Forecast Dispersion in Finite-Player Forecasting Games*

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Abstract

We study forecast dispersion in a finite-player forecasting game modeled as an aggregate game with payoff externality and dispersed information. In the game, each agent cares about being accurate as well as about the distance of his forecast from the average forecast; and with a finite number of agents, the agents can strategically control that average. We show that the finiteness of the number of agents weakens the strategic effect induced by the underlying preference. We find that when each agent prefers to be close to the average forecast, the presence of strategic manipulation of the average forecast contributes to a higher forecast dispersion; when instead each agent wants to be distinctive from the average, the opposite is true.

Keywords: forecast dispersion, finite-player, aggregate games, coordination, incomplete information.

JEL Classification Codes: C72, D82, D83, E37.

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1 Introduction

In many situations, market participants make decisions based on economic forecasts. For example, production, investment, employment, or budget planning decisions depend crucially on forecasts about the relevant economic variables. Hence, dispersion in forecasts can be undesirable in environments where the accuracy of forecasts plays an important role. In this sense, lowering forecast dispersion might be a natural goal for policy makers, and the study of the persistent dispersion in the predictions of future economic conditions has received scholarly attention (e.g., Andrade and Le Bihan, 2013; Ehrbeck and Waldmann, 1996; Ottaviani and Sørensen, 2006, among many others). Our goal in this paper is to extend our understanding of forecast dispersion by examining the *finite-player* equilibrium properties in forecasting games, which have not been explored by previous literature.

In economic forecasting, an agent desires to give an accurate prediction that correctly estimates the economic fundamentals under dispersed information. Different agents may obtain different information about the unknown state of the economy from news media reports, public announcements of government agencies or central banks, and private sources. With dispersed information, an agent often cares about the aggregate (or average) prediction that reflects the agents' different information about the fundamentals, generating a coordination motive. These features induce strategic behavior of agents, and the strategic effects in such environments are well-documented in the context of games with a continuum of agents (e.g., Angeletos and Pavan, 2007).

But if there is a finite number of agents, so that an agent can influence the average forecast by changing his own forecast, then there is an additional channel of strategic effect. The equilibrium behavior will then reflect not only coordination—i.e, the agents' desire to move toward or away from the average forecast—but also their ability to strategically manipulate that average. The study of finite-player forecasting games enables us to disentangle these two aspects of strategic interactions among agents and their effects on forecast dispersion. Our approach is thus relevant to oligopolistic settings with a relatively small number of forecasters and provides a micro foundation for explaining forecast dispersion in such environments.

For example, one can think of the forecasting game as being played by a group of financial analysts, each of whom desires to accurately estimate a company's earnings over the coming years. Holding the accuracy fixed, each analyst might prefer that his or her forecast is further away from the so-called consensus forecast—the average forecast of all the forecasts from individual analysts tracking a particular stock. In some other cases, analysts might prefer to herd toward the consensus forecast.[1] What is important is that the consensus forecast could be the average of 30 different forecasts or, for a smaller company's stock, the average of just two analyst forecasts.[2] The smaller is the number of analysts tracking the company, the greater is the extent to which each analyst can strategically influence the consensus forecast. Our focus is on the properties of such strategic effects on the dispersion of forecasts.

To model the forecasting games that are of direct interest to this paper, we use the framework of aggregate games as defined in Martimort and Stole (2012). In aggregate games, each player's payoff is a function of his or her own strategy and some aggregator of the strategy profile of all players.[3] In our forecasting game, each forecaster receives two signals—private and public—about the true fundamentals. A forecaster's strategy is a mapping from his or her private and public signals to a prediction of the fundamentals. Each forecaster's payoff depends on the distance of the forecaster's prediction from the fundamentals and from the aggregator, for which we use the average forecast across the population.[4] The forecaster prefers to be close to the fundamentals, but we do not require that the forecaster prefers to be close to the average. Indeed, the forecaster might want to make an accurate prediction that is distinct from the "herd."

[1]Croushore (1997) notes that "some participants might shade their forecasts more toward the consensus (to avoid unfavorable publicity when wrong), while others might make unusually bold forecasts, hoping to stand out from the crowd."

[2]See McClure, Ben. "Earnings Forecasts: A Primer." *Stocks Articles on Investopedia.com.* http://www.investopedia.com/articles/stocks/06/earningsforecasts.asp (Accessed February 9, 2017).

[3]Frankel, Morris and Pauzner (2003) consider games in which a player's payoff depends on the strategy profile of *other* players. Such games would not immediately belong to the aggregate game class, and we prefer to define the forecasting game as an aggregate game in the context of the example previously given where the agents' "collective" estimate of the unknown economic condition matters. See also Martimort and Stole (2011) for an example in which debt rating agencies assess a firm's underlying probability of default taking into account the average assessment.

[4]The class of aggregate games with additively separable aggregators, such as the mean, are studied in Acemoglu and Jensen (2013) and Cornes and Hartley (2012). The general definition of aggregate games with a linear aggregate is treated in Martimort and Stole (2012). See these works and the references cited therein for many examples and specific applications of aggregate games.

Our first result characterizes the best responses of forecasters: the optimal forecast takes the form of a linear combination of private and public signals, where the weights on the signals depend on the precision of the signals and the degree of forecasters' private value to coordinate. Importantly, this private value of coordination subsumes both the forecasters' incentive to herd or exaggerate and their incentive to strategically control the average forecast. The former incentive, which we call the *herding motive*, arises from the preference structure; whereas the latter, which we call the *market power motive*, is only present in games with a finite set of players. To highlight the differences in the nature of strategic behavior, we identify a benchmark model of forecasting games with a continuum of forecasters who have no market power, as well as a pure-prediction benchmark in which forecasters only care about being correct with no strategic behavior of any kind.

We then proceed by analyzing the strategic behavior in the finite-player model and explore the strategic effects on forecast dispersion. We find that in a game where the forecasters want to herd around the average forecast, each forecaster assigns *less value to aligning* his or her forecast with others; in equilibrium, each forecaster adjusts upward his or her reliance on private information, as compared to the continuum-player model. This adjustment represents the "market power" distortion of the forecaster's behavior that weakens the underlying degree of herding motive. That is, any forecaster's inclination to herd is not entirely transferred to his or her reliance on public information given the non-negligible influence that any agent's forecast, thus his or her private information, can have on the average forecast. Consequently, the dispersion of forecasts in the finite-player model with herding is higher than in the continuum-player model. The opposite is true in a game where the forecasters want to be distinctive from the average forecast. In either case, the size of the difference is larger with a fewer number of forecasters, i.e., stronger the market power.

The effects of strategic behavior on forecast dispersion depend not only on the underlying primitives of the forecasting market but also on the market size. Hence, in order to render appropriate policy implications for reducing forecast dispersion, the research needs to distinguish between the use of finite-player and continuum-player models for representing forecasting markets in addition to obtaining accurate estimates of the parameters that shape the forecasting market. As equivocal as our results may seem, the payoff to our theoretical analysis on the finite-player equilibrium properties is that it offers a micro framework to identify what can go when and emphasizes that understanding forecast dispersion depends crucially on whether one considers a large or small number of forecasters.

Our paper is related to a number of different strands in the literature. First, this paper connects to the line of research that studies the empirical properties of forecasts (e.g., Andrade and Le Bihan, 2013; Coibion and Gorodnichenko, 2012; Ehrbeck and Waldmann, 1996; Mankiw, Reis and Wolfers, 2004; Ottaviani and Sørensen, 2006; and Patton and Timmermann, 2010). Within this line of literature, the most relevant comparison is with Ottaviani and Sørensen (2006). These authors suggest that the observed forecast dispersion in the survey data might be the outcome of strategic behavior, and develop theories of strategic forecasting in a model with dispersed private information. We also consider forecasting games with dispersed information and strategic behavior to examine the dispersion across forecasts. The key difference is that their account of strategic behavior as herding or exaggeration around the prior mean of the uncertain state does not depend on the presence of multiple forecasters; we extend further by considering strategic behavior that arises from the element of a small number of forecasters who can strategically influence the average forecast. Our analysis suggests that the assumption of strategic interactions among a finite number of forecasters might help explain some empirical properties of survey forecasts that are hard to understand otherwise.[5]

The modeling approach in our paper relates to a large literature on aggregate games (e.g., Acemoglu and Jensen, 2013; Cornes and Hartley, 2012; and Martimort and Stole, 2012, among many others). Leaving aside their theoretical underpinnings, we adopt the concept of aggregate games as defined in their work. Our forecasting game is an example of aggregate games with the additional properties that the agents have dispersed information and the payoff structure allows for externalities. These features are reminiscent of the broad body of literature on games with externalities and dispersed information (e.g., Amador and

[5]Ehrbeck and Waldmann (1996) provide evidence of forecasters' strategic incentives in choosing forecasts. Andrade and Le Bihan (2013) find the relatively low level of disagreement among forecasters observed in the ECB Survey of Professional Forecasters in comparison to forecast dispersion that should be observed if one resorted only on dispersed information; and suggest that, as a way to account for this discrepancy, modeling uncertainty or strategic interactions between forecasters might provide a better match of the empirical patterns of forecasts.

Weill, 2010; Angeletos and Pavan, 2004, 2007; Colombo, Femminis and Pavan, 2014; Morris and Shin, 2002; and Ui, 2014). This line of studies generally focus on the effects of payoff or informational externalities under dispersed information in environments where there is a large number of small players as a measure-one continuum. We instead consider a finite set of players, which allows for a richer set of externality effects. In addition to the payoff externality, each player can exert externality on all other players by strategically influencing the average forecast, i.e., exercising market power. In such environments, we focus on the effects of market power on the dispersion of equilibrium behavior across agents.[6]

The rest of the paper is organized as follows. Section 2 defines finite-player forecasting games, equilibrium, and benchmark models. Section 3 presents our main results on the finite-player properties of strategic effects on forecast dispersion. Section 4 summarizes our results and briefly addresses a few directions for future work. The appendix contains some proofs omitted from the text.

2 Forecasting Game

In this section, we consider forecasting games in the context of aggregate games with the additional requirement that agents have a coordination motive and dispersed information, and define two benchmarks.

In the economy there is a finite number of agents (forecasters), each of whom is indexed by i, and the number of agents is $n \in \mathbb{N}$ where $n \geq 2$. We represent the true fundamentals of the economy with an exogenous random variable $\theta \in \mathbb{R}$ drawn from an improper distribution over the real line. Each agent i chooses a prediction of θ, which we denote as forecast $a_i \in \mathbb{R}$, and receives a payoff u_i. This payoff depends on the agent's own forecast and an aggregate of all agents' forecasts, the property of which defines an aggregate game. We consider the average forecast across the population $A_n \equiv \frac{1}{n}\sum_{i=1}^{n} a_i$ as the aggregator.

We focus on forecasting games in which each agent i cares both about being correct, generating a fundamental motive to be close to the true θ, and about his distance to the ag-

[6] Bergemann, Heumann and Morris (2015) also study games with a finite number of agents and noisy signals, but their focus is on how the nature of the information structure affects the equilibrium in the presence of the interaction of market power and learning externalities.

gregator A_n, which generates a coordination motive.[7] For tractability, we assume that agent i's preferences are quadratic to ensure linearity in the best responses. Formally, combining the two elements of the agents' preferences and adopting a quadratic specification, agent i's payoff is given by $u_i(a_i, A_n, \theta) = -\frac{1}{2}\left((1-r)(a_i - \theta) + r(a_i - A_n)\right)^2$, or equivalently,

$$u_i(a_i, A_n, \theta) = -\frac{1}{2}\left(a_i - (1-r)\theta - rA_n\right)^2, \tag{1}$$

where the parameter $r \in (-1, 1)$ gives the weight that the agent puts on the aggregator relative to the fundamentals.[8]

The parameter r captures whether and how strongly the agents want to be close to or distinctive from the average forecast. If $r = 0$, then each agent cares only about being close to the true θ. When $r > 0$, an agent benefits the most by making an accurate prediction that is closer to the average forecast. For example, in some situations forecasters may fear a reputation loss from moving away from the "herd". On the other hand, $r < 0$ characterizes a forecasting game in which each agent desires to correctly predict the underlying fundamentals but also prefers to be distinctive from the average forecast. In situations where forecasters compete in contests with prizes allocated to the best performers, forecasters may gain by moving away from the herd and winning the prize while maintaining a balance between being both accurate and distinctive.

We assume that agents do not observe the realization of the true θ but instead observe noisy private and public signals that are informative about the underlying fundamentals. Each agent i observes a private signal x_i and a public signal p characterized as

$$x_i = \theta + (\alpha_{x,i})^{-1/2}\varepsilon_i \tag{2}$$

and

$$p = \theta + (\alpha_p)^{-1/2}\varepsilon, \tag{3}$$

where ε_i and ε are, respectively, idiosyncratic and common noises that are independent of each other as well as of θ, and both follow $\mathbb{N}(0, 1)$. We let $\alpha_{x,i}$ and α_p denote the precision

[7] For ease of exposition, we use male pronouns for the agent.

[8] The equilibrium is unique if and only if $r < 1$.

of private and public signals, respectively.

For tractability, we maintain $\alpha_{x,i} = \alpha_{x,j}$ for $i \neq j$ to focus on symmetric equilibria. Because a change in a_i exerts a non-negligible effect on the average forecast A_n in the finite-player model, in any symmetric equilibrium the aggregate variable A_n is a function of $(\mathbf{x}, p)$ where $\mathbf{x} = (x_1, \ldots, x_n)$ is an arbitrary private signal profile. The following definition of a linear equilibrium is standard.

Definition 1. *A linear equilibrium is a strategy profile* $\mathbf{a} = (a_1, \cdots, a_n)$ *where* $a_i : \mathbb{R}^2 \to \mathbb{R}$ *for each* $i \in \{1, \ldots, n\}$ *such that for all* i*,* a_i *is linear in* x_i *and* p*; and for all* (x_i, p)*, each* a_i *satisfies*

$$a_i(x_i, p) = arg\max_{a'} \mathbb{E}\left[u_i(a', A_n(\mathbf{x}, p), \theta)|x_i, p\right], \tag{4}$$

where $A_n(\mathbf{x}, p) = \frac{1}{n}\sum_{i=1}^{n} a_i(x_i, p)$ *for all* $(\mathbf{x}, p)$*.*

Agent i's best response in the utility maximization problem (4) is determined by the first-order condition $a_i(x_i, p) = \mathbb{E}\left[(1-r)\theta + rA_n(\mathbf{x}, p)|x_i, p\right]$ for all (x_i, p), which can be rewritten as:

$$a_i(x_i, p) = \mathbb{E}\left[(1-\gamma)\theta + \gamma A_{n-i}((x_j)_{j\neq i}, p)|x_i, p\right], \ \forall (x_i, p), \tag{5}$$

where $A_{n-i}((x_j)_{j\neq i}, p) \equiv \frac{1}{n-1}\sum_{j\neq i} a_j(x_j, p)$ and $\gamma \equiv \frac{r(n-1)}{n-r}$. The parameter $\gamma \in (-1, 1)$ denotes the *equilibrium degree of coordination*, which measures how agents value aligning their forecasts with the forecasts of others in equilibrium. The agents' forecasts are strategic complements if $\gamma > 0$ and strategic substitutes if $\gamma < 0$.

Proposition 1. *For any given value of* (θ, p)*, a linear equilibrium exists and is unique. The equilibrium forecast of agent* i *is given by*

$$a_i(x_i, p) = \lambda(\gamma)x_i + (1-\lambda(\gamma))p, \ \forall i \in \{1, \ldots, n\}, \tag{6}$$

where $\lambda(\gamma) = \frac{\alpha_x}{\alpha_x + \frac{1}{1-\gamma}\alpha_p}$*. In the equilibrium the average forecast is*

$$A_n(\mathbf{x}, p) = \lambda(\gamma)\theta + (1-\lambda(\gamma))p + \lambda(\gamma)(\alpha_x)^{-1/2}\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i. \tag{7}$$

The equilibrium strategy (6) dictates how the agents allocate their use of private and public information. The equilibrium allocation is characterized by the coefficient $\lambda(\gamma)$, which measures the sensitivity of the equilibrium forecasts to private information relative to public information. This coefficient depends on the relative precision of private information to public information and on the degree of coordination γ.

In the specification of the game, each agent cares about the distance of his forecast from the average of n forecasts as well as having some control over that average. These aspects of the model are intrinsically linked in subtle ways, together creating strategic behavior. Our primary goal is to disentangle the strategic effects due to the finiteness of the number of agents from those due the intrinsic payoff structure of the model; thus gauging to what extent the finiteness affects forecast dispersion. For this purpose, we find it useful to classify two benchmark models. The first benchmark is the forecasting game with $r = 0$, referred to as a simple-prediction model. In this benchmark, the forecasters only care about being correct on the true fundamentals, so there is no strategic behavior featured in equilibrium. The second benchmark is the forecasting game with a continuum of agents uniformly distributed over the unit interval $[0, 1]$, referred to as a continuum-player model.

Proposition 2. *As n tends to infinity, the equilibrium of our finite-player forecasting game converges to the equilibrium of the continuum-player model.*

In the manner of replicating our forecasting game as the number of forecasters increases, we obtain a well-defined continuum limit forecasting game. Hence, the continuum-player model is an approximation to the large forecasting game with a large number of small forecasters. The continuum-player model can be interpreted as a perfectly competitive forecasting market in which forecasters are "price" takers; whereas our finite-player game represents an oligopolistic market in which a small number of forecasters can control "price," where "price" connotes the average forecast. In this sense, the continuum-player model serves as our benchmark when the consideration of strategic manipulation of the average forecast is not of central concern to the analysis. Comparing equilibrium of the finite-player model to this benchmark thus isolates any discrepancy between the two models' strategic incentives of forecasters in choosing their forecasts.

3 Strategic Behavior and Forecast Dispersion

In this section, we first decompose γ to understand the strategic incentive that is only present in the finite-player model. We then investigate the strategic behavior in equilibrium by comparing $\lambda(\gamma)$ in our model to the corresponding parameters in the benchmarks. Finally, we examine the effects of strategic behavior on forecast dispersion.

3.1 Strategic Incentives in the Finite-Player Model

Our finite-player forecasting game imparts two types of strategic incentives. The first type is induced from the preference structure of the model that the agents prefer either to herd around the average forecast or to be distinctive from the herd, which we call the *herding motive*. The second type arises from the finite-player feature of the model that each agent's forecast is influential on the average forecast A_n. For the present context, we say that an agent exercises *market power* if, by changing his own forecast, he can strategically manipulate the average forecast; and call such incentive the *market power motive*. We use the partial derivative $\partial A_n/\partial a_i = 1/n$ as a measure for market power of agent i. The parameter γ then reflects not only the agents' desire to herd around or be distinctive from the average forecast but also the market power of the agents—their ability to strategically control the average.

To distinguish the two notions of strategic incentives, we decompose γ as follows:

$$\gamma = r + r^* \tag{8}$$

where $r^* \equiv \frac{-r(1-r)}{n-r} \in (-1, 1)$. The first term r in (8) denotes the *degree of the herding motive* that is intrinsic in our forecasting game; it captures the importance that the agents are inclined to attach to the average forecast. It is trivial to see that $sign(\gamma) = sign(r)$.[9] The second term—the discrepancy between γ and r—is what we call the *residual degree of coordination*. This residual degree is linked to the number of agents in the forecasting game. Given the degree of the herding motive, the magnitude of the residual degree decreases as

[9]When the agents prefer to herd their forecasts around the average ($r > 0$), then in equilibrium each agent would want to make his forecast similar to the other agents' forecasts ($\gamma > 0$); when the agents prefer to be distinctive from the average ($r < 0$), then in equilibrium each agent would wish to make his forecast unique relative to those of others ($\gamma < 0$).

the number of agents increases. Hence stronger market power (in terms of a higher $1/n$) increases the magnitude of the residual degree.

It is immediate to see that $\lim_{n\to\infty}\gamma = r$ because the residual degree dissipates in the limit, that is, $\lim_{n\to\infty} r^* = \lim_{n\to\infty} \frac{-r(1-r)}{n-r} = 0$. Hence, when the forecasting game has an infinite number of agents, only the herding motive matters; so the parameter r effectively represents the equilibrium degree of coordination in the continuum-player model. The following lemma summarizes the comparison between the equilibrium degree of coordination in our forecasting game and that of the continuum-player model.

Lemma 1. *For any given n such that $2 \leq n < \infty$,*

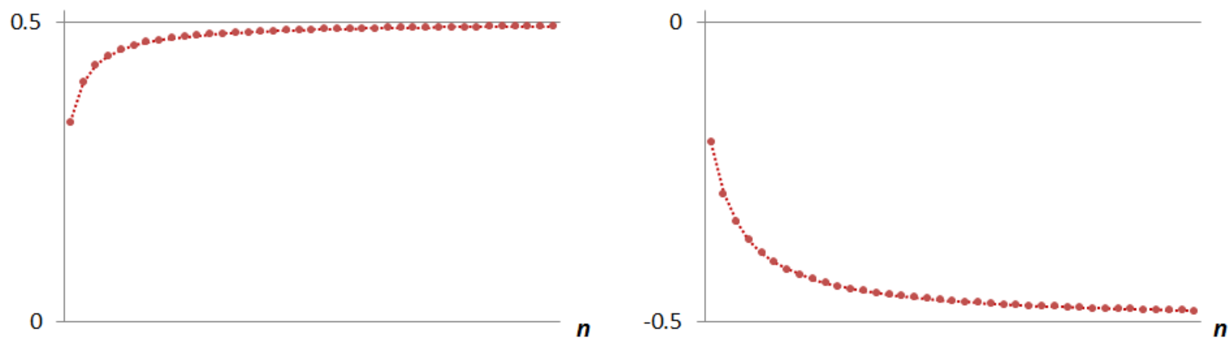
(i) $\gamma < r$ when $r > 0$, where $\frac{\partial\gamma}{\partial n} > 0$;

(ii) $\gamma > r$ when $r < 0$, where $\frac{\partial\gamma}{\partial n} < 0$.

Proof. $\gamma = \frac{r(n-1)}{n-r} = r - \frac{r(1-r)}{n-r} \lesseqgtr r$ if and only if $r \gtreqless 0$ because $(1-r) > 0$ and $n \geq 2$. Also by differentiating γ with respect to n given r, we obtain $\frac{\partial\gamma}{\partial n} = \frac{r(1-r)}{(n-r)^2}$, which is strictly positive when $r > 0$ and strictly negative when $r < 0$. □

Lemma 1 establishes that, relative to the continuum-player benchmark, (i) each player puts less private value on aligning his forecast with those of others in the finite-player game with positive degree of herding; whereas (ii) each player puts less private value on differentiating his forecast from others' forecasts in the finite-player game with negative degree of herding. In either case, the weaker equilibrium degree of coordination is essentially due to the presence of market power in games with finite players. Figure 1 illustrates Lemma 1 for the cases of $r = 0.5$ and $r = -0.5$.

Lemma 1 is a direct implication of the finite-player model in comparison to the continuum-player benchmark. With an infinite number of agents, the agents are infinitesimal so that each agent cannot influence the average forecast. Without market power, the equilibrium degree of coordination is exactly the underlying degree of the herding motive, r. The parameter r measures the slope of the best responses with respect to the mean of individual forecasts in the population in which the idiosyncratic noises disappear in the mean forecast as n goes to infinity. However, with a finite number of agents, a change in agent i's choice of forecast a_i exerts a non-negligible effect on the average forecast of the population. In



(A) When $r = 0.5$ **(B)** When $r = -0.5$

Figure 1. The equilibrium degree of coordination γ

equilibrium each agent takes into account such externality by appropriately adjusting how he values aligning his forecast with those of others. In fact, the choices of others are summarized in the average of their forecasts, A_{n-i}, which is contaminated with the other agents' private noises. Consequently, the externality causes each agent to put less value on what other agents do. That is, the slope of the best responses with respect to the average of the other agents' forecasts flattens. This adjustment is captured by the second term r^* in (8), where the size of such adjustment is larger with fewer agents.

3.2 Strategic Behavior in Equilibrium

We now investigate how the two strategic incentives—herding motive and market power motive—affect the equilibrium behavior, in particular, the equilibrium use of information.

The coefficient $\lambda(\gamma) = \frac{\alpha_x}{\alpha_x + \frac{1}{1-\gamma}\alpha_p}$ in the equilibrium strategy (6) depends on the relative precision of the two signals as well as on the equilibrium degree of coordination γ.[10] Hence, the equilibrium strategy in the finite-player forecasting game respects both the herding and market power motives that are together captured by the equilibrium degree of coordination. We can disentangle the herding and market power effects on the equilibrium behavior by

[10]For any given α_x and α_p, agents rely more on public information relative to private information (i.e., $\lambda(\gamma)$ is lower) when the degree of coordination is higher. Angeletos and Pavan (2007) obtain the same result for similar environments but with a continuum of players.

decomposing $\lambda(\gamma)$ as follows:

$$\lambda(\gamma) = \lambda + \underbrace{\frac{\frac{\gamma}{n-1}\lambda(1-\lambda)}{1+\frac{\gamma}{n-1}\lambda}}_{\equiv\lambda^*} \tag{9}$$

where $\lambda \equiv \frac{\alpha_x}{\alpha_x + \frac{1}{1-r}\alpha_p}$.

Notice first that the second term in (9) denoted by λ^* dissipates as $n \to \infty$ because $\lim_{n\to\infty} \gamma = r$ and λ does not depend on n. So when the forecasting game gets larger, the weight that the agents put on private information in equilibrium converges to λ. This parameter λ represents the sensitivity of the equilibrium forecasts to private information in the continuum-player benchmark (See the proof of Proposition 2). The term λ^* thus measures the "residual" sensitivity of the equilibrium forecasts to private information as compared the continuum-player benchmark where the agents have no market power.

We can further break down λ into two terms as $\lambda = \delta + \left[-\frac{r\delta(1-\delta)}{1-r\delta}\right]$, where $\delta \equiv \frac{\alpha_x}{\alpha_x+\alpha_p}$ captures the relative precision of private information. Note that $\lambda(\gamma) = \lambda = \delta$ for $r = 0$. If the forecasters only care about being correct (i.e., $r = 0$), then there are neither herding motive nor market power motive; hence no strategic behavior featured in the equilibrium use of information. The two types of information are then given weights that are commensurate with their precision. The term δ thus represents the sensitivity of the equilibrium forecasts to private information in the simple-prediction benchmark.

We formalize the comparison of the relative sensitivity of the equilibrium to private information between our game and the two benchmarks in the following lemma.

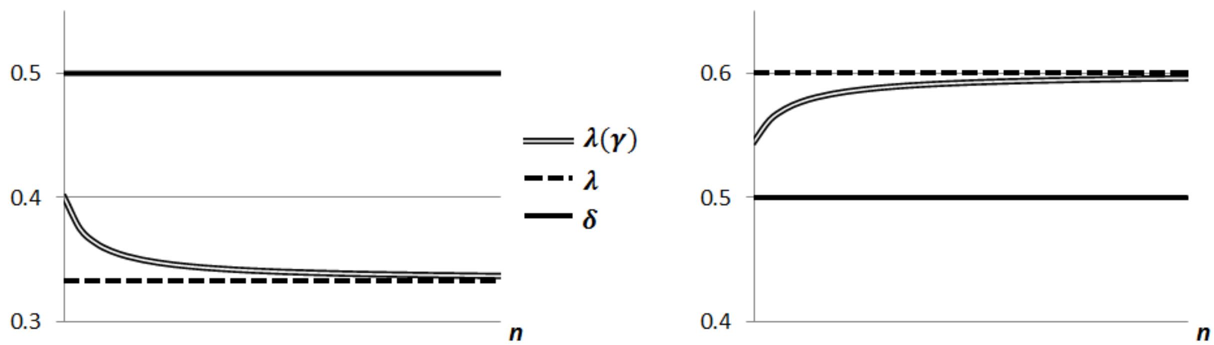
Lemma 2. *For any given α_x and α_p and for any given n such that $2 \leq n < \infty$,*

(i) $\lambda < \lambda(\gamma) < \delta$ when $r > 0$, where $\frac{\partial\lambda(\gamma)}{\partial n} < 0$;

(ii) $\delta < \lambda(\gamma) < \lambda$ when $r < 0$, where $\frac{\partial\lambda(\gamma)}{\partial n} > 0$.

Proof. We can see from (9) that $\lambda(\gamma) \lesseqgtr \lambda$ iff $\gamma \lesseqgtr 0$, which holds iff $r \lesseqgtr 0$, respectively, for any given finite $n \geq 2$. Also by differentiating $\lambda(\gamma) = \frac{\alpha_x}{\alpha_x + \frac{n-r}{n(1-r)}\alpha_p}$ with respect to n for any given r, α_x, and α_p, we obtain $\frac{\partial\lambda(\gamma)}{\partial n} = \frac{-r\frac{\alpha_x\alpha_p}{n^2(1-r)}}{\left(\alpha_x + \frac{n-r}{n(1-r)}\alpha_p\right)^2}$, which is strictly positive if $r < 0$ and is strictly negative if $r > 0$. Further, $\lambda(\gamma) = \frac{\alpha_x}{\alpha_x + \frac{1}{1-\gamma}\alpha_p} \leq \frac{\alpha_x}{\alpha_x+\alpha_p} = \delta$ if and only if $\gamma \geq 0$, which holds iff $r \geq 0$, where the equality is true iff $r = 0$. □

Lemma 2 implies that in the finite-player forecasting game where the agents want to be both accurate and herding (resp. accurate and distinctive), each agent adjusts upward (resp. downward) his reliance on private information relative to the continuum-player benchmark, but not above (resp. below) the level of reliance in the simple-prediction benchmark. This adjustment measured by the residual λ^* in (9) represents what we call the *market power distortion* in the equilibrium choice of information, where $\lambda^* > 0$ when $r > 0$ and $\lambda^* < 0$ when $r < 0$. The size of such distortion is larger with fewer agents, thus with stronger market power. Figure 2 illustrates Lemma 2 for the cases of $r = 0.5$ and $r = -0.5$ with $\delta = 0.5$.



(A) When $r = 0.5$ **(B)** When $r = -0.5$

Figure 2. The sensitivity to private information with $\delta = 0.5$

The reasoning behind on how the weights are adjusted follows from considering the best response of an agent in a finite-player model. Suppose that other than agent 1, the agents use the strategy $a_j(x_j, p) = \lambda x_j + (1-\lambda)p,\ \forall j \in \{2, \ldots, n\}$, where $\lambda = \frac{\alpha_x}{\alpha_x + \frac{1}{1-r}\alpha_p}$. The mean of these agents' forecasts is $A_{n-1}((x_j)_{j\neq i}, p) = \frac{1}{n-1}\sum_{j\neq 1} a_j(x_j, p) = \lambda \frac{1}{n-1}\sum_{j\neq 1} x_j + (1-\lambda)p$. Then agent 1's best response is

$$\begin{aligned} a_1(x_1, p) &= \mathbb{E}\left[(1-\gamma)\theta + \gamma A_{n-1}((x_j)_{j\neq 1}, p)|x_1, p)\right] \\ &= (1-\gamma+\gamma\lambda)\mathbb{E}[\theta|x_1, p] + \gamma(1-\lambda)p \\ &= \lambda' x_1 + (1-\lambda')p \end{aligned}$$

where $\lambda' = (1-\gamma(1-\lambda))\delta$ and $\delta = \frac{\alpha_x}{\alpha_x+\alpha_p}$. The λ' is strictly higher than λ if and only if $\gamma < r$, which holds in the case of positive herding ($r > 0$) by Lemma 1. Thus, when $r > 0$, if agent

1 expects that other agents use the strategy that puts the weight $\lambda = \frac{\alpha_x}{\alpha_x + \frac{1}{1-r}\alpha_p}$ on private information, then the best response for agent 1 is to put a weight on private information λ' that is higher than λ. But all of the other agents also have the same incentives to do so. Hence, this incentive leads every agent to "strategically" use more private information in equilibrium compared to when there is an infinite number of agents.

Perhaps the better reasoning simply follows from the fact that each agent's forecast has a non-negligible effect on the average forecast. That is, with a finite number of agents, the average forecast is contaminated with the agents' private noises. Accordingly, when the agents want to be close to the average forecast ($r > 0$), the importance that any agent is inclined to attach to the average forecast is not entirely transferred to the agent's weight on public information, given the influence that any agent's private information can have on the average forecast in the presence of market power. The opposite happens when the agents want to move away from the average forecast ($r < 0$). The finiteness of the number of agents induces each of them to assign a higher weight on public information relative to the case with an infinite number of agents. These results can be interpreted as the weakening of the "pre-existent" herding effect on the equilibrium use of information due to market power.

3.3 The Effect of Strategic Behavior on Forecast Dispersion

Following from the foregoing analysis, we present our main result on the equilibrium level of dispersion that is measured by the variation in the equilibrium forecasts across agents.

Each agent i chooses his optimal forecast as characterized by (6) in the finite-player model, which can be rewritten as:

$$a_i = \theta + \lambda(\gamma)(\alpha_x)^{-1/2}\varepsilon_i + (1 - \lambda(\gamma))(\alpha_p)^{-1/2}\varepsilon \tag{10}$$

where $\lambda(\gamma) = \frac{\alpha_x}{\alpha_x + \frac{1}{1-\gamma}\alpha_p}$ and $\gamma = \frac{r(n-1)}{n-r}$. As is evident from (10), idiosyncratic noise ε_i generates dispersion under incomplete information. Therefore, the equilibrium level of forecast dispersion for any given realizations of θ and p can be characterized by the following expression for the finite-player model:[11]

$$Var(a_i|\theta, p) = \left(\lambda(\gamma)(\alpha_x)^{-1/2}\right)^2. \quad (11)$$

We can see from (11) that the equilibrium level of forecast dispersion depends on the sensitivity of individual forecasts to private information relative to public information $\lambda(\gamma)$. This weight incorporates the market power distortion, the magnitude of which is inversely proportional to the number of forecasters. Thus for any given levels of signal precisions and the degree of the herding motive, the equilibrium dispersion of forecasts in the finite-player model also depends upon the strength of market power. We examine the market power effect by showing how the number of agents affects the dispersion of forecasts and by comparing the forecast dispersion in the finite-player model to that in the continuum-player benchmark given by $\left(\lambda(\alpha_x)^{-1/2}\right)^2$.

Proposition 3. *For any given α_x, α_p, and r, the finite-player model's forecast dispersion falls with an increase in n if and only if $r > 0$; and increases with an increase in n if and only if $r < 0$. Further, relative to the continuum-player benchmark, the forecast dispersion in the finite-player game is higher when $r > 0$ but lower when $r < 0$.*

Proof. An observation of (11) yields $\frac{\partial Var(a_i|\theta,p)}{\partial n} \propto \frac{\partial \lambda(\gamma)}{\partial n}$. Thus by Lemma 2, as n increases, $Var(a_i|\theta, p)$ increases iff $r < 0$ and decreases iff $r > 0$. Further, for any given α_x and α_p, $\left(\lambda(\gamma)(\alpha_x)^{-1/2}\right)^2 > \left(\lambda(\alpha_x)^{-1/2}\right)^2$ iff $\lambda(\gamma) > \lambda$, which happens iff $r > 0$. □

Proposition 3 implies that, as the market power (measured in terms of $1/n$) increases, the forecast dispersion increases when the forecasters wish to herd around the average forecast whereas it decreases when they wish to be distinctive from the average.[12] Public information is a relatively better predictor of the average forecast than private information. But with a

[11] Our characterization of forecast dispersion follows Angeletos and Pavan's (2004) definition of the cross-sectional heterogeneity in equilibrium actions.

[12] When the game has no herding motive ($r = 0$) and so forecasts are strategically independent ($\gamma = 0$), each forecaster uses an equilibrium strategy that is independent of the number of forecasters in the game. Thus in the pure-prediction benchmark, a change in the number of forecasters has no effect on the equilibrium level of forecast dispersion given by $\left(\delta(\alpha_x)^{-1/2}\right)^2$. Following from Lemma 2, the forecast dispersion in the finite-player model is lower (resp. higher) when $r > 0$ (resp. $r < 0$) as compared to the dispersion in the pure-prediction benchmark.

small number of forecasters in the game, the average forecast contains the forecasters' private noises. Hence, relative to a larger forecasting game, the forecasters who wish to herd around the average forecast find it optimal to rely less on public information, which leads to a higher disagreement among forecasters. When instead the forecasters want to move away from the average forecast, they find it optimal to rely more on public information, generating a lower disagreement among forecasters than in the large forecasting game.

In sum, the market power effect on forecast dispersion runs counter to the role of the underlying herding motive in the forecasting game. In a game with positive herding motive (i.e., preference for herding), the presence of market power contributes to a higher forecast dispersion. In a game with negative herding motive (i.e., preference for being distinctive), the presence of market power contributes to a lower forecast dispersion. We emphasize that understanding strategic interactions among players and their effects on forecast dispersion depends crucially on whether one considers a large or small set of players.

In the context that takes into account the finite-player externality (or market power) in forecasting, the disagreement among forecasters observed in oligopolistic settings where agents prefer to stand out from the herd is lower in comparison to the disagreement that should be observed if one resorted to the assumption of large, competitive settings. Our analysis suggests that modeling strategic interactions among a finite set of forecasters might provide a better match of some empirical patterns of forecast dispersion observed in the survey data (e.g., ECB Survey of Professional Forecasters) than the previous works that try to replicate them. As such, further analysis is needed to accurately estimate the underlying primitives using the forecasting model of an appropriate size.

4 Concluding Remarks

In this paper, we investigate the finite-player equilibrium properties for the class of forecasting games in which each forecaster cares both about being correct on the true fundamentals and about his distance to the average forecast. In particular, we isolate the differences in the nature of strategic interactions between forecasters when the set of players is finite with respect to the case with a continuum of payers. We find that the strategic incentive that is only present in games with a finite number of agents, which we call the "market power" motive, causes each forecaster to behave strategically in a way that weakens the role played by the underlying herding or exaggeration motive in the forecasting game. Such consideration leads to the discrepancy between the forecast dispersion that should be observed in a setting with a small number of forecasters (where each forecaster can strategically influence the average forecast, thus exerting market power) and the forecast dispersion observed in a setting with a large number of small forecasters. Our analysis identifies a micro-theoretic structure for explaining forecast dispersion, and further signifies the importance of studying forecast dispersion in the context of a finite economy. For future work, a fruitful analysis is to estimate the key parameters in our model by using survey data on n number of professional forecasters. By doing so, the insights of our theoretical analysis could be carried over to the empirical analysis of the source of persistent forecast dispersion.

The framework we focus on is amenable to a number of extensions that are relevant for different applications. We conclude this paper by mentioning a few of these directions. First, the role of information on cross-sectional dispersion of actions as well as on aggregate volatility has been extensively studied by Angeletos and Pavan (2004) in environments with an infinite number of players in which there is no market power effect. As a complementary analysis, we can readily investigate how forecast dispersion changes due to more precise information in the context of the finite-player model. The information effect per se, however, is not central to our analysis of the strategic effects on forecast dispersion; hence we do not expound on it in this paper but relegate the formal analysis to online supplementary materials. Second, we could extend the forecasting game to a dynamic version in which the fundamental variable of the forecasters' interests follows an AR (1) process. In such extension, we obtain the analogous versions of our results, and our main themes are robust. We could also allow for the consideration of reputation in a dynamic forecasting game. A simple way to introduce such a consideration is to assume that the agent derives some benefit from being correct in the previous period. Other extensions include allowing for correlation between the private signals or for an endogenous information acquisition motive. Our model could readily accommodate each of these extensions with some added nuances but without adding commensurate insights.

Appendix: Omitted Proofs

Proof of Proposition 1. The proof follows the similar arguments in Morris and Shin (2002) and Angeletos and Pavan (2007) but with a finite number of players. The expected utility of forecaster i conditional on signals x_i and p and assuming forecaster j, $j \neq i$, uses strategy a_j, is

$$\mathbb{E}[u_i|x_i,p] = \mathbb{E}\left[-\frac{1}{2}(a_i - (1-r)\theta - rA_n(\mathbf{x},p))^2|x_i,p\right].$$

Given x_i and p, the first-order condition for the optimization of the finite-player forecasting game gives the optimal forecast of

$$a_i(x_i,p) = (1-r)\mathbb{E}[\theta|x_i,p] + r\mathbb{E}[A_n(\mathbf{x},p)|x_i,p]. \tag{12}$$

Given linearity, it is natural to look for equilibrium forecast decisions that are linear in x_i and p so that $a_i = \kappa_0 x_i + \kappa_1 p$, where κ_0 and κ_1 are constants determined in equilibrium. Substituting $\mathbb{E}[\theta|x_i,p] = \delta x_i + (1-\delta)p$, where $\delta = \frac{\alpha_x}{\alpha_x+\alpha_p}$, and $A_n(\mathbf{x},p) = \frac{1}{n}\sum_{i=1}^n a_i(x_i,p)$, we have:

$$a_i(x_i,p) = (1-r)(\delta x_i + (1-\delta)p) + \frac{r}{n}a_i + \frac{r}{n}\sum_{j\neq i}\mathbb{E}[a_j|x_i,p].$$

Plugging in the candidate equilibrium strategy:

$$a_i(x_i,p) = (1-r)(\delta x_i + (1-\delta)p) + \frac{r}{n}(\kappa_0 x_i + \kappa_1 p) + \frac{r}{n}\sum_{j\neq i}\mathbb{E}[\kappa_0 x_j + \kappa_1 p|x_i,p].$$

Note that $\mathbb{E}[x_j|x_i,p] = \mathbb{E}[\theta|x_i,p]$, which yields:

$$\begin{aligned} a_i(x_i,p) &= (1-r)(\delta x_i + (1-\delta)p) + \frac{r}{n}(\kappa_0 x_i + \kappa_1 p) + \frac{r}{n}(n-1)\kappa_1 p + \frac{r}{n}(n-1)\kappa_0(\delta x_i + (1-\delta)p), \\ &= \left((1-r)\delta + \frac{r}{n}\kappa_0 + \frac{r(n-1)}{n}\kappa_0\delta\right)x_i \\ &\quad + \left((1-r)(1-\delta) + \frac{r}{n}\kappa_1 + \frac{r(n-1)}{n}\kappa_1 + \frac{r(n-1)}{n}\kappa_0(1-\delta)\right)p. \end{aligned}$$

It follows that $a_i(x_i, p) = \kappa_0 x_i + \kappa_1 p$ constitutes a linear equilibrium if and only if κ_0 and κ_1 solve

$$\kappa_0 = (1-r)\delta + \frac{r}{n}\kappa_0 + \frac{r(n-1)}{n}\kappa_0\delta,$$
$$\kappa_1 = (1-r)(1-\delta) + \frac{r}{n}\kappa_1 + \frac{r(n-1)}{n}\kappa_1 + \frac{r(n-1)}{n}\kappa_0(1-\delta),$$

which gives the following:

$$\kappa_0 = \frac{\alpha_x}{\alpha_x + \frac{n-r}{n(1-r)}\alpha_p} \text{ and } \kappa_1 = \frac{\frac{n-r}{n(1-r)}\alpha_p}{\alpha_x + \frac{n-r}{n(1-r)}\alpha_p}.$$

This is clearly a unique symmetric linear equilibrium. Let $\lambda(\gamma) \equiv \frac{\alpha_x}{\alpha_x + \frac{1}{1-\gamma}\alpha_p}$ where $\gamma \equiv \frac{r(n-1)}{n-r}$ (so that $\frac{1}{1-\gamma} \equiv \frac{n-r}{n(1-r)}$). Then $a_i(x_i, p) = \lambda(\gamma)x_i + (1-\lambda(\gamma))p$ for all $i \in \{1, \ldots, n\}$ as in (6), and the average forecast (7) follows. Also note that (12) can be rewritten as

$$a_i(x_i, p) = (1-r)\mathbb{E}[\theta|x_i, p] + \frac{r}{n}a_i(x_i, p) + \frac{r(n-1)}{n}\mathbb{E}[A_{n-i}((x_j)_{j\neq i}, p)],$$
$$= \frac{n(1-r)}{n-r}\mathbb{E}[\theta|x_i, p] + \frac{r(n-1)}{n-r}\mathbb{E}[A_{n-i}((x_j)_{j\neq i}, p)],$$

confirming that the first-order condition can be rewritten as in (5). □

Proof of Proposition 2. We begin by defining the environment of the continuum-player model and characterizing the equilibrium forecast and the equilibrium level of forecast dispersion in such model. In an economy with a continuum of agents uniformly distributed over the unit interval $[0, 1]$, we retain the specifications of the model in Section 2 but with the average forecast counterpart as $A \equiv \int a d\Psi(a)$ where $\Psi(a)$ is the cumulative distribution function for individual forecasts across the population. In the continuum-player model, because the private noises ε_i are assumed to be i.i.d. across agents, in any symmetric equilibrium the aggregate variable A is a function of (θ, p) alone. Thus $A(\theta, p) = \int_x a(x, p)d\bar{\Psi}(x|\theta, p)$ for all (θ, p) where $\bar{\Psi}(x|\theta, p)$ denotes the conditional cumulative distribution function of x given (θ, p). Given x and p, the first-order condition for the optimization problem, $\max\ \mathbb{E}\left[-\frac{1}{2}(a - (1-r)\theta - rA)^2|x, p\right]$, gives the optimal forecast of

$$a(x, p) = (1-r)\mathbb{E}[\theta|x, p] + r\mathbb{E}[A(\theta, p)|x, p], \tag{13}$$

for all (x, p) where $A(\theta, p) = \mathbb{E}[a(x, p)|\theta, p]$. The proof is similar to the proof of Proposition 1. We guess that $a = \kappa_0^{'}x + \kappa_1^{'}p$, where $\kappa_0^{'}$ and $\kappa_1^{'}$ are constants. Then, $A(\theta, p) = \kappa_0^{'}\theta + \kappa_1^{'}p$, and (13) reduces to $a(x, p) = (1-r)\mathbb{E}[\theta|x, p] + r\kappa_0^{'}\mathbb{E}[\theta|x, p] + r\kappa_1^{'}p$. Substituting in $\mathbb{E}(\theta|x, p) = \delta x + (1-\delta)p$ where $\delta = \frac{\alpha_x}{\alpha_x+\alpha_p}$, we obtain $a(x, p) = (1-r+r\kappa_0^{'})\delta x + ((1-r)(1-\delta) + r\kappa_0^{'}(1-\delta) + r\kappa_1^{'})p$. It follows that $a(x, p) = \kappa_0^{'}x + \kappa_1^{'}p$ constitutes a linear equilibrium if and only if $\kappa_0^{'}$ and $\kappa_1^{'}$ solve $\kappa_0^{'} = (1-r)\delta + r\delta\kappa_0^{'}$ and $\kappa_1^{'} = (1-r)(1-\delta) + r\kappa_0^{'}(1-\delta) + r\kappa_1^{'}$. Equivalently, $\kappa_0^{'} = \frac{\alpha_x}{\alpha_x+\frac{1}{1-r}\alpha_p}$ and $\kappa_1^{'} = 1 - \kappa_0^{'}$. Therefore, it follows that, for any given value of (θ, p), the equilibrium forecast of agent i in the continuum-player model is given by

$$a(x, p) = \lambda x + (1-\lambda)p \tag{14}$$

where $\lambda \equiv \frac{\alpha_x}{\alpha_x+\frac{1}{1-r}\alpha_p}$; and the equilibrium average forecast is $A(\theta, p) = \lambda\theta + (1-\lambda)p$. This is clearly a unique symmetric linear equilibrium in the continuum-player forecasting game. As is evident from (14), idiosyncratic noise ε_i generates dispersion; and the equilibrium level of forecast dispersion for any given realizations of θ and p in the continuum-player model is given by $V(a|\theta, p) = (\lambda(\alpha_x)^{-1/2})^2$. Now the proof for convergence follows from noticing that $\lim_{n\to\infty}\lambda(\gamma) = \lim_{n\to\infty}\frac{\alpha_x}{\alpha_x+\frac{1}{1-\gamma}\alpha_p} = \frac{\alpha_x}{\alpha_x+\frac{1}{1-r}\alpha_p} = \lambda$ because $\gamma \equiv \frac{r(n-1)}{n-r} \to r$ as $n \to \infty$. Thus,

$$\lim_{n\to\infty} a_i(x_i, p) = \lim_{n\to\infty} \lambda(\gamma)x_i + (1-\lambda(\gamma))p = \lambda x + (1-\lambda)p = a(x, p).$$

The average forecast in finite-player equilibrium is given by

$$A_n = \lambda(\gamma)\theta + \lambda(\gamma)(\alpha_x)^{-1/2}\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i + (1-\lambda(\gamma))p,$$

whereas in continuum-player equilibrium it is given by $A = \lambda\theta + (1-\lambda)p$. Considering replica forecasting games as n goes to infinity, the average error in the private signals of the agents converges to zero by the law of large numbers. Hence, A_n tends to A as n goes to infinity. □

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