in this problem?

For the remaining question, you do not need to use the above production function. Now suppose that there exists a government so that households should pay lump-sum tax,  $T$ , to the government. The government then spends the collected tax to buy some consumption goods i.e.  $G = T$ . Now, the (private) saving is given as  $S = s(Y - T)$ .

3. Suppose again that the economy is at the steady-state. Economist X comes to the policymaker of this economy and suggests to increase government spending in order to increase demand for goods, which will result in higher output. Because the government prefers balanced budget,  $G = T$  should still hold. If you are the policymaker, will you follow his advice? Please use the diagram with capital on the horizontal axis, instead of the formula, to solve this problem and provide your reasoning. (Hint: There might be multiple steady-state equilibria in this case (equilibrium may not exist, which we are not interested in. So please disregard this case) but I would like you to focus on the equilibrium whose capital is high.<sup>1</sup>)

### Exercise E (Solow Model: Effects of War)

Consider the basic Solow model whose production function is given as  $F(K_t, L_t)$  where it is increasing, continuous, concave, and exhibits constant returns to scale property. Here,  $K_t$  is the level of capital stock and  $L_t$  is the size of working population which is assumed to be constant over time. i.e.  $L_t = L$  for all  $t$ . Suddenly, a war hits the economy at time 0. Assume the economy was in the steady-state before the war. Suppose that the war destroys some fractions of capital stock so that  $K_0$ , the steady state level of capital stock, reduces to  $K_0 < K^*$ .

1. What is the immediate impact of the war on total capital and on capital per worker?

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<sup>1</sup>This is because the low steady-state equilibrium is unstable while the high steady-state equilibrium is stable.

2. What happens subsequently to capital per worker in the post-war economy? Discuss with (1) the diagram where  $k$  (capital per worker) is on the horizontal axis and with (2) another diagram with  $t$  (time) on the horizontal axis and  $k$  on the vertical axis.
3. In the short run, does capital grow at a higher or lower rate?