Supplementary Materials for

"Forecast Dispersion in Finite-Player Forecasting Games"*

Jin Yeub Kim[†] Myungkyu Shim[‡]

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Abstract

In Online Appendix A, we examine the conditions under which dispersion increases due to more precise information in our finite-player forecasting game. The critical thresholds of the conditions for which the dispersion decreases in the finite-player game differ from those in the continuum-player game.

In Online Appendix B, the conditions for non-decreasing dispersion are derived for games where private information acquisition is allowed. Greater precision of public information can increase dispersion in the finite-player game while it decreases dispersion in the continuum-player game (and vice versa).

Reducing forecast dispersion might be an important policy objective, but our comparative statics results imply that disclosure of more precise public and private information does not necessarily lower forecast dispersion. We provide relevant discussions in Online Appendix C.

*This manuscript is not intended for publication, but provides supplementary materials for the main text.

[†]Department of Economics, University of Nebraska-Lincoln, 1240 R Street, Lincoln, NE 68588-0489, USA. E-mail: shiningjin@gmail.com, Webpage: https://sites.google.com/site/jinyeubkim

[‡]School of Economics, Sogang University, 35 Baekbeom-Ro, Mapo-Gu, Seoul, 04107, South Korea. E-mail: audrb917@gmail.com, Webpage: http://myungkyushim.weebly.com

Online Appendix A. Role of Dispersed Information

As a complementary analysis to Angeletos and Pavan (2004), we investigate how forecast dispersion changes due to more precise information in the context of the finite-player model.[1] The results on information effects per se resembles those in Angeletos and Pavan (2004) to a large extent. More interesting is the comparison of the conditions under which dispersion increases between our finite-player game and the continuum-player benchmark.

For non-decreasing dispersion, we want $dVar(a_i|\theta,p) \geq 0$. By total differentiating (11) in the main text, we obtain

$$dVar(a_i|\theta,p) = \left[\frac{\left(\alpha_x + \frac{1}{1-\gamma}\alpha_p\right)^2 - 2\alpha_x\left(\alpha_x + \frac{1}{1-\gamma}\alpha_p\right)}{\left(\alpha_x + \frac{1}{1-\gamma}\alpha_p\right)^4}\right] d\alpha_x + \left[\frac{-2\alpha_x\frac{1}{1-\gamma}\left(\alpha_x + \frac{1}{1-\gamma}\alpha_p\right)}{\left(\alpha_x + \frac{1}{1-\gamma}\alpha_p\right)^4}\right] d\alpha_p$$

Setting this to be greater than or equal to zero implies that the dispersion of forecasts weakly increases if and only if

$$\left(\alpha_p - (1-\gamma)\alpha_x\right) d\alpha_x \geq 2\alpha_x d\alpha_p, \tag{A.1}$$

or equivalently,

$$(1 - 2\lambda(\gamma))d\alpha_x \geq \frac{2\lambda(\gamma)}{1-\gamma} d\alpha_p. \tag{A.2}$$

If $\lambda(\gamma) \geq \frac{1}{2}$, then (A.2) cannot hold, whereas the condition could hold if $\lambda(\gamma) < \frac{1}{2}$.

The following proposition specifies the partial effects of the information precision on the forecast dispersion in both the finite-player and continuum-player models.

Proposition A.1. *(i) The dispersion of forecasts decreases with an increase in α_p for any given α_x in both the finite-player game and the continuum-player benchmark. (ii) The dispersion of forecasts increases with an increase in α_x for a given α_p if and only if $\delta < \frac{1}{2-\gamma}$ for the finite-player game and $\delta < \frac{1}{2-r}$ for the continuum-player benchmark.*

[1] By examining the effect of more precise information on aggregate volatility, we find that an increase in the dispersion of individual forecasts is not always associated with an increase in the volatility of the average forecast. We omit the details of the formal analysis, available upon request.

Proof. Part (i) is obvious from equation (11) because for any given α_x, a higher α_p decreases $\lambda(\gamma)$ (or λ for the continuum-player benchmark). For part (ii), set $d\alpha_p = 0$ in (A.2). The condition holds with strict inequality iff $(1 - 2\lambda(\gamma)) > 0$. This condition can be rewritten as $\frac{\alpha_x}{\alpha_x+\alpha_p} < \frac{1}{2-\gamma}$ (or $\frac{\alpha_x}{\alpha_x+\alpha_p} < \frac{1}{2-r}$ for the continuum-player benchmark), where $\delta = \frac{\alpha_x}{\alpha_x+\alpha_p}$. □

Lemma 1 states that $\gamma > r$ and γ decreases with n when $r < 0$. Hence, when the forecasters prefer to be distinctive from the average forecast, the hurdle rate of the relative precision of private information such that more precise private information would make lower dispersion is higher in the finite-player forecasting game than in the continuum-player benchmark; that is $\frac{1}{2-\gamma} > \frac{1}{2-r}$. Similarly, because $\gamma < r$ and γ increases with n when $r > 0$, the smaller and more oligopolistic is the forecasting game (i.e., the stronger is the market power), the lower is the hurdle rate such that more precise private information would make lower dispersion.

Online Appendix B. Private Informaton Acquisition

The development of technology enables economic agents to obtain more transparent and precise public information. For example, government agencies and central banks disseminate information about their policies, and this information is easily accessible through the mass media with high public visibility. Part (i) of Proposition A.1 then implies that, regardless of whether the forecasting market is large or small, any type of policy that involves the provision of more precise public information for a fixed level of private information precision unambiguously decreases forecast dispersion. However, in many environments, we can reasonably expect that economic forecasters—professional or not—can exert some effort to collect more precise information from an independent private source, in addition to receiving publicly available information.

We can extend our model by endogenizing private information choice following Colombo, Femminis and Pavan (2014), Hellwig and Veldkamp (2009), and Róndina and Shim (2015). Consider an information-acquisition stage prior to the forecasting game: In the first stage, each agent i chooses the precision of the private information. In the second stage, each agent i observes the realizations of the signals (x, p) and then chooses his forecast a_i.

In the second stage, for any α_x and α_p, the unique equilibrium strategy is given by (6) for all i. When all agents acquire private information of precision α_x, then all agents follow such strategy given α_x. Thus the expected payoff at the first stage when an agent acquires private information of precision α_x conditional on the equilibrium strategies at the second stage is given by:

$$U_i(\alpha_x) \equiv \mathbb{E}[u_i^*] - \frac{(1-r)^2}{2} C(\alpha_x), \tag{B.1}$$

where u_i^* denotes the equilibrium payoff at the second stage holding fixed the equilibrium forecasts specified in (6), and $C(\alpha_x)$ denotes the cost of acquiring private information of precision α_x, where the term $\frac{(1-r)^2}{2}$ is for normalization. We assume that $C(0) = 0$, $C^{'}(\cdot) > 0$, and $C^{''}(\cdot) > 0$. Given the equilibrium forecasts, the ex-ante expected equilibrium payoff is $\mathbb{E}[u_i^*] = -\frac{(1-r)^2}{2} \frac{\alpha_p + \left(1 + \frac{\gamma^2}{n-1}\right)\alpha_x}{((1-\gamma)\alpha_x + \alpha_p)^2}$.[2]

Hence in the first stage, each agent chooses α_x so as to maximize his expected equilibrium payoff net of the cost of information acquisition, that is,

$$\alpha_x^* = \arg\max_{\alpha_x} \ U_i(\alpha_x), \tag{B.2}$$

where α_x^* denotes the precision of private information acquired in equilibrium. Then we obtain the following lemma.

Lemma B.1. *If $r < \frac{1}{2}$, then the precision of private information α_x^* that each agent acquires in the unique symmetric equilibrium is implicitly given by $\frac{\partial \mathbb{E}[u_i^*]}{\partial \alpha_x}|_{\alpha_x = \alpha_x^*} = \frac{(1-r)^2}{2} C'(\alpha_x^*)$. The equilibrium precision of the private information α_x^* decreases with an increase in the precision of public information α_p.*

Proof. In any symmetric equilibrium, the precision of private information acquired in equilibrium must satisfy the first-order condition of the information-acquisition stage: $\frac{\partial \mathbb{E}[u_i^*]}{\partial \alpha_x} =$

[2] The derivation of the ex-ante expected payoff is due to technical computation, so omitted here.

$\frac{(1-r)^2}{2}C'(\alpha_x)$. We have $\frac{\partial \mathbb{E}[u_i^*]}{\partial \alpha_x} = \frac{(1-r)^2}{2}G(\alpha_x)$ where $G(\alpha_x) \equiv \frac{(1-\gamma)\left(1+\frac{\gamma^2}{n-1}\right)\alpha_x+\left(1-2\gamma-\frac{\gamma^2}{n-1}\right)\alpha_p}{((1-\gamma)\alpha_x+\alpha_p)^3}$. Then if $r < \frac{1}{2}$, then $G(0) > 0$ for $\alpha_p > 0$. Further, $\lim_{\alpha_x \to \infty} G(\alpha_x) = 0$. Therefore, the continuity of $G(\alpha_x)$ ensures the existence of equilibrium. In addition, $G'(\alpha_x) > 0$ for $\alpha_x < \frac{(3\gamma-1)+2\frac{\gamma^2}{n-1}}{(1-\gamma)\left(1+\frac{\gamma^2}{n-1}\right)}\alpha_p$ and $G'(\alpha_x) < 0$ for $\alpha_x > \frac{(3\gamma-1)+2\frac{\gamma^2}{n-1}}{(1-\gamma)\left(1+\frac{\gamma^2}{n-1}\right)}\alpha_p$. Thus by the intermediate value theorem, there is a unique $\alpha_x > 0$ that solves $G(\alpha_x) = C'(\alpha_x)$. By differentiating $G(\alpha_x)$ with respect to α_p, we obtain $\frac{\partial G(\alpha_x)}{\partial \alpha_p} < 0$. Thus $\frac{d\alpha_x^*}{d\alpha_p} < 0$. □

The results of this analysis basically follow from Colombo, Femminis and Pavan's (2014) result on the crowding-out effect of public information: More precise public information reduces the agents' incentives to obtain better private information, and thus crowds out the agents' acquisition of private information in equilibrium.[3] Then the proposition on the dispersion of forecasts follows.

Proposition B.1. *When the endogenous private information choice is allowed, the dispersion of forecasts increases with an increase in α_p only if $\lambda^*(\gamma) \equiv \frac{\alpha_x^*}{\alpha_x^*+\frac{1}{1-\gamma}\alpha_p^*} > \frac{1}{2}$ (or equivalently, $\delta^* \equiv \frac{\alpha_x^*}{\alpha_x^*+\alpha_p} > \frac{1}{2-\gamma}$) where $\lambda^*(\gamma)$ is the equilibrium degree of coordination and δ^* is the relative precision of private information given the precision of private information α_x^* acquired in equilibrium.*

Proof. If α_x is endogenous, then the dispersion of forecasts increases in α_p if and only if $\frac{dVar(a_i|\theta,p)}{d\alpha_p}|_{\alpha_x=\alpha_x^*} = \frac{\partial Var(a_i|\theta,p)}{\partial \alpha_x}|_{\alpha_x=\alpha_x^*}\frac{d\alpha_x^*}{d\alpha_p} + \frac{\partial Var(a_i|\theta,p)}{\partial \alpha_p}|_{\alpha_x=\alpha_x^*} > 0$. This condition is equivalent to $(\alpha_p - (1-\gamma)\alpha_x^*)\frac{d\alpha_x^*}{d\alpha_p} > 2\alpha_x^*$. Because $\frac{d\alpha_x^*}{d\alpha_p} < 0$ by Lemma B.1, the inequality holds only if $(\alpha_p - (1-\gamma)\alpha_x^*) < 0$. This necessary condition is equivalent to $\delta^* = \frac{\alpha_x^*}{\alpha_x^*+\alpha_p} > \frac{1}{2-\gamma}$. □

This result implies that, when $r < 0$, the maximum rate of the relative precision of private information such that more precise public information would maker lower dispersion is higher in the finite-player game than in the continuum-player game; that is $\frac{1}{2-\gamma} > \frac{1}{2-r}$. When $r > 0$, this threshold is lower the smaller is the forecasting game.

[3] Also see Róndina and Shim (2015) show that an increase in the precision of private information can reduce the precision of public information.

The conditions under which dispersion increases both due to more precise private information (when information signals are exogenous) and due to more precise public information (when private signal is endogenous) are somewhat tedious. But beyond what those technical expressions entail, one crude implication is that greater precision of information can increase dispersion in the finite-player game while it decreases dispersion in the continuum-player game (and vice versa), given the same level of relative precision of private information in both games. This further signifies the importance of studying forecast dispersion in the context of a finite economy.

Online Appendix C. Provision of Public Information

Dispersion in forecasts can be undesirable in environments where the accuracy of forecasts plays an important role. Then, lowering forecast dispersion might be a natural goal for policy makers. We briefly discuss whether the enhanced dissemination of public information can be beneficial in such sense.

The Proposition A.1 part (i) implies that any type of policy that involves the provision of more precise public information improves welfare in the sense that forecast dispersion decreases, which holds for a fixed level of private information precision.[4] Proposition B.1 suggests that when forecasters can exert effort to acquire the private information before observing the realization of the public signal, then more precise public information can be "undesirable" in the sense that it increases dispersion when the forecasters already rely too much on private information. This result conversely implies that the provision of more precise public information improves welfare only when the forecasters in the economy are not very sensitive to private information. The direction and magnitude of these effects differ between small versus large forecasting markets.

One implication is that regardless of whether acquisition efforts for private information are taken into account or not, the enhanced dissemination of public information or more

[4] On the other hand, the improved precision of private information can increase forecast dispersion for some range of parameters.

transparent communication by government agencies and central banks might not always be an effective policy if its goal is to lower forecast dispersion. The effectiveness depends crucially on the underlying primitives and the size of the economy. Thus the difficulty that policy makers face arises from the fact that they need to accurately estimate the degree of reliance on the private source of information by forecasters given the market size in order to execute the policy of providing more precise public information and to accomplish the desired goal of lowering dispersion. This task is necessary to identify the source of persistent forecast dispersion and to render appropriate policies of public information dissemination, but is beyond the scope of our methodology in the main text.

Before concluding this manuscript, it is worth noting that economic agents also might not fully seek available public information when making predictions about the economy, the so-called "rational inattention" as argued by Binder (2014) and Rudebusch and Williams (2009). In particular, Rudebusch and Williams (2009) find a puzzling observation that "[f]or over two decades, researchers have provided evidence that the yield curve, specifically the spread between long- and short-term interest rates, contains useful information for signaling future recessions[, but] forecasters appear to have generally placed too little weight on the yield spread when projecting declines in the aggregate economy." This argument provides an alternative explanation as to why dispersion among forecasters persists even when the precision of public information improves and as to why disclosing more precise public information might not be an effective policy in lowering dispersion. Although public information is in theory easily accessible, forecasters might incur costs from proactively using the available public information in making predictions. Thus an agent might be rationally inattentive to more precise public information even when it is readily available if the cost of using such information is too high; rather, the forecasters stick to only the previously available (possibly worse) public information. If this is the case, then enhanced dissemination of public information or more transparent communication by government agencies and central banks might not always be socially desirable.

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