Econ 1335: Advanced Topics in Macroeconomics

Problem Set 1

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The solutions are due *Tuesday September 29* in class. Enjoy!

Exercise A: A Real Business Cycle Model with Various Shocks

Consider a social planner who faces the following problem.

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \psi B_t \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{1}$$

subject to

$$\begin{aligned} (1) \ C_t + I_t &= Z_t K_t^{1-\alpha} H_t^{\alpha} \\ (2) \ K_{t+1} &= Q_t I_t + (1-\delta) K_t \end{aligned}$$

where $\beta \in [0,1]$ is the discount factor, $\phi > 0$ is the inverse Frisch labor supply elasticity, $\psi > 0$ is the scaling parameter, and $\alpha \in (0,1)$ is the labor income share. There are three exogenous shocks; a preference shock, B_t, a neutral technology shock, Z_t, and an investment-specific technology shock, Q_t. Each shock X_t follows an AR (1) process:

$$\ln X_t = \rho_X \ln X_{t-1} + \varepsilon_t^X \tag{2}$$

where $\varepsilon_t^X \sim$ i.i.d. $\mathbb{N}(0, \sigma_X^2)$. Hence, we implicitly assume that the steady state value of each shock is one. i.e. $X = 1$ where X is the steady state value of the variable X_t.

(i) Derive the equilibrium conditions of this economy and provide the economic meaning of each equation.

(ii) Only for this question, assume $\delta = 1$ (full depreciation) and $B_t = Q_t = 1$ for all t. Further, assume that the disutility from working takes the form of $\psi B_t \ln(1 - H_t)$ instead of $\psi B_t \frac{H_t^{1+\phi}}{1+\phi}$. Now

we can find the closed-form solution of the problem. In particular, derive the close-form solution for C_t, H_t, and K_{t+1} as a function of parameters and state variable K_t.

(iii) Go back to the initial problem (ignore (ii)). Find the steady state of the economy.

(iv) Log-linearize the equilibrium conditions around the steady state.

(v) Assume $\beta = 0.99$, $\psi = 4$, $\phi = 1$, $\alpha = 2/3$, and $\rho_X = 0.99$. Suppose that the only shock in this economy is the neutral technology shock, Z_t, and $\sigma_Z = 0.01$ (other shocks are shut down). Using the dynare program, (1) report the impulse response functions of key macro variables (output, consumption, hours worked, investment, and capital) with respect to the shock and (2) report the second moment of this economy (in terms of relative to output volatility).

(vi) Do the same exercise in (v) with (1) investment-specific technology shock (Q_t) and (2) preference shock (B_t) only. What are the main differences?

(vii) Fixing other parameters, but vary the inverse Frisch elasticity (ϕ) from 1 to 1/10, 5, 10 and compare the impulse response of hours worked and labor productivity to the neutral technology shock (Z_t). What is the lesson from this exercise?

Exercise B: Government Spending Shock in the Neo-classical Business Cycle Models

Consider a following representative consumer's problem.

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{3}$$

subject to

$$C_t = W_t H_t + T_t + \Pi_t$$

where W_t is the hourly wage rate, Π_t is the profit from the firm's problem and T_t is the lump-sum tax. A representative firm facing production function $Y_t = H_t^{\alpha}$ maximizes its profit in the competitive labor market. Finally, the government provides a (useless) public good under the balanced budget rule ($T_t = G_t$). In particular, we set $G_t = \gamma_t Y_t$ where γ_t is an exogenous process

$$\gamma_t = \gamma^{1-\rho} \gamma_{t-1}^{\rho} \exp \varepsilon_t \tag{4}$$

where $\gamma = G/Y < 1$ is the steady state value and ε_t is an i.i.d. shock with zero mean.
Finally, the resource constraint is $C_t + G_t = Y_t$.

(i) Define the competitive equilibrium of this economy and list the equilibrium conditions that fully characterize the equilibrium.

(ii) Solve for C_t, H_t, and Y_t as functions of parameters and state variable γ_t. Suppose that there is a positive government spending shock so that γ_t increases. What happens to consumption, hours worked, and output of this economy? In the data, consumption and output comove. Can the government spending shock produce the comovement observed in the data?

(iii) Suppose that the utility function takes the alternative form; $\ln(C_t + \psi G_t) - \frac{H_t^{1+\phi}}{1+\phi}$ where $\psi \in (0, 1)$. i.e. the public good also provides a positive utility to the consumer. One can think of food stamp as an example. Is this economy able to reproduce the comovement between consumption and output in the data?

(iv) Now we consider a modified economy where the production function is given by $Y_t = \gamma_t^{1-\alpha} H_t^{\alpha}$ while G_t cannot affect utility ($\psi = 0$ from (iii)). i.e. the public good now improves the productivity of the economy (eg. construction of high way or railroad). What is the relationship between consumption and government spending shock?

(v) Read "Government Spending and Private Activity" by Valerie Ramey (2012) and briefly discuss your results through (ii) to (iv).[1]

[1]Recall that the Old style Keynesian model argues that government spending can stimulate both consumption and output at least in the short-run.