Frequency-Specific Effects of Macroprudential Policies*

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Abstract

Are macroprudential policies effective tools for stabilization and for welfare improvement? The answer to this question crucially depends on the frequency and the sector that we consider. Using a financial-sector augmented New Keynesian model, we find that a set of macroprudential policies that are commonly used both in theory and in practice have different frequency-specific effects on the economy: loan volatility is reduced across all frequencies while inflation rate volatility is amplified across all frequencies, when a countercyclical capital requirement policy is implemented. In contrast, when the Taylor rule is extended to respond to loan growth, loan is stabilized only at the relatively high frequencies while output volatility increases across all frequencies. Lastly, the loan-to-value(LTV) ratio regulation does not have much impact on our model economy. These problems cannot be solved by implementing policies more aggressively. Hence, our findings unveil the design limits of policies of our interest; they can be effective at the targeted sectors and frequencies while having significant perverse effects on other sectors and frequencies. We further analyze the spectral welfare gains and show the welfare analysis without considering frequency-specific effects can be misleading.

JEL classification: E32, E44, E58

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1 Introduction

The worldwide recession from the Global Financial Crisis was different from other recessions in the history since the Great Depression, because it was initiated from the collapse of financial market. A number of researches have investigated the cause of the financial disruption. Among them, Borio (2012) and Drehmann, Borio, and Tsatsaronis (2012) argue that exist financial cycles driving the movements of financial markets, independent of the traditional business cycle. They define financial cycles as "the self-reinforcing interactions between perceptions of value and risk, risk-taking and financing constraints which translate into financial booms and busts". Not only financial cycles are shown to have much lower frequencies (8 to 32 years per cycle) than usual business cycles have (1.5 years to 8 years per cycle), the infrequent downturns of financial cycles, compared to business cycles, can initiate financial crises. They warn the possible divergence between business and financial cycles and emphasize the role of the monetary policy to take a balanced approach when stabilizing both cycles with one policy tool. For example, they suggest the monetary authority should be ready to tighten whenever financial imbalances show signs of building up, even if inflation appears to be under control in the near term.[1]

Meanwhile, other researchers (Bernanke (2013), Yellen (2014)) point out the limitation of the monetary policy tool when targeting multiple policy objectives. They rather propose additional policy tools, so called macroprudential policies, designed to strengthen the financial system's resilience to economic downturns and other adverse aggregate shocks, and actively limit the build-up of financial risks (BIS (2010)) at the financial cycle frequency. Hence, the goal of macroprudential policies is different from that of the traditional monetary policy, which is designed to stabilize output and inflation at the business cycle frequency.

This debate brings our attention to the method for policy evaluation which enables us to compare the policy effects across various frequencies. Since the financial shocks (or cycles) might affect the macroeconomy more at the lower frequency, the traditional approach for evaluating policy effectiveness, which limits its focus to the effect of policies at the business cycle frequency, does not work. In order to overcome such a problem, this paper introduces an alternative approach; we analyze "frequency-specific" effects of macroprudential policies, similar to Brock, Durlauf, and Rondina (2008). We compute the variance of key macro variables at different frequencies so that frequency-specific effects of macroprudential polices are detected. Our analysis is not restricted to the analysis of the variance at different frequencies; we also conduct a "spectral welfare analysis", following Otrok (2001), by comparing the frequency-specific

[1] BIS (2014).

life-time welfare under different policies.

In doing so, we consider a version of New Keynesian model with a financial sector, which extends the business cycle model introduced in Iacoviello (2014); the financial sector consists of retail banks (receive deposits from patient households and lend to impatient households and investment banks) and investment banks (obtain funds from retail banks and lend to entrepreneurs), which is similarly designed to that of Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Hence, financial frictions are incorporated in the model as the borrowing (or capital adequacy regulation) constraints of different agents, which is the usual balance sheet channel that amplifies the propagation of shocks. Following Iacoviello (2014), we have the two classes of shocks; non-financial shocks (aggregate TFP shock, investment-specific technology shock, aggregate demand shock, and monetary policy shock) which are common shocks assumed in the business cycle literature and financial shocks including default shocks (transfers of wealth from savers to borrowers when the borrowers are default), loan-to-value shocks (changes in maximum loan-to-value ratios), and housing demand shocks (changes in the price of housing). The financial shocks are important since they approximately accounted for two-thirds of the output drops during the Great Recession, in which the financial sector played an important role for the propagation of the negative shock. Different from Iacoviello (2014), price stickiness in the goods market is further introduced so that the monetary policy has a role in our model economy.

Three policy tools, that are widely used both in the previous literature and in practice, are introduced for evaluation. The first policy is an extended Taylor-rule monetary policy[2] that aims to stabilize not only inflation and GDP gap, but also loan growth. Second policy is the counter-cyclical capital requirement regulation, which is the core feature of BASEL III, so that the banks should accumulate more buffers in good times for the possible losses in bad times. The third policy is to tighten loan-to-value (henceforth LTV) ratio on impatient households and entrepreneurs so that they cannot borrow as they want for the given level of collateral value.

The steps for evaluating the performances of different policies are as follows. With the model introduced in Section 3, we first simulate the economies under different policy regimes for several times to obtain time series of key macroeconomic variables. We then analyze in Section 5 the extent to which each policy is effective in lowering frequency-specific variances, which enables us to compare the frequency-specific effects of policies. In doing so, we compute the spectral density of each simulated series, following Otrok (2001), since the spectral density provides us the variance of the series at each frequency so that

[2]In this paper, we classify the extended monetary policy as a macroprudential policy for the purpose of distinguishing from typical monetary policy, although it is rather considered as another version of monetary policy in practice.

we can examine whether the policy effectiveness is heterogeneous across different frequencies. After taking the average of the spectral densities at each frequency over the simulated series, we can finally obtain the expected frequency-specific variances under different policy regimes. Spectral welfare analysis is then conducted in Section 6; we first apply the band pass filter (Baxter and King (1999)) to the simulated time series. Then we obtain filtered time series at each frequency. Spectral utility of the average (representative) consumer is then computed from the filtered time series for each frequency band.

Our quantitative experiments provide several notable findings. First, tightening LTV ratio is nearly ineffective both in stabilizing key macro variables and in raising welfare of the average consumer almost at every frequency. Second, the extended Taylor rule has several perverse effects on the volatility of the key aggregate variables mostly at lower frequencies; output fluctuations are amplified at almost every frequency while the negative effect is the greatest at the lower frequencies; it amplifies loan fluctuations at relatively lower frequency so that it achieves the original goal of macroprudential policies only at higher frequency (higher than 8 years/cycle) and amplifies the volatility of inflation rate almost across all frequencies, especially at frequencies lower than 8 years/cycle. In this sense, the positive effects of extended Taylor rule is limited. Even worse, welfare deteriorates at almost all frequencies. Third, the countercyclical capital requirement policy is very successful in stabilizing loan fluctuations across all frequencies so that it satisfies the original goal described in BIS (2010), unlike the two policies above. However, it also incurs sizable costs; it amplifies both output and inflation fluctuations at every frequency. The adverse effects on these variables are more severe at higher frequencies, which is in contrast to the extended Taylor rule. In addition, the spectral welfare analysis reveals that the policy lowers welfare at many frequencies; interestingly, this is in contrast to the average welfare gain estimated without considering the frequency-specific effects; the countercyclical capital requirement policy is welfare-improving on average. This further highlights the importance of frequency-specific analysis when evaluating macroprudential policies.

Our findings have several important policy implications. First, even when the loan market is stabilized thanks to the macroprudential policies, the volatility of other key macro variables that are especially important for the real sector increases. In other words, if the output stabilization is the implicit dual-objective of macroprudential policies, current well-known policies are not successful. More importantly, these policies amplify the volatility of inflation at all frequencies, which is exactly contrary to the main mandate of central banks. Second, more aggressive macroprudential policies amplify negative effects on output stabilization while their positive effects on loan stabilization become greater. These implications are preserved if we take an spectral welfare approach to evaluate macroprudential policies. In other words, our findings raise the importance of carefully designed macroprudential policies in order to minimize such adverse effects. Lastly, various approaches to evaluate effectiveness of macroprudential policies robustly recommend the central banks not to respond directly to financial market fluctuations (extended Taylor rule in our framework).

Related Literature. Our paper is related to the literature that analyzes the role and the effectiveness of macroprudential policies. The conventional method to evaluate the performances of policies adopted by the previous literature is the welfare-cost approach; they compute the value of lifetime utility under different policy regimes and compare them using the compensational variation in terms of consumption.[3] One stream of literature focuses on measuring the welfare cost of policies. Van Den Heuvel (2008) measures the welfare cost of bank capital requirements and shows that the regulations produce 0.1% to 1% loss in consumption in the U.S economy. Nguyen (2014) applies a general equilibrium model to the dynamic banking sector to show that the increase in bank capital requirements to the optimal level can produce welfare gains greater than 1% of lifetime consumption. Another stream of the literature tries to answer in which situations macroprudential policies are effective. Benes and Kumhof (2015) shows countercyclical bank capital requirements can create a precautionary motive to banks when the creditworthiness (or riskiness) of borrowers depreciates. Bailliu, Meh, and Zhang (2015) compare different sets of macroprudential regimes and find that welfare gains are largest when macroprudential policies react to financial shocks rather than productivity shocks. Lastly, a group of literature search for the optimal coordination between monetary and macroprudential policies. Quint and Rabanal (2014), and Suh (2012) find the optimal simple rule for monetary and macroprudential policies in the Euro Area and the U.S, respectively. Collard, Dellas, Diba, and Loisel (2014), and Angeloni and Faia (2013) support the view that implementing macroprudential policies along with monetary policies is important due to risk-taking behaviors by banks. On the other hand, Kiley and Sim (2014) find that an optimal monetary policy without macroprudential policies is sufficient to ensure efficiency even under the financial shock. Woodford (2012) suggests a modified inflation targeting framework to take account of financial stability concerns alongside traditional stabilization objectives. Our approach is unique since we focus on the frequency-specific effects of implementing macroprudential policies while the studies mentioned above die not consider the possible frequency-specific effects.

This paper is also related to the literature that applies the notion of design limit approach to macroeconomics. Brock, Durlauf, and Rondina (2008) and Brock, Durlauf, and Rondina (2013) show that unless the central bank implements a policy that is optimally designed to stabilize the economy at every

[3] See Schmitt-Grohé and Uribe (2006) for details of the approach.

frequency, the monetary policy can have unexpected negative effects at certain frequencies. The main difference between our paper and their series of papers is that we study the frequency-specific effects of macroprudential policies using a medium-scale New Keynesian model with financial frictions while they consider the small-scale New Keynesian model to derive the optimal monetary policy; to our best knowledge, our paper is the first to consider the possible design limit of macroprudential policies.

Our analysis is further related to the literature on spectral welfare analysis. To our best knowledge, Otrok (2001) is the only paper in this regard. While the methodology is identical, our paper is distinctive from his paper as he considered a simple partial equilibrium model of consumers while we consider a full-blown general equilibrium model.

The remainder of this paper is organized as follows. We first introduce the notion of frequency-specific effects of policy in Section 2. Main model is then introduced in Section 3 with parameterization and preliminary analysis in Section 4. Key findings from our model are presented in Section 5 and Section 6. In Section 7, we conclude the paper.

2 Frequency-Specific Effects: a Brief Description

In this section, we introduce the primary concepts and steps taken in our main quantitative exercises.[4] Suppose that we have a covariance-stationary macro variable $\{Y_t\}_{t=-\infty}^{\infty}$, which is defined in the *time domain*. This variable oscillates over time so that it can be described as the weighted sum of periodic functions of the form cosine and sine functions. Then the spectral density function of the time series Y_t, $s_Y(\omega)$, can be described as follows.

$$s_Y(\omega) = \frac{1}{2\pi}\left[\sum_{k=-\infty}^{\infty} \lambda_k \exp(-i\omega k)\right] \tag{2.1}$$

where $\omega \in [0, \pi]$ is the frequency, λ_k is the k-th autocovariance of Y_t, and $i = \sqrt{-1}$. Then using De Moivere's theorem, symmetry of autocovariance, and properties of cosine and sine functions, we can obtain the spectral density of the following form:

$$s_Y(\omega) = \frac{1}{2\pi}\left[\lambda_0 + 2\sum_{k=1}^{\infty} \lambda_k \cos(\omega k)\right] \tag{2.2}$$

The spectral density function provides the information on the extent to which a specific frequency contributes to the variance of the series. To see this, we plot the spectral density of (1) a white noise

[4]Some parts of this section are from Hamilton (1994).

process (Figure 2.1a) and (2) an AR (1) process (Figure 2.1b). In particular, we set the standard deviation of the innovation terms in each series as 0.038 and the persistence term ρ as 0.95 for the AR (1) process.[5] The horizontal axis denotes the frequency from low (0.13) to high (3.14) and the vertical axis denotes the spectral density corresponding to each frequency. Since the white noise process is i.i.d. across time, the contributions of variances at each frequency are equivalent in Figure 2.1a. However, for AR (1) process, the long-run frequency contributes more to the dynamics of the simulated series since it is generated to be very persistent over time.

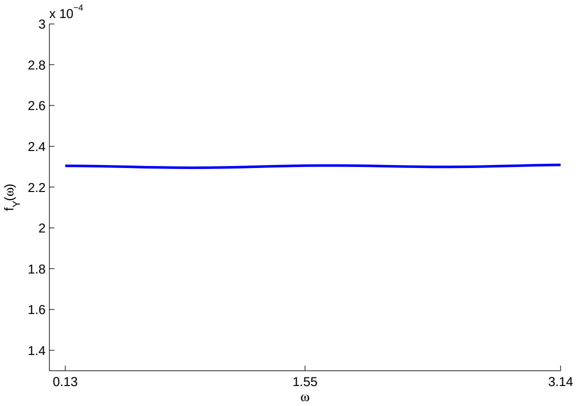

Figure 2.1a: White Noise

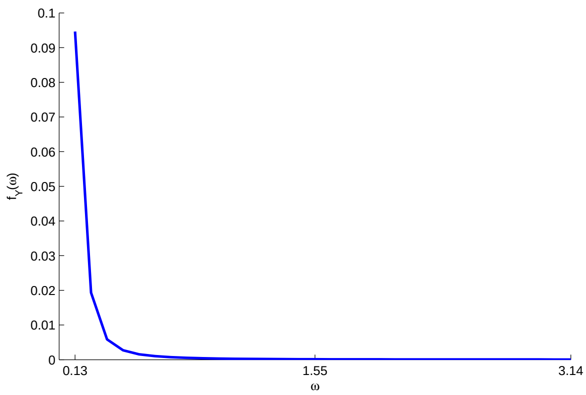

Figure 2.1b: AR(1) Process

Figure 2.1: Spectral Density Functions: Examples

One good property of the spectral density is that the sum of all spectral density is equal to the variance of the variable. Formally,

$$\mathbb{V}(Y_t|R_i) = \int_{-\pi}^{\pi} S_{Y_t|R_i}(\omega)d\omega \tag{2.3}$$

where R_i is the policy regime i under which the time series Y_t is simulated and $\mathbb{V}$ denotes variance. Hence, we can interpret spectral density at each frequency as the variance at each frequency.

We note that main objective of the (macroprudential) policies is to stabilize the economy. Put it differently, the spectral density can be used to evaluate the effectiveness of the policies, particularly on the heterogeneous effects on the variance at different frequency. This is important since a policy that is intended to stabilize the economy at some specific frequencies - a macroprudential policy is designed to stabilize the economy at the relative low frequency - may have an adverse effect at the different frequency. This property is known as the "design limit" of policy. For instance, Brock, Durlauf, and Rondina (2008) considers an example that shows a policy that is supposed to minimize the overall variance of variables

[5] Simulated series are with $T = 1,000$ (total period) and $N = 1,000$ (numbers of simulation).

can increase the variance of the series at the relatively short frequency. The finding that the welfare cost from different policies can vary across frequencies as the utility function is not time-separable (Otrok (2001)) further raises the importance of our approach when it comes to studying the frequency-specific effects of macroprudential policies.

In this paper, we compute two frequency-specific effects of macroprudential policies; (1) frequency-specific variance and (2) frequency-specific welfare. The steps to evaluate the effectiveness of different macroprudential policies on variance can be described as follows.

1. Simulate the model economy exposed to all exogenous shocks in different policy regimes.
2. Compute the spectral density of the simulated series, take the average across simulations, and compare the density functions obtained from different policy regimes.

We first point that all exogenous shocks are included in our quantitative exercise; one might argue that only one exogenous shock should be introduced when simulating the model economy, productivity shock for instance, so that we can examine the effectiveness of a policy when the specific shock is particularly considered. We decide to introduce all exogenous shocks in the simulation by the following reasons. First, as is well-known and will be shown later, macroprudential policies do not work in the economy in which financial shocks are excluded. Hence, analysis with non-financial shocks only will be not useful for our purpose. Second, it is not clear which financial shock to be included in the simulation since contribution of financial shocks to the fluctuations of the aggregate economy is not clearly studied. For instance, historical variance decomposition of the model in Iacoviello (2014) (Figure 3) shows that importance of a particular shock is different across variables. In addition, the macroprudential policy in the real world is not set to respond to a single shock; the policy works whenever the financial market fluctuates. Hence, it seems to be more natural to consider all exogenous shocks in the simulation.

In particular, we will consider the effectiveness of macroprudential policies in comparison with the benchmark economy without any such policies. If the policy is effective at some frequency, the spectral density will become lower than that from the benchmark economy. If it has an adverse effect, the spectral density will be above that obtained from the benchmark economy. Therefore, if a policy is effective in stabilizing the economy as a whole, the spectral density of key macro variables, such as output, loan, consumption, and inflation rate, will be below the corresponding spectral density from the benchmark economy, which will be studied in details in Section 5.2.

The steps we take to conduct spectral welfare analysis is described as follows.

1. Simulate the model economy exposed to all exogenous shocks in different policy regimes.

2. Apply band pass filter (Baxter and King (1999)) to the series obtained in step 1 and obtain filtered series.

3. Compute the spectral utility for each frequency band i and then compute the average life-time values under different policy regimes.

Again, we compare the life-time value associated with a macroprudential policy and compare the welfare gain (loss) with the benchmark economy without such a policy. Details of the analysis will be discussed in Section 6.

3 The Model

The model introduced in this section is not particularly new by itself, but includes the key features of models with financial frictions. In particular, our model builds upon the model developed by Iacoviello (2014)[6]; we incorporate New-Keynesian features to the original setup by Iacoviello (2014), similar to Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Our strategy to keep the model consistent with the previous literature is in order to minimize the model-specific factors that can possibly affects equilibrium behaviors.

The economy consists of patient households, impatient households, entrepreneurs, retail banks, investment banks, retailers, and monetary authorities. Two financial intermediaries have different roles in the economy; retail banks lend funds to both impatient households and investment banks where they use deposits from patient households. Investment banks, however, obtain fund only from retail banks and lend to entrepreneurs. In order to obtain the hump-shape behavior of macro variables, habit formation and various adjustment costs are introduced.

3.1 Households There is the measure of one patient household and another measure of one impatient household. As usually assumed in the literature, patient households have a higher discount factor than impatient households, namely $\beta_s > \beta_b > 0$. Hence, in equilibrium only patient households save while impatient households borrow.

3.1.1 Patient Households The representative patient households (saver), denoted as s, solve the following expected lifetime utility maximization problem by choosing optimal consumption C_t^s, hours

[6] A model in Iacoviello (2005) also shares similar features.

worked N_t^s, housing H_t^s, capital holding K_t^s and saving in the bank d_t^s, taking prices as given:

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \left[\varepsilon_t^c \ln \left(C_t^s - hC_{t-1}^s \right) + \varepsilon_t^c \varepsilon_t^h \nu_h^s \ln H_t^s - \nu_n^s \frac{(N_t^s)^{1+\phi}}{1+\phi} \right] \tag{3.1}$$

where $\beta_s \in (0,1)$ is a discount factor of patient households, $h \in [0,1]$ is a parameter that governs habit formation, $\phi > 0$ is the inverse Frisch elasticity, and $\nu_h^s > 0$ (resp. $\nu_n^s > 0$) is a relative utility parameter from housing (resp. working). ε_t^c is an exogenous shock to preference for consumption and housing jointly,[7] and ε_t^k is an investment-specific technology shock. ε_t^h is an exogenous shock to housing preference, one of the financial shocks in our model economy.

Budget constraints for the patient households are as follows.

$$C_t^s + \frac{K_t^s}{\varepsilon_t^k} + p_t^H \left(H_t^s - H_{t-1}^s \right) + d_t + AC_{d^s,t} + AC_{K^s,t} = w_t^s N_t^s + r_t^d d_{t-1} + \left(r_t^K + \frac{1-\delta}{\varepsilon_t^k} \right) K_{t-1}^s \tag{3.2}$$

where the price of consumption goods is normalized to 1 ($P_t \equiv 1$), p_t^H is the real price of housing, r_t^d is a gross real interest rate from the deposit. Households rent capital to entrepreneurs at the rental rate r_t^K, and receive the real wage w_t^s for labor supply. We define $AC_{x,t}$, a convex real external adjustment cost for any variable x_t, as follows:

$$AC_{x,t} = \frac{\iota_x}{2} \frac{\left(x_t - x_{t-1} \right)^2}{x} \tag{3.3}$$

where $\iota_x \geq 0$ is an adjustment cost parameter and x is the steady state level for x_t.

3.1.2 Impatient Households

Similar to patient households, the representative impatient households (borrowers), denoted as b, also choose the optimal level of consumption, C_t^b, hours worked N_t^s, and housing stock H_t^b. As the discount factor of impatient households β_b is smaller than that of patient households, they prefer spending to saving and borrow from the banking sector to fund their spending. However, due to the financial friction, they cannot borrow as much as they want and lenders (retail banks) ask for collateral to secure loans. Since the only asset of impatient households is housing stock, the level of new bank loans depends on the discounted value of the house they own.

[7] Iacoviello (2014) interprets it as an aggregate spending shock.

The problem of impatient households can be written as follows:

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t \left[\varepsilon_t^c \ln \left(C_t^b - h C_{t-1}^b \right) + \varepsilon_t^c \varepsilon_t^h \nu_h^b \ln H_t^b - \nu_n^b \frac{\left(N_t^b\right)^{1+\phi}}{1+\phi} \right] \tag{3.4}$$

subject to

$$C_t^b + p_t^H \left[H_t^b - H_{t-1}^b \right] + r_t^b l_{t-1}^b + AC_{l^b,t} = w_t^b N_t^b + l_t^b + \varepsilon_t^b \tag{3.5}$$

$$l_t^b \leq \rho_b l_{t-1}^b + (1-\rho_b) \left[\gamma_t^{Hb} \mathbb{E}_t \frac{p_{t+1}^H H_t^b}{r_{t+1}^b} - \varepsilon_t^b \right] \tag{3.6}$$

where l_t^b denotes bank loans, paying a gross interest rate r_t^b, and w_t^b is the real wage rate. $\varepsilon_t^b \geq 0$ is a default shock for impatient households, which is another financial shock in our model; this can be interpreted as a wealth redistribution shock between borrowers and lenders since this shock increases the net wealth of impatient household (borrower) while it lowers the net wealth of retail banks (lenders).[8] Contrary to Iacoviello (2014), we assume that the default shock also negatively affects the borrowing constraint of the impatient households in order to capture the idea that the default on existing loans can limit the level of new loans.

Equation (3.6) is the borrowing constraint of impatient households, where $\rho_b \in [0,1]$ allows for the slow adjustment of bank loans over time.[9] Borrowers cannot borrow more than the fraction of γ_t^{Hb} of the expected value of their housing stock. Here we assume that this constraint is imposed by government policies, so called LTV ratio regulation.

The LTV ratio regulation γ_t^{Hb} is composed of two parts as follows:

$$\gamma_t^{Hb} = \gamma_0^{Hb} \varepsilon_t^{lb} - \gamma_1^{Hb} \left(\frac{p_t^H}{p^H} - 1 \right) \tag{3.7}$$

where the first term is a constant LTV ratio regulation and the other term is a time-varying regulation. γ_0^{Hb} in the first term is the constant maximum LTV ratio cap, imposed by the policy, while ε_t^{lb} captures lenders' subjective perceptions of the riskiness of the housing stock. We call this shock a risk perception shock (or LTV shock). The time-varying LTV ratio regulation is one of the popular macroprudential tools to stabilize housing prices. If $\gamma_1^{Hb} > 0$, the LTV cap becomes tighter (lower) as housing prices increase. That is, it becomes more difficult for impatient households to borrow from banks with the

[8] See Iacoviello (2014) for more discussions.

[9] For the parameters to govern slow adjustment of loans, see Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015).

collateral (housing) she/he holds.

3.2 Entrepreneurs A continuum of entrepreneurs, denoted as e, produces intermediate goods X_t^e and sell at a price of p_t^X in a competitive market. They hire workers and combine them with housing stock H_{t-1}^e and capital (both produced by themselves, K_{t-1}^e, and rent from patient households, K_{t-1}^s). The Cobb-Douglass production technology can be written as:

$$X_t = \varepsilon_t^z \left(\left(K_{t-1}^e\right)^{\omega^k} \left(K_{t-1}^s\right)^{1-\omega^k} \right)^{\alpha} \left(H_{t-1}^e\right)^{\nu} \left(\left(N_t^s\right)^{\omega^n} \left(N_t^b\right)^{1-\omega^n} \right)^{(1-\alpha-\nu)} \tag{3.8}$$

where ε_t^z is a neutral productivity shock and $(1-\omega^k)$ and ω^n are shares of patient households' capital and labor, respectively.

Similarly to impatient households, entrepreneurs also face a borrowing constraint when making financing decisions. Their utility function is:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t \log \left(C_t^e - hC_{t-1}^e\right) \tag{3.9}$$

where $\beta_e < \beta_s$ is assumed and C_t^e is the consumption of the entrepreneur. They are subject to the following constraints:

$$\begin{aligned} C_t^e + \frac{K_t^e}{\varepsilon_t^k} + p_t^H[H_t^e - H_{t-1}^e] + w_t^s N_t^s + w_t^b N_t^b + r_t^K K_{t-1}^s + r_t^e l_{t-1}^e + AC_{K^e,t} + AC_{l^e,t} \\ = p_t^X X_t + \frac{1-\delta}{\varepsilon_t^k} K_{t-1}^e + l_t^e + \varepsilon_t^e \end{aligned} \tag{3.10}$$

$$l_t^e \leq \rho_e l_{t-1}^e + (1-\rho_e) \left(\gamma_t^{He} \mathbb{E}_t \frac{p_{t+1}^H H_t^e}{r_{t+1}^e} + \gamma_t^{Ke} K_t^e - \gamma_t^{Ne}(w_t^s N_t^s + w_t^b N_t^b) - \varepsilon_t^e \right) \tag{3.11}$$

Equation (3.10) is the budget constraint of the representative entrepreneur where r_t^e is a gross real interest rate on entrepreneur loans l_t^e. Similarly to equation (3.6), ε_t^e is a default shock to entrepreneurs, which captures losses on banks and gains from entrepreneurs. Equation (3.11) is the borrowing constraint for entrepreneurs. Contrary to impatient households, entrepreneurs can use both housing and capital stocks as collateral when borrowing from banks. γ_t^{He} and γ_t^{Ke} are the ratio of housing and capital they can pledge, respectively. γ_t^{He} shares the same implication with the LTV ratio regulation on impatient households' housing stock.

$$\gamma_t^{He} = \gamma_0^{He}\varepsilon_t^{le} - \gamma_1^{He}\left(\frac{p_t^H}{p^H} - 1\right) \tag{3.12}$$

However, the amount of loan capacity decreases due to the working capital assumption. Similarly to Iacoviello (2014), Aoki, Benigno, and Kiyotaki (2009), and Neumeyer and Perri (2005), entrepreneurs are assumed to pay for some portion of wage bills in advance, *i.e* $\gamma_t^{Ne} \in (0,1]$. We assume $\gamma_t^{Ke} = \gamma_0^{Ke}\varepsilon_t^{le}$ and $\gamma_t^{Ne} = \gamma_0^{Ne}\varepsilon_t^{le}$, where ε_t^{le} is a risk perception shock which is applied to housing stock, capital and wage at the same time.

3.3 Retail Banks

Retail banks, denoted as r, collect deposits from patient households and lend to impatient households l_t^b and investment banks l_t^i. As we assume $\beta_r < \beta_s$, retail banks prefer debt to equity.[10] To prevent banks from high leverage, regulators impose a cap on banks' capital ratio relative to the total asset. It is called as the minimum capital requirement.

The utility maximizing problem of retail banks is:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_r^t \log\left(C_t^r - hC_{t-1}^r\right) \tag{3.13}$$

subject to the following constraints:

$$C_t^r + l_t^b + l_t^i + r_t^d d_{t-1} + AC_{d^r,t} + AC_{l^{ir},t} + AC_{l^{br},t} = d_t + r_t^i l_{t-1}^i + r_t^b l_{t-1}^b - \varepsilon_t^b - \varepsilon_t^i \tag{3.14}$$

$$l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i \geq \rho_r(l_{t-1}^b + l_{t-1}^i - d_{t-1} - \varepsilon_{t-1}^b - \varepsilon_{t-1}^i) + (1-\rho_r)[\eta_t^b l_t^b + \eta_t^i l_t^i - \varepsilon_t^b - \varepsilon_t^i] \tag{3.15}$$

where r_t^i is a gross real interest rate on loans to investment banks. ε_t^b and ε_t^i in the budget constraint (3.14) are defaults shocks on loans to households and investment banks, which lower the level of bank equity. Equation (3.15) is the bank capital requirement regulation constraint. If we assume $\rho_r = 0$ and $\eta_t^b = \eta_t^i$ for simplicity, it can be rewritten as

$$\frac{(equity)}{(total\ assets)} = \frac{l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i}{l_t^b + l_t^i} \geq \eta_t^b$$

which means retail banks should retain a certain level of equity, proportional to assets.

[10] The preference of debt over equity can also be introduced by tax treatment on debt, equity dilution cost, or liquidity premium on deposits.

Similarly to the LTV ratio regulation, the capital requirement regulation also consists of two parts as follows:

$$\eta_t^j = \eta_0^j + \eta_1^j \left(\frac{l_t/Y_t}{l/Y} - 1 \right) \quad \text{where } \ j = \{b, i\} \tag{3.16}$$

where the first term is a constant capital requirement regulation and the next term is a time-varying regulation. η_0^j in the first term is the constant minimum capital requirement. The time-varying capital requirement regulation is called a counter-cyclical capital requirement regulation. It requires banks to hold more equity when loans expand much faster than output. That is, the policy is counter-cyclically tightened when the credit expands. Assuming $\eta_1^j > 0$, η_t^j is positively related to the deviation of loan to GDP ratio from the steady state value. In the extreme case when $\eta_t^j = 0$, the particular asset j is considered to be riskless.

3.4 Investment Banks Investment banks, denoted as i, obtain funds from the retail banks and lend to entrepreneurs. Investment banks are also subject to capital requirement regulation. The utility maximization problem of the representative investment bank is given by

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \log \left(C_t^i - hC_{t-1}^i \right) \tag{3.17}$$

subject to the following constraints:

$$C_t^i + l_t^e + r_t^i l_{t-1}^i + AC_{l^e,t} + AC_{l^i,t} = l_t^i + r_t^e l_{t-1}^e + \varepsilon_t^i - \varepsilon_t^e \tag{3.18}$$

$$l_t^i \leq \rho_i l_{t-1}^i + (1 - \rho_i)[(1 - \eta_t^e) l_t^e + \varepsilon_t^i - \varepsilon_t^e] \tag{3.19}$$

Budget constraint (3.18) and capital requirement constraint (3.19) (written in a form of borrowing constraint[11] are similar to other agents' constraints. We will skip the definition of η_t^e, which is exactly same with η_t^i and η_t^b.

3.5 Retailers Monopolistic competitive retailers purchase goods from the entrepreneurs in a competitive market and differentiate them into intermediate goods, as in the typical New Keynesian literature. The technology is linear: $Y_t(z) = X_t^e - F(z)$ where $F(z)$ are fixed costs to make the steady-state profit

[11] It is equivalent to $\frac{(equity)}{(total\ assets)} = \frac{l_t^e - l^i + \varepsilon_t^i - \varepsilon_t^e}{l_t^e} \geq l_t^e$, if $\rho^i = 0$

of the retailer zero. Then retailers sell intermediate goods, $Y_t(z)$, to the final goods-producing firm at a price of $P_t(z)$. Final output Y_t is given by

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.20}$$

where $\varepsilon > 1$.

The cost minimization problem of the final goods-producing firm yields the inverse demand function

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t \tag{3.21}$$

and the aggregate price index

$$P_t = \left(\int_0^1 P_t(z)^{1-\varepsilon} dz\right)^{\frac{1}{1-\varepsilon}} \tag{3.22}$$

Each retailer chooses optimal price $P_t(z)$; following Calvo (1983), the retailer can adjust the price with probability $1-\theta$. If the retailer is not able to adjust its price, $P_t(z) = P_{t-1}(z)$. Each retailer maximizes its market value:

$$\max_{P_t(z)} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_t^s \left[P_t(z) Y_t(z) - P_t^X X_t^e\right] \tag{3.23}$$

subject to the equation (3.21). The optimal price level for firm z in period t is:

$$P_t^*(z) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X}{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j}} \tag{3.24}$$

We assume a symmetric equilibrium case where $P_t^* = P_t^*(z), \forall z$, thus the aggregate price level evolves according to

$$P_t = \left[(1-\theta)(P_t^*)^{1-\varepsilon} + \theta\left(P_{t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.25}$$

3.6 Monetary Authority

We assume that the monetary authority conducts a monetary policy following the extended Taylor rule, which incorporates the loan (or credit) as an additional determinant of the policy rate $R^i_{t+1} = r^i_t \mathbb{E}_t \pi_{t+1}$, the nominal inter-bank interest rate:

$$\frac{R^i_t}{R^i} = \left[\left(\frac{R^i_{t-1}}{R^i} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \left(\frac{l_t}{l} \right)^{\gamma_L} \right]^{1-\rho_R} \right] \varepsilon^R_t \tag{3.26}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is a gross inflation rate, $l_t = l^b_t + l^e_t + l^i_t$ is a total loan in the economy. Variables without time subscript indicates steady-state levels. ρ_R is a smoothing parameter of policy rate and γ_π, γ_Y, and γ_L are feed-back parameters of corresponding variables. It becomes a standard Taylor rule if we set $\gamma_L = 0$.

3.7 Housing Market

We assume that housing supply is exogenously given as $\bar{H}$. Then the housing market clearing condition is given by

$$\bar{H} = H^s_t + H^b_t + H^e_t \tag{3.27}$$

In what follows, we normalize $\bar{H}$ as one, without loss of generality.

3.8 Exogenous Shocks

We have four non-financial shocks (Spending Shock (ε^c_t), Investment-specific technology shock(ε^k_t), TFP shock (ε^z_t) and monetary policy shock (ε^R_t)) and six financial shocks (housing demanding shock (ε^h_t), three default shocks (ε^b_t, ε^e_t, and ε^i_t), two risk perception shocks (ε^{lb}_t and ε^{le}_t)) hence 10 exogenous shocks as total. For $x \in \{c, k, z, R, h, lb, le\}$, the exogenous shock process ε^x_t is assumed to follow an AR (1) process:

$$\log \varepsilon^x_t = \rho^x \log \varepsilon^x_{t-1} + u^x_t \tag{3.28}$$

where u^x_t is the i.i.d. shock that is normally distributed with mean 0 and variance σ_x. Default shocks $x \in \{b, e, i\}$, are defined as level instead of log level.

4 Calibration and Preliminary Analysis

4.1 Parameterization

In calibrating parameters, we use the estimated values from Iacoviello (2014) as the parameters which are common between our and his model. If the parameters were not present in his paper, we might use parameter values that are generally used in the literature. For instance, we set the patient households discount factor at 0.9925 to target 3% annual risk-free interest rate. As in Iacoviello (2014), our value for capital depreciation is higher than the typical number in the literature, 0.025, because housing is the additional factor of production which does not depreciate. Following the standard NK-DSGE literature, the elasticity of substitution for intermediate varieties, ε, is calibrated as 11 to target the steady state mark-up at 10%. The coefficients in the Taylor rule are also usual numbers to ensure the unique equilibrium of the model. Parameters related to macroprudential policies will be described later. Table 4.1 shows our benchmark calibration for the parameters and Table 4.2 represents the parameterization for exogenous shocks used in our model.

4.2 Basic Results: Impulse Response Functions Throughout our analysis, we will compare four model economies which only differ in the macroprudential policies implemented. The benchmark economy (Model 1) is set to have 70% LTV ratio ($\gamma^{Hb} = 0.7$), 8% constant minimum capital requirement, and the monetary policy neutral to loan changes. Other economies have different policy measures as follows:

- Model 1: No macroprudential policy (benchmark economy)
- Model 2: Extended Taylor rule ($\gamma^L = 0.0125$)
- Model 3: Counter-cyclical capital requirement ($\eta_j^1 = 0.25$ for $j = b, i, e$)
- Model 4: Time-varying LTV ratio regulation on housing ($\gamma_1^{Hb} = \gamma_1^{Eb} = 0.3$)

The parameter values are chosen in the following sense. In Model 2, the responsiveness to loans from the central bank is the same as to output. In Model 3, the capital requirement increases by 0.25% in response to 1% increase in loans. In Model 4, LTV regulation decreases by 3% in response to 10% increase in housing prices.

Before we present our main results, we first show the impulse response functions of our model economies to selected exogenous shocks (TFP shock, risk perception shock (on entrepreneur), default shock (on entrepreneur), housing preference shock, and monetary policy shock) to check if the models behave well consistently with the usual economic intuition and different policies result in the different impulse responses to the variables. As will be turned out later, the intuition discussed below is useful to understand our main findings.

4.2.1 Comparing Effects of Different Policies with IRFs First of all, Figure 4.1 is the collection of impulse response functions to one-time-one-unit shock to the aggregate productivity. As usually argued in the literature, different sets of macroprudential policies do not have much impact on

Table 4.1: Benchmark Calibration (Benchmark Economy)

Parameter	Value	Description
β_s	0.9925	Discount factor, patient household
β_b	0.94	Discount factor, impatient household
β_e	0.94	Discount factor, entrepreneur
β_r, β_i	0.945	Discount factor, banks
ν_s^h,ν_b^h	0.075	Housing preference parameter
ν_s^n,ν_b^n	2	Labor preference parameter
ϕ	1	Inverse Frisch elasticity
δ	0.035	Rate of capital depreciation
h	0.8	Habit formation
α	0.35	Total capital share in production
ω_n	0.67	Wage share of patient household
ω_k	0.64	Capital share of patient household
ν	0.04	Housing share in production
ε	11	Elasticity of substitution for intermediate varieties
θ	0.78	Calvo Parameter
ι_{K^s}	1.73	Capital adjustment cost, household
ι_{K^e}	0.59	Capital adjustment cost, entrepreneur
ι_{d^s}	0.10	Deposit adjustment cost, household
ι_{d^r}	0.14	Deposit adjustment cost, bank
ι_{l^b}	0.37	Household loan adjustment cost, household
$\iota_{l^{br}}$	0.47	Household loan adjustment cost, retail bank
ι_{l^e}	0.07	Entrepreneur loan adjustment cost, entrepreneur
$\iota_{l^{ei}}$	0.06	Entrepreneur loan adjustment cost, investment bank
$\iota_{l^{ir}}$	0.47	Interbank loan adjustment cost, retail bank
ι_{l^i}	0.05	Interbank loan adjustment cost, investment bank
ρ_b	0.70	Speed of deleveraging, impatient household
ρ_e	0.65	Speed of deleveraging, entrepreneur
ρ_r	0.24	Speed of loan adjustment, retail bank
ρ_i	0.70	Speed of loan adjustment, investment bank
γ_0^{Hb}, γ_0^{He}	0.7	LTV ratio on housing
γ_0^{Ke}	0.9	LTV ratio on entrepreneur capital
η_0^b	0.08	Minimum capital requirement, households loan
η_0^e	0.08	Minimum capital requirement, entrepreneur loan
η_0^i	0.08	Minimum capital requirement, interbank loan
ρ^R	0.75	Interest rate inertia, monetary policy
γ^π	1.5	Inflation targeting parameter, monetary policy
γ^Y	0.125	Output targeting parameter, monetary policy
γ^L	0	Financial targeting parameter, monetary policy

the response of macro variables when the real shock hits the economy. Shapes are consistent with the usual intuition; key variables all increase due to high productivity in this economy.

Figure 4.2 is the collection of impulse response functions to one-time-one-unit shock to risk perception

Table 4.2: Benchmark Calibration: Exogenous Shocks

Parameter	Value	Description
ρ^C	0.994	Autocorr. of spending shock
ρ^K	0.916	Autocorr. of investment-specific technology shock
ρ_z	0.839	Autocorr. of TFP shock
ρ_H	0.932	Autocorr. of housing demand shock
ρ_b	0.969	Autocorr. of default shock (impatient HH)
ρ_e	0.992	Autocorr. of default shock (entrepreneur)
ρ_i	0.916	Autocorr. of default shock (investment bank)
ρ_{lb}	0.839	Autocorr. of LTV shock (impatient HH)
ρ_{le}	0.873	Autocorr. of LTV shock (entrepreneur)
σ_C	0.025	s.d of spending shock
σ_K	0.025	s.d of investment-specific technology shock
σ_z	0.007	s.d. of TFP shock
σ_H	0.0348	s.d. of housing demand shock
σ_b	0.0013	s.d. of default shock (impatient HH)
σ_e	0.0011	s.d. of default shock (entrepreneur)
σ_i	0.0011	s.d. of default shock (investment bank)
σ_{lb}	0.0115	s.d. of Risk perception(LTV) shock (impatient HH)
σ_{le}	0.0204	s.d. of Risk perception(LTV) shock (entrepreneur)

(LTV) on entrepreneurs. Since positive shock to the risk perception of entrepreneurs means that they can borrow more with the same asset values, this shock stimulates the economy. Overall, different macroprudential policies are still not effective to lower the responsiveness of the economy to the shock. One noticeable observation is that inflation moves in opposite direction when the extended Taylor rule is implemented (Model 2): this comes from the fact that interest rates further change from the changes in loan size. Interest rates increase more in this case so that the incentive to consume decreases. However, the incentive of consumers for consumption smoothing, which is enhanced by the habit formation, requires less changes in interest rates. As a result, the inflation rate needs to decrease in equilibrium so that interest rates do not much change by the Taylor rule. The response of consumption is greater than that under different policies, which is directly following. Lastly the greater response of consumption and output in Model 3 versus Model 1 comes from the successfully controlled loan market; the circumstance where less loan in equilibrium implies less deposit is required by financial intermediary so that patient households consumption instead increases with more incomes. In what follows, explanations on the effects of the extended Taylor rule on the inflation rate and those of countercyclical capital requirement policies are omitted since intuitions are the same.

Figure 4.3 plots impulse response functions to the positive shock to the default of entrepreneurs; as this

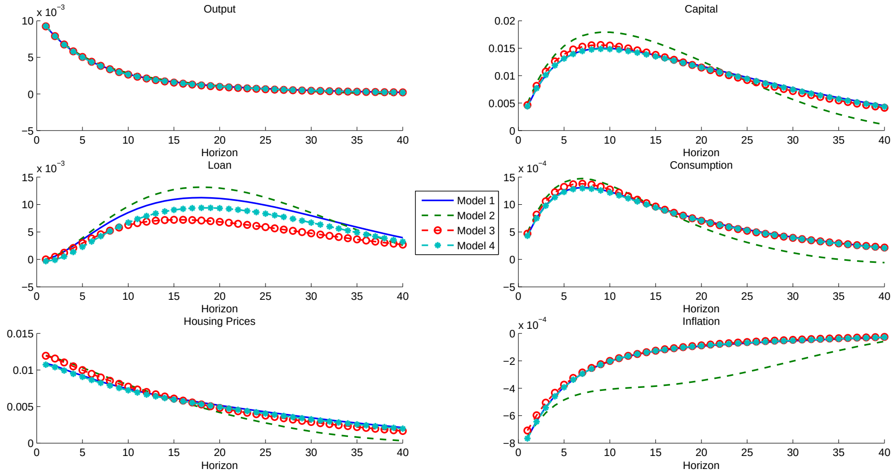

Figure 4.1: **Impulse Response Functions: Productivity Shock**
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

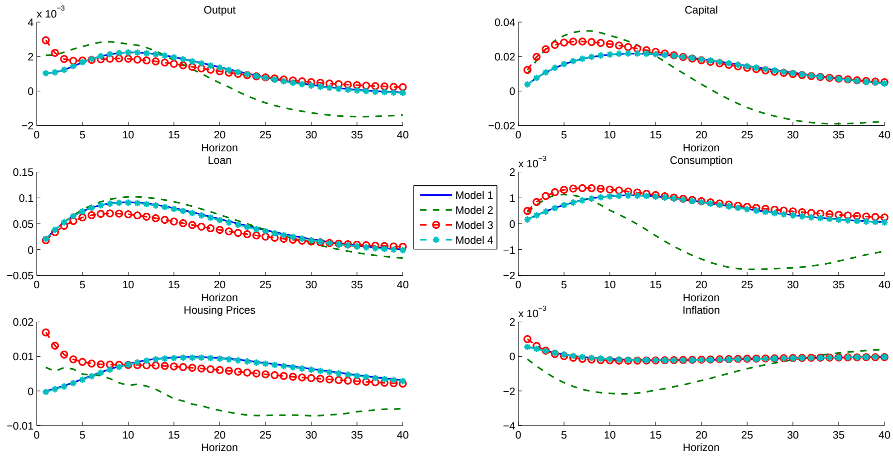

Figure 4.2: Impulse Response Functions: Risk Perception Shock to Entrepreneur
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

is a negative redistribution shock to banks, they will lower loans, which triggers economic downturns. While some macroprudential policies (countercyclical capital requirements in particular) seem to be effective in lowering loan fluctuations, most other policies are not effective. Figure 4.4 presents impulse response functions to the shock to the housing preference of households. Given fixed housing supply, the increase in housing demand means soaring housing prices, which grows the wealth of average agents in this economy. Hence, the economy experiences boom. Note again, macroprudential policies are not effective in terms of lowering the responsiveness of the economy to the exogenous shocks. Lastly, Figure 4.5 shows the impulse responses to the positive monetary policy shock. Higher policy rates usually dampen the economy; all variables exhibit patterns commonly observed in the recession, and different macroprudential policies do not show different patterns.

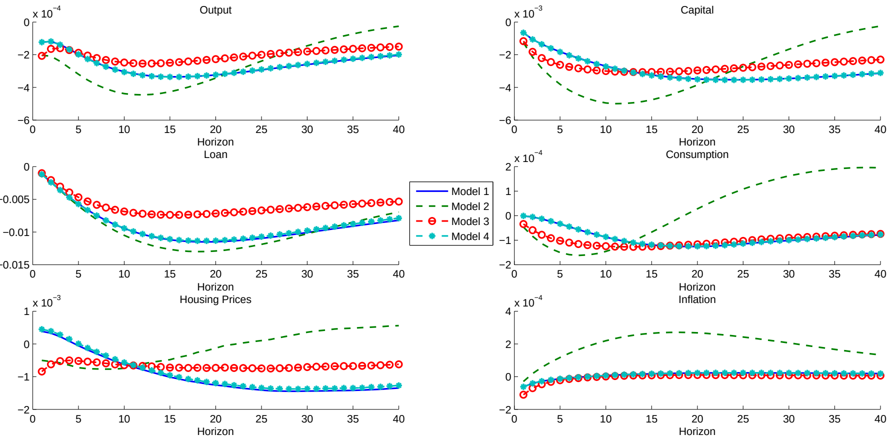

Figure 4.3: Impulse Response Functions: Default Shock to Entrepreneur
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

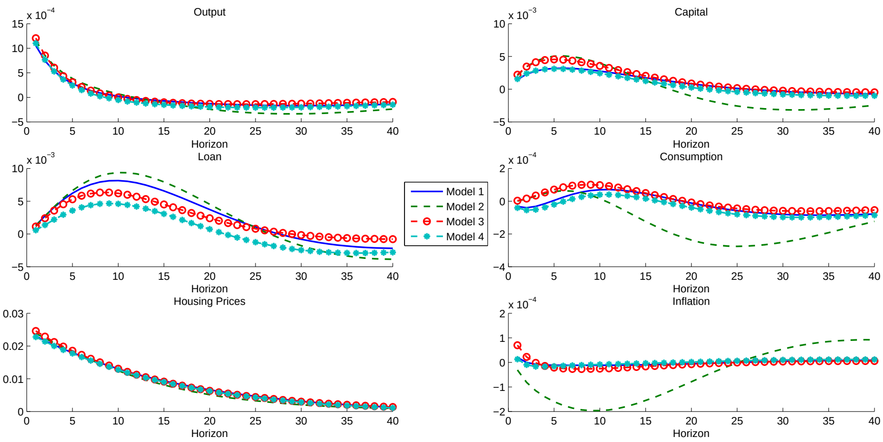

Figure 4.4: Impulse Response Functions: Housing Preference Shock
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

Therefore, the quick preview of the effectiveness of different policies with impulse response functions shows that in most cases macroprudential policies do not achieve their goals aiming to lower the ef-

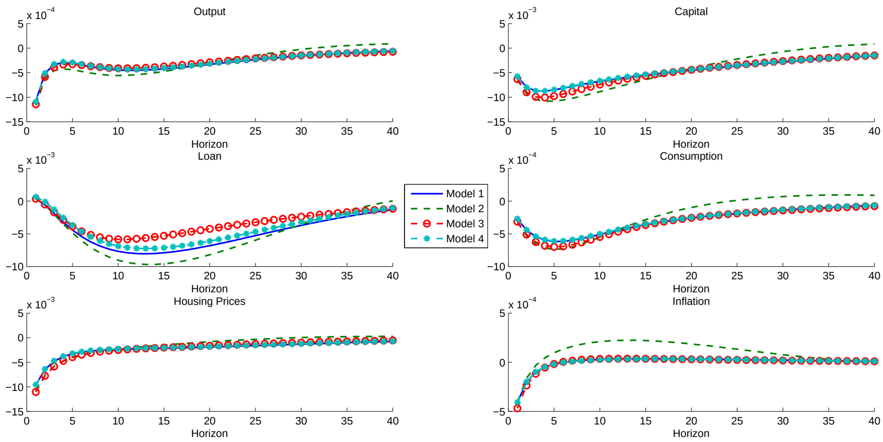

Figure 4.5: Impulse Response Functions: Monetary Policy Shock

Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

fects of exogenous shocks. If any, it is mostly observed from the policy requiring banks to accumulate countercyclical capital buffers, or when the financial shock hits the economy.

4.2.2 Effectiveness of More Aggressive Policy

In this section, we evaluate the performance of more aggressive macroprudential policies. In particular, we consider the macroprudential policies on countercyclical capital requirements since it seems to be more effective than other policies in stabilizing loan fluctuations. Figure 4.6 to 4.9 show the impulse response functions to the productivity shock, default shock to entrepreneurs, housing preference shock, and monetary policy shock, respectively. The thick blue line represents the responses from the benchmark economy, the dotted green line represents the economy with the weak macroprudential policy ($\eta_j^1 = 0.25$ for $j = b, i, e$), and the circled red line represents the economy with the aggressive macroprudential policy ($\eta_j^1 = 0.5$ for $j = b, i, e$ hence the coefficient is twice from the weak policy economy).

We first note that in terms of lowering loan responses, the aggressive macroprudential policy is mostly effective. In other words, the aggressive policy achieves its goal to stabilize the financial market more effectively than the weak policy does. However, the negative impacts on the real sector are also amplified (see Figure 4.7 to 4.9); this comes from the fact that smaller response in loan results in smaller response in deposit, which thus increases the responsiveness of consumption and output as discussed above.

5 Frequency-Specific Effects on Variance

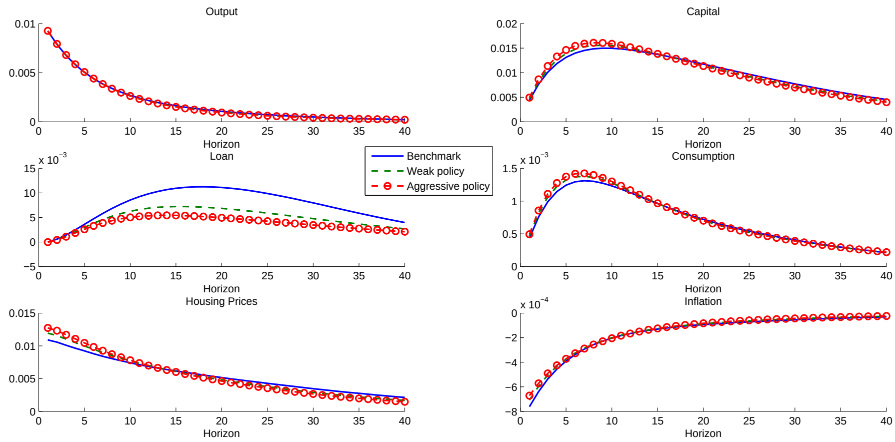

Figure 4.6: Impulse Response Functions: Productivity Shock

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

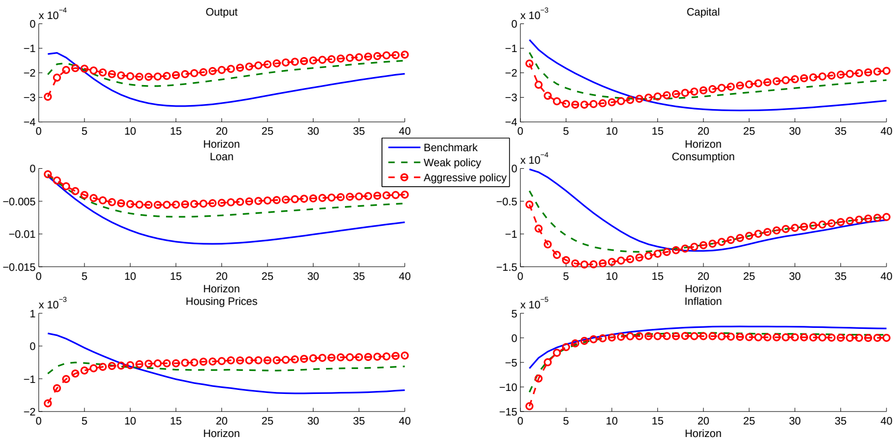

Figure 4.7: Impulse Response Functions: Default Shock to Entrepreneur

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

5.1 Volatility at Business Cycle Frequency Before we proceed to analyze the frequency-specific effectiveness of macroprudential policies, as a benchmark to our main analysis, we first report the standard deviation of the key variables at the business cycle frequency. In particular, we simulate the model economy 1,000 times with each simulation setting the total period at $1,024$. We then filter each of the series with $\kappa = 200$ by applying the band-pass filter (Baxter and King (1999)) to obtain the series of frequency between 2 years/cycle and 8 years/cycle where κ is the number of leads/lags used in the

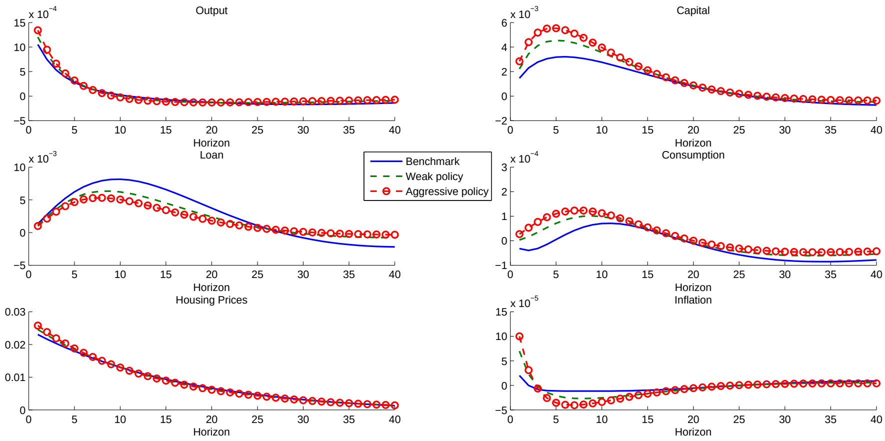

Figure 4.8: Impulse Response Functions: Housing Preference Shock

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

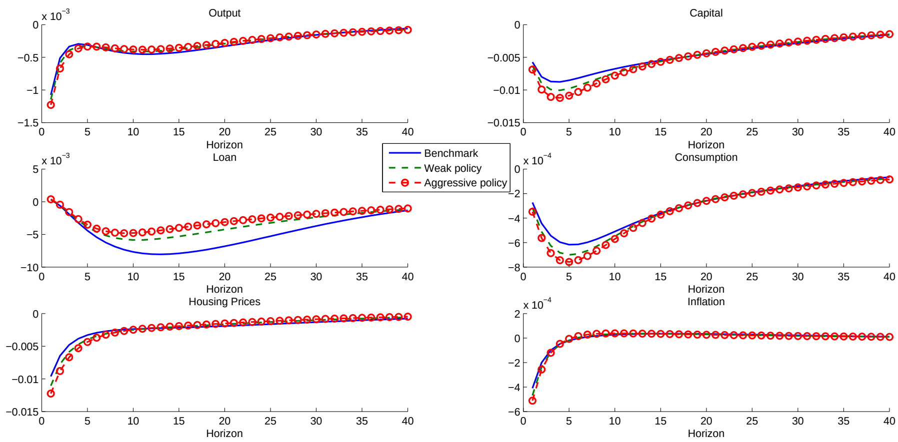

Figure 4.9: Impulse Response Functions: Monetary Policy Shock

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

approximation of the filtering. We then compute the standard deviation of each series and take the average of standard deviations from each simulation. Table 5.1 summarizes the results; as we are interested in the performance of the macroprudential policies relative to the benchmark economy without such policies, we compute the standard deviation of key variables relative that obtained in the benchmark economy.

Several observations are noteworthy. First, the macroprudential policy that tightens the LTV ratio in response to housing prices (Model 4) is not essentially effective at all. Second, the extended Taylor rule

Table 5.1: Relative Standard Deviation of Key Variables

	std(Y)	std(C)	std(π)	std(L)	std(L/Y)	std(p^H)
Model 2	1.01	1.17	1.15	1.03	1.01	1.13
Model 3	1.00	1.05	1.14	0.87	0.93	1.07
Model 4	1.00	1.00	1.00	0.99	0.99	1.00
Model 5	1.03	1.08	1.23	0.77	0.88	1.20

Note: All values are relative to the benchmark economy. Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, Model 4 is the economy with tight LTV on housing. Model 5 is the economy with aggressive countercyclical capital requirement policy.

(Model 2) much more amplifies the variance of consumption than other policies when compared to the benchmark economy, while it does not achieve its original goal of stabilizing the financial market. Even worse, the volatility of inflation rate also increases dramatically. Hence, from the traditional perspective on the role of central bank, which emphasizes output and inflation stabilities, it is not recommended for the central bank to directly take loan into account. Third, stabilizing the financial market at the business cycle frequency is the most successful in the economies with countercyclical capital requirement policies (Model 3 (weak policy) and Model 5 (aggressive policy)), while the output fluctuations are somewhat similar to the benchmark economy. Finally, the aggressive macroprudential policy (Model 5) is more effective in the financial market stabilization than the weak policy (Model 3) at the cost of amplifying the fluctuation in the real sector.

5.2 Frequency-Specific Effects of Different Policies We now turn to our main analysis, which analyzes the frequency-specific effects of macroprudential policies. In so doing, we take the steps described in Section 2. Since our main purpose here is to evaluate the performance of different policies versus the benchmark economy without any macroprudential policy, we compute the spectral density of each variable of the specific policy regime and compare the results to those of the benchmark model. We particularly consider following macro variables in this exercise; output, loan, consumption, loan to output ratio, housing price, and inflation rate. Figure 5.1 to 5.6 show the results. In each figure, the vertical axis denotes the spectral density relative to model 1 at each frequency and the horizontal axis denotes frequencies from low to high frequency.

Firstly, when the extended Taylor rule is implemented in the economy, output fluctuations are amplified at every frequency while the negative effect is greater at the relatively low frequency. In contrast, the countercyclical capital requirement policy is also overall ineffective in stabilizing output compared to the benchmark economy, but the negative effect is much smaller than the economy with the extended Taylor rule. In addition, the output fluctuations are more amplified at the relatively high frequency. Therefore, this policy, even though it could be effective in stabilizing the financial market as shown later, clearly has a slight negative effect on the real sector, which is one of the design limit of the macroprudential policy. This finding is in line with our previous discussions on impulse response functions; lower loan volatility would also decrease deposit volatility, which would amplify the consumption fluctuation and then the output fluctuation. Consumption volatility exhibits similar patterns; it is dramatically exacerbated under the extended Taylor rule. Almost at every frequency, the relative magnitude of the consumption volatility is about thirty percent higher than that of the benchmark economy. This is because interest rates directly respond to loan so that the Euler equation more affects consumption than economies under different policy regimes. In contrast, the LTV policy (Model 4) does not affect consumption and output fluctuations once again.

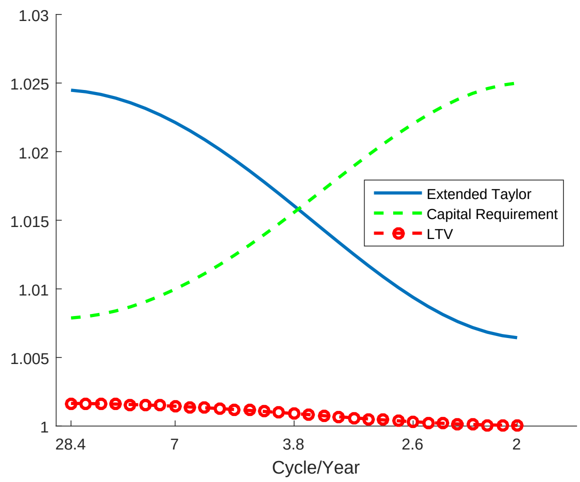

Figure 5.1: Frequency-Specific Effects: Output
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

How about loan fluctuations, which is the main objective of macroprudential policies? Figure 5.3 shows the result; importantly, the extended Taylor rule satisfies the main goal only at the relatively high frequency (higher than 8 years/cycle). It rather amplifies the financial market fluctuations at the relatively low frequency. In the sense that the financial cycle exhibits a much lower frequency (8 to 32 years/cycle), this implies that the extended Taylor rule does not achieve its goal to stabilize the financial cycles. Together with our observations from Figure 5.1, this further implies that the extended Taylor rule affects output and loan fluctuations in the opposite direction to the original objective of the macroprudential policy. On contrary, the countercyclical capital requirement policy is very effective in

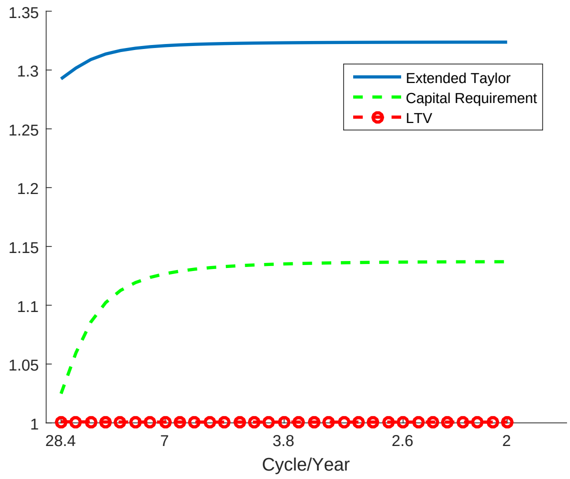

Figure 5.2: Frequency-Specific Effects: Consumption

Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

stabilizing loan fluctuations regardless of the frequency. However the output fluctuations are amplified at every frequency under this policy. This implies that the macroprudential policies that effectively stabilize the financial market pay the cost to destabilize the goods market. In the mean time, the policy to tighten LTV ratio on housing (Model 4) is not that effective in the financial market as well as in the goods market. This means that tightening LTV ratio is not overall effective in any market in our model economies. The observation from Figure 5.5 is consistent with the above discussions. Figure 5.4 and 5.5 provide the similar findings.

We also point out that tightening LTV ratio is not overall effective since it is not the policy that reacts to the changes in loan; other policies, in contrast, react to the changes in loan (potentially loan-to-output ratio). Rather, LTV policy responds to housing prices, but its propagation is relatively week compared to the effects of the overall loan on the decision of the banks. In contrast, the countercyclical capital requirement policy is effective since it really reacts to the fluctuations in loan to GDP ratio. It is effective in lowering loan fluctuations at every frequency since banks accumulate enough capital during boom times in preparation for possible losses during recessions, which results in less changes in the overall loan level.

Lastly, we also consider the effects of macroprudential policies on inflation rate (Figure 5.6). First, we can observe that the negative effect of the extended Taylor rule (Model 2) on inflation rate volatility is observed for the frequency lower than about 3 years/cycle. The high frequency which cannot be observed from the business cycle analysis can actually lower the volatility of inflation rate while the variance of

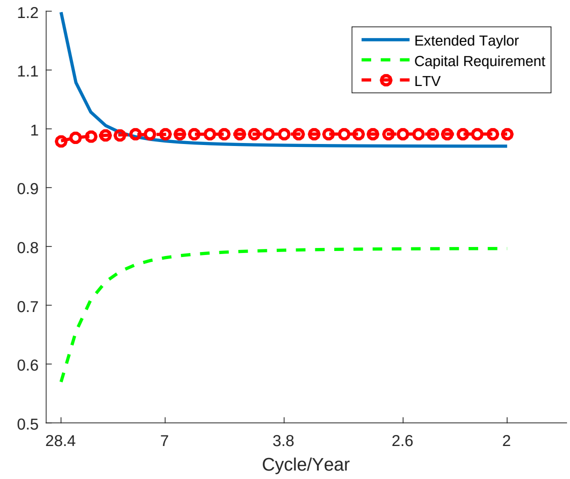

Figure 5.3: Frequency-Specific Effects: Loan

Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

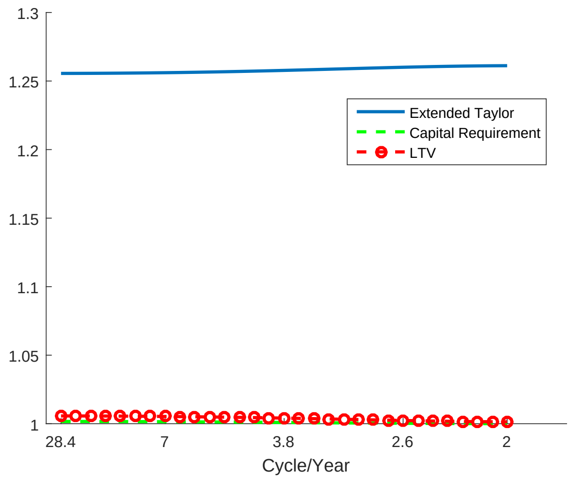

Figure 5.4: Frequency-Specific Effects: Housing Price

Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

inflation rate becomes much greater as we consider lower frequency.[12] Therefore the negative effect on the inflation rate volatility is maximized at the lowest frequency under the extended Taylor rule. The intuition for this result is in line with our discussions on the impulse response functions in Section 4.2.1; as the interest rate directly responds to loan in this economy, the inflation rate should adjust in order for consumption smoothing. As a result, the inflation rate volatility becomes higher in this economy.

[12] If Hodrick-Prescott filter is used to obtain the filtered data, this problem becomes exaggerated since it does not filter out movement at the very high frequency.

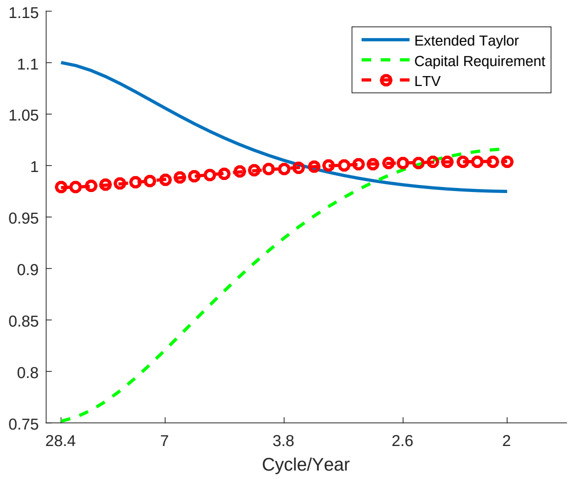

Figure 5.5: Frequency-Specific Effects: Loan to Output
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

The countercyclical capital requirement policy (Model 3) on the fluctuations of inflation rate is further amplified at every frequency. Hence, our result provides an important lesson for the central bank; the currently well-known macroprudential policies can ruin the goal of the central bank that aims to stabilize the inflation rate.

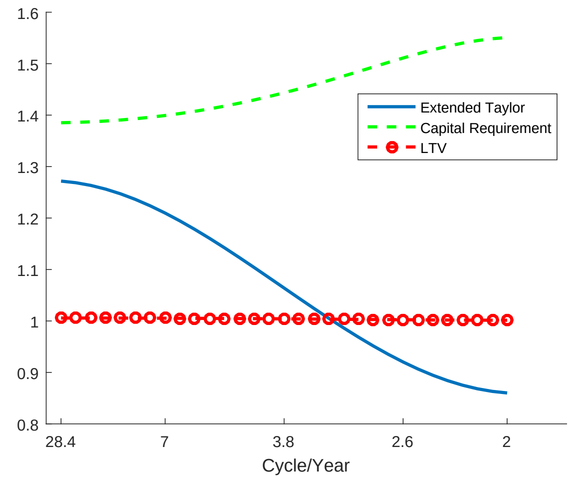

Figure 5.6: Frequency-Specific Effects: Inflation
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

In summary, our exercise implies that the currently well-known macroprudential policies can be effective in stabilizing the financial market, especially when the countercyclical capital requirement policy is implemented. However, there is a substantial cost to achieve its effectiveness in the financial sector; it can amplify the fluctuations in the real sector or in inflation rate. Especially the performance of the extended Taylor rule is worse than that of countercyclical capital requirement policy in many dimensions. Contrary to these policies, tightening LTV ratio on housing does not much affect the economy compared to other policies. Hence, the design limit of macroprudential policies in our model is two folds. First, the financial sector stabilization is associated with the real sector (or inflation rate) destabilization. Second, the effectiveness of policy is different across frequencies; for instance, when the extended Taylor rule is implemented, the inflation rate volatility increases as the frequency becomes lower while the opposite is observed in the financial market.

Next, we study if the effects of the aggressive macroprudential policy can be different from the less aggressive macroprudential policy. As in the previous section, we only consider the countercyclical capital requirement policy since it is the most effective policy in terms of the loan market stabilization. We again compute the relative spectral densities of the economy with the weak macroprudential policy ($\eta_j^1 = 0.25$ for $j = b, i, e$, thick blue line) and with the aggressive macroprudential policy ($\eta_j^1 = 0.5$ for $j = b, i, e$, circled green line), and plot the spectral densities in Figure 5.7.

Two conclusions can be drawn from the figures. First, the aggressive policy is more effective in stabilizing the financial market than the weak one as expected. This achievement, however, leads to amplified fluctuations in the real sector compared to the weak policy. Particularly, the output, consumption, and inflation rate volatility at each frequency are much more exacerbated under the aggressive policy. Even worse, the negative effects on the real sector increase as frequencies become higher. If monetary authorities care about the aggregate fluctuations at the business cycle frequency, which is usually defined as the fluctuations between about 2 years/cycle and 8 years/cycle, the aggressive macroprudential policy is not recommended; it increases output volatility compared to the benchmark economy approximately by 5% point, consumption volatility approximately by 20% point, and inflation rate volatility by more than 50% point at the business cycle frequency.

6 Frequency-Specific Effects on Welfare

One might raise a concern that the variance does not directly translate into the welfare (for instance, more volatile economy can be welfare-improving (mean effect, Cho, Cooley, and Kim (2015))). This section deals with such a concern by conducting a spectral welfare analysis as in Otrok (2001). In doing so, we consider a representative (average) consumer whose utility function takes the following form:

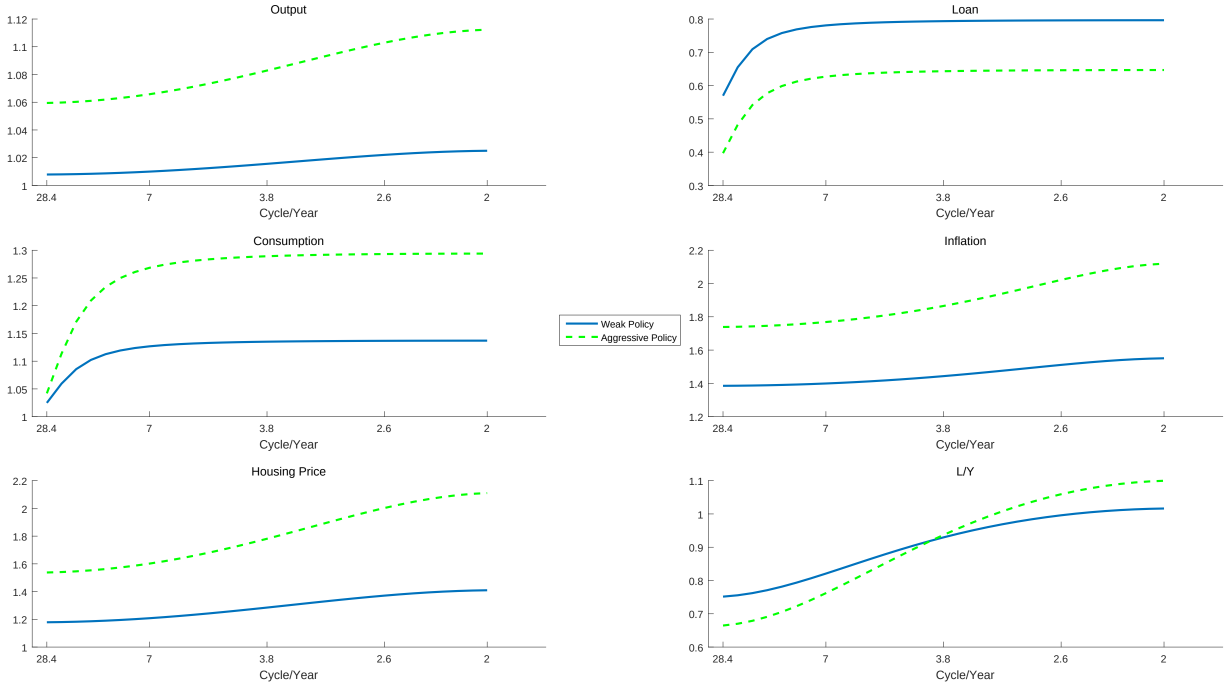

Figure 5.7: Frequency-Specific Effects: Weak vs. Aggressive Policy
Note: Relative volatility compared to model 1 at each frequency. 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 1$ for $j = b, i, e$.

$$\mathbb{V}^r = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t^c \ln \left(C_t^r - hC_{t-1}^r \right) - \nu_n \frac{(N_t^r)^{1+\phi}}{1+\phi} \right] \tag{6.1}$$

where r denotes a policy regime r, C_t denotes aggregate consumption, and N_t denotes aggregate hours worked. Since we have four types of agents (household, entrepreneur, and banks) that maximize their own utility functions, we aggregate consumption of different agents in order to obtain C_t^r so that it represents aggregate consumption of this economy. N_t^r, hours worked under regime r, is also obtained by aggregating hours worked by patient household and that by impatient household.[13] We note that aggregate housing demand is not included in the above utility function as the aggregate housing supply is assumed to be fixed as one so that its value does not change across the policy regime. Parameters for the spectral welfare exercise are taken from Table 4.1; especially for β, we choose the average value of β between patient and impatient household, which does not change our conclusion reported below.

Our objective is to evaluate performances of different macroprudential policies compared to the bench-

[13]Similar strategy is taken by Suh (2012).

mark economy without any macroprudential policy, hence we compare the value, V^r, the life-time value associated with Model $r > 1$ with the value associated with Model 1 (benchmark economy), V^1, by considering the following λ adjusted value function:

$$\mathbb{V}^{r,\lambda} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t^c \ln(1+\lambda) \left(C_t^r - hC_{t-1}^r \right) - \nu_n \frac{(N_t^r)^{1+\phi}}{1+\phi} \right] \tag{6.2}$$

where λ measures the welfare gain (or loss) in terms of consumption variation.

Let $r = 1$. Using the property of log function,

$$\mathbb{V}^{1,\lambda} = \frac{\mathbb{E}_0(\varepsilon_t^c)}{1-\beta} \ln(1+\lambda) + \underbrace{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t^c \ln \left(C_t^1 - hC_{t-1}^1 \right) - \nu_n \frac{\left(N_t^1\right)^{1+\phi}}{1+\phi} \right]}_{\equiv \mathbb{V}^1} \tag{6.3}$$

Then for $r > 1$, λ solves $\mathbb{V}^{1,\lambda} = \mathbb{V}^r$ so that it satisfies the following formula:

$$\lambda \approx (1-\beta)(\mathbb{V}^r - \mathbb{V}^1) \tag{6.4}$$

where we can ignore $\mathbb{E}_0(\varepsilon_t^c)$ as it is constant across different regimes and is a constant ($\mathbb{E}_0(\varepsilon_t^c) = \exp\left(\frac{1}{2}\frac{\sigma_c^2}{1-(\rho_c)^2}\right) > 0$).

We note that λ is positive (resp. negative) when the macroprudential policy is welfare-improving (resp. welfare detrimental). In particular, as we are interested in the frequency-specific welfare gains, we compute λ^i for each frequency i by computing the spectral utility functions. The steps to compute the welfare formula (6.4) is described in Section 2.

6.1 Average Welfare Gains We first compute the welfare gain by directly using the simulated series, not the filtered series; hence, the welfare gain is not frequency-specific but is consistent with the previous literature. We call this measure average welfare gain (loss). Results are reported in Table 6.1:

Table 6.1: Welfare Analysis: Average Welfare Gains

	Welfare Gains (%)
Extended Taylor rule	−0.4247
Countercyclical capital requirement	0.0302
Time-varying LTV ratio regulation on housing	−0.0328
Aggressive countercyclical capital requirement	0.0321

Note: We multiply 100 to λ.

Several points are noteworthy here. As can be inferred from our previous analysis, the Extended

Taylor rule is, in general, welfare-detrimental. While the size is much smaller, LTV policy is also welfare-detrimental. On contrary, countercyclical capital requirement policy is welfare-improving and the size of the welfare gain is comparable to that obtained by Suh (2012). The last row shows the welfare gain from more aggressive countercyclical capital requirement policy, which confirms the above finding that this policy is welfare-improving. As a whole, Table 6.1 shows that, consistently with the findings reported in the previous section, the countercyclical capital requirement policy is the best among the macroprudential policy tools considered in our paper.

6.2 Frequency-Specific Welfare Gains This section presents our main results on frequency-specific welfare gains. Figure 6.1 shows the welfare gain (loss) of each policy at different frequency and Figure 6.2 compares the welfare gains (loss) of weak and aggressive countercyclical capital requirement policy. We first point out that unlike Otrok (2001), the average welfare gain obtained in the previous section is not additively decomposed into the spectral welfare gains reported in this section. In Otrok (2001), the welfare loss from the economic fluctuations can be summarized by the consumption volatility as denoted by Lucas (1987). As a result, the sum of the spectral welfare cost, which is also a (linear) function of frequency-specific variance, which is an additive decomposition of the overall variance, is equal to the average welfare cost. This is not the case in our experiment; the value function is not a linear function of consumption volatility as the model is second-order approximated at the equilibrium (Schmitt-Grohé and Uribe (2004)).

Key observations can be summarized as follows. First, the LTV regulation barely affects the welfare gain or loss, a consistent finding with our previous analysis. Second, the welfare loss from the extended Taylor rule is not concentrated at a specific frequency band; rather, it is broadly observed along the frequency bands. Interestingly, there is a welfare gain at the relatively low frequency. At the business cycle frequency (2-8 years), however, there is a welfare loss. Lastly, similar to the Extended Taylor rule, the welfare loss from the countercyclical capital requirement policy is dispersed along the frequency. Except at the frequency of 7-10 years/cycle, there is a welfare loss from the policy. Moreover, we can observe from Figure 6.2 that aggressive countercyclical capital requirement policy (dotted green line) widens the welfare loss at every frequency when compared to the less aggressive policy (solid blue line). This is also a finding consistent with Figure 5.7; more aggressive policy makes the real sector more volatile so that welfare loss from the policy increases at every frequency.

Our findings through this section raise a cautionary note on the welfare analysis with macroprudential policies; even a policy seems to be effective, such as increasing the average welfare (Table 6.1), it may

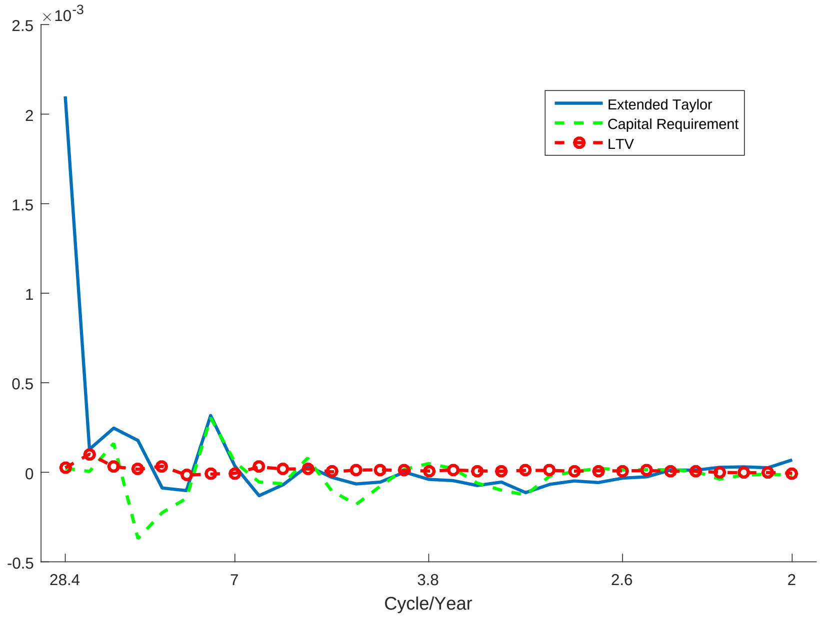

Figure 6.1: Spectral Welfare Analysis: Comparison across Policy Regimes

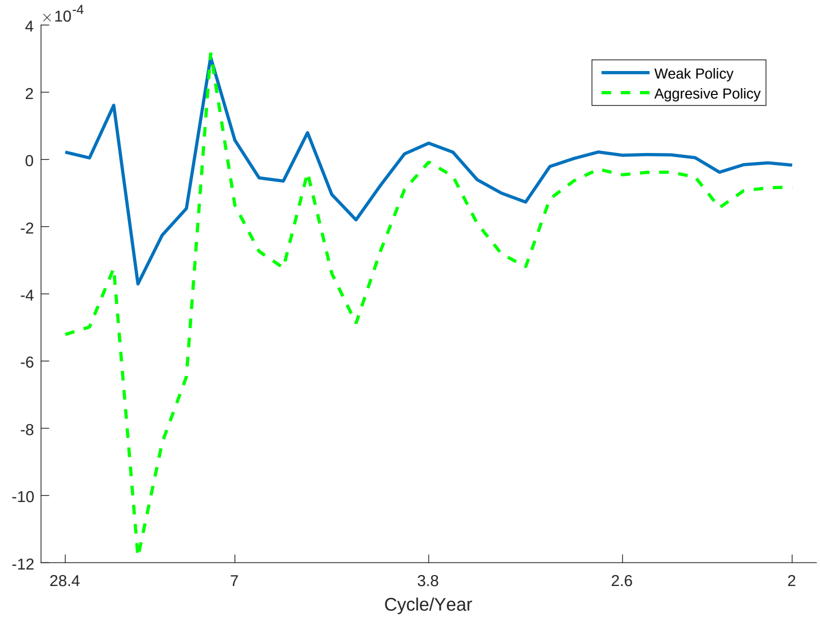

Figure 6.2: Welfare Analysis: Effects of Aggressive Countercyclical Capital Requirement Policy

not hold with the spectral welfare analysis. In other words, the effectiveness of macroprudential policies on the aggregate economy should be carefully examined as their effects are frequency-specific.

7 Conclusion

We evaluate the performances of various macroprudential policies in this paper within the financial sector augmented New Keynesian model. Our results from the conventional macroprudential policy are somewhat negative. A policy that makes LTV ratio respond to housing prices does not change the equilibrium properties of the model. Moreover, the extended Taylor rule is not recommended since its perverse effects on the real sector and inflation rate are observed at almost every frequency. On the other hand, the countercyclical capital requirement policy achieves its original goal of the stabilization of the financial market at the cost of destabilizing the inflation rate and consumption. Our findings also shed a light on the ongoing debate whether the monetary authority should react to the loan market fluctuations (extended Taylor rule) or we need an independent policy tool (counter-cyclical requirement policy for instance); our analysis suggests that it might not be a good idea for the central bank to respond directly to financial market fluctuations.

One possibility to understand our results is that the macroprudential policies considered in our analysis are not optimally designed; they are originally intended to stabilize the financial market, hence there can exist an unexpected cost associated with such policies, as shown in our paper. Therefore, the future research needs to find the optimally designed macroprudential policy so that it can achieve its main goal while minimizing adverse effects. Other possibility is that the model we consider in this paper is not able to fully capture the positive effects of such policies on the real sector. In this regard, the studies with alternative models could be another direction of the future research.

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A Equilibrium Conditions

A.1 Patient Household

λ_t^s the Lagrangian multiplier attached to the budget constraint.

$$[C_t^s] \qquad \lambda_t^s = \frac{\varepsilon_t^c}{C_t^s - hC_{t-1}^s} - h\beta_s \mathbb{E}_t \left[\frac{\varepsilon_{t+1}^c}{C_{t+1}^s - hC_t^s}\right] \tag{A.1}$$

$$[K_t^s] \qquad \frac{1}{\varepsilon_t^k} + \frac{\partial AC_{K^s,t}}{\partial K_t^s} = \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s}\left(r_{t+1}^K + \frac{1-\delta}{\varepsilon_{t+1}^K}\right)\right] \tag{A.2}$$

$$[H_t^s] \qquad p_t^H = \frac{\varepsilon_t^c \varepsilon_t^h \nu_h^s}{H_t^s \lambda_t^s} + \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} p_{t+1}^H\right] \tag{A.3}$$

$$[N_t^s] \qquad w_t^s = \frac{\nu_n^s (N_t^s)^\phi}{\lambda_t^s} \tag{A.4}$$

$$[d_t^s] \qquad 1 + \frac{\partial AC_{d,t}}{\partial d_t} = \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} r_{t+1}^d\right] \tag{A.5}$$

$$[\lambda_t^s] \quad C_t^s + \frac{K_t^s}{\varepsilon_t^k} + p_t^H \left[H_t^s - H_{t-1}^s\right] + d_t + AC_{d^s,t} + AC_{K^s,t} = w_t^s N_t^s + r_t^d d_{t-1} + \left(r_t^K + \frac{1-\delta}{\varepsilon_t^k}\right) K_{t-1}^s \tag{A.6}$$

A.2 Impatient Household

λ_t^b (resp. μ_t^b) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^b] \qquad \lambda_t^b = \frac{\varepsilon_t^c}{C_t^b - hC_{t-1}^b} - h\beta_b \mathbb{E}_t \left[\frac{\varepsilon_{t+1}^c}{C_{t+1}^b - hC_t^b} \right] \tag{A.7}$$

$$[H_t^b] \qquad p_t^H = \frac{\varepsilon_t^c \varepsilon_t^h \nu_h^b}{H_t^b \lambda_t^b} + \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1}^b}{\lambda_t^b} p_{t+1}^H \right] + \mu_t^b (1-\rho_b) \gamma_t^b \mathbb{E}_t \left[\frac{p_{t+1}^H}{r_{t+1}^b} \right] \tag{A.8}$$

$$[N_t^b] \qquad w_t^b = \frac{\nu_n^b (N_t^b)^\phi}{\lambda_t^b} \tag{A.9}$$

$$[l_t^b] \qquad 1 - \frac{\partial AC_{l^b,t}}{\partial l_t^b} = \mu_t^b + \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1}^b}{\lambda_t^b} (r_{t+1}^b - \rho_b \mu_{t+1}^b) \right] \tag{A.10}$$

$$[\lambda_t^s] \qquad C_t^b + p_t^H \left[H_t^b - H_{t-1}^b \right] + r_t^b l_{t-1}^b + AC_{l^b,t} = w_t^b N_t^b + l_t^b + \varepsilon_t^b \tag{A.11}$$

$$[\mu_t^b] \qquad l_t^b = \rho_b l_{t-1}^b + (1-\rho_b) \left[\gamma_t^{Hb} \mathbb{E}_t \frac{p_{t+1}^H H_t^b}{r_t^b} - \varepsilon_t^b \right] \tag{A.12}$$

A.3 Entrepreneur λ_t^e (resp. μ_t^e) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^e] \qquad \lambda_t^e = \frac{1}{C_t^e - hC_{t-1}^e} - \beta_e \mathbb{E}_t \frac{h}{C_{t+1}^e - hC_t^e} \tag{A.13}$$

$$[H_t^e] \qquad p_t^H = \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} p_{t+1}^H (1 + r_{t+1}^H) \right] + \mu_t^e (1-\rho_e) \gamma_t^{He} \frac{p_{t+1}^H}{r_{t+1}^e} \tag{A.14}$$

$$[K_t^e] \qquad \frac{1}{\varepsilon_t^k} + \frac{\partial AC_{K^e,t}}{\partial K_t^e} = \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} (1 + r_{t+1}^K - \delta) \right] + \mu_t^e (1-\rho_e) \gamma_t^{Ke} \tag{A.15}$$

$$[N_t^s] \qquad (1 + (1-\rho_e)\gamma_t^{Ne}) w_t^s N_t^s = (1-\alpha-\nu)\omega^n p_t^X X_t \tag{A.16}$$

$$[N_t^b] \qquad (1 + (1-\rho_e)\gamma_t^{Ne}) w_t^b N_t^b = (1-\alpha-\nu)(1-\omega^n) p_t^X X_t \tag{A.17}$$

$$[r_t^K] \qquad r_t^K = \alpha(1-\omega^k) p_t^X X_t / K_{t-1}^s \tag{A.18}$$

$$[l_t^e] \qquad 1 - \frac{\partial AC_{l^e,t}}{\partial l_t^e} = \mu_t^e + \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} (r_{t+1}^e - \rho_e \mu_{t+1}^e) \right] \tag{A.19}$$

$$[\lambda_t^e] \quad C_t^e + \frac{K_t^e}{\varepsilon_t^k} + p_t^H [H_t^e - H_{t-1}^e] + w_t^s N_t^s + w_t^b N_t^b + r_t^K K_{t-1}^s + r_t^e l_{t-1}^e + AC_{K^e,t} + AC_{l^e,t}$$
$$= p_t^X X_t + \frac{1-\delta}{\varepsilon_t^k} K_{t-1}^e + l_t^e + \varepsilon_t^e \tag{A.20}$$

$$[\mu_t^e] \qquad l_t^e = \rho_e l_{t-1}^e + (1-\rho_e) \left(\gamma_t^{He} \mathbb{E}_t \frac{p_{t+1}^H H_t^e}{r_{t+1}^e} + \gamma_t^{Ke} K_t^e - \gamma_t^{Ne} (w_t^s N_t^s + w_t^b N_t^b) - \varepsilon_t^e \right) \tag{A.21}$$

where $X_t = \varepsilon_t^z \left(K_{t-1}^e\right)^{\alpha\omega^k} \left(K_{t-1}^s\right)^{\alpha(1-\omega^k)} \left(H_{t-1}^e\right)^{\nu} \left(N_t^s\right)^{(1-\alpha-\nu)\omega^n} \left(N_t^b\right)^{(1-\alpha-\nu)(1-\omega^n)}$. From the firm's production function, $p_t^H r_t^H = \nu p_t^X X_t / H_{t-1}^e$

A.4 Retail Banks λ_t^r (resp. μ_t^r) the Lagrangian multiplier attached to the budget constraint (resp. capital requirement constraint).

$$[C_t^r] \qquad \lambda_t^r = \frac{1}{C_t^r - hC_{t-1}^r} - \beta_r \mathbb{E}_t \frac{h}{C_{t+1}^r - hC_t^r} \tag{A.22}$$

$$[d_t] \qquad 1 - \frac{\partial AC_{d^r,t}}{\partial d_t} = \mu_t^r + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r}(r_{t+1}^d - \rho_r \mu_{t+1}^r)\right] \tag{A.23}$$

$$[l_t^b] \qquad 1 + \frac{\partial AC_{l^{br},t}}{\partial l_t^b} = \mu_t^r(1-(1-\rho_r)\eta_t^b) + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r}(r_t^b - \rho_r \mu_{t+1}^r)\right] \tag{A.24}$$

$$[l_t^i] \qquad 1 + \frac{\partial AC_{l^{ir},t}}{\partial l_t^i} = \mu_t^r(1-(1-\rho_r)\eta_t^i) + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r}(r_t^i - \rho_r \mu_{t+1}^r)\right] \tag{A.25}$$

$$[\lambda_t^r] \qquad C_t^r + l_t^b + l_t^i + r_t^d d_{t-1} + AC_{d,t} + AC_{l^i,t} + AC_{l^b,t} = d_t + r_t^i l_{t-1}^i + r_t^b l_{t-1}^b - \varepsilon_t^b - \varepsilon_t^i \tag{A.26}$$

$$[\mu_t^r] \quad l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i = \rho_r(l_{t-1}^b + l_{t-1}^i - d_{t-1} - \varepsilon_{t-1}^b - \varepsilon_{t-1}^i) + (1-\rho_r)[\eta_t^b l_t^b + \eta_t^i l_t^i - \varepsilon_t^b - \varepsilon_t^i] \tag{A.27}$$

(A.28)

A.5 Investment Banks λ_t^i (resp. μ_t^i) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^i] \qquad \lambda_t^i = \frac{1}{C_t^i - hC_{t-1}^i} - \beta_i \mathbb{E}_t \frac{h}{C_{t+1}^i - hC_t^i} \tag{A.29}$$

$$[l_t^e] \qquad 1 + \frac{\partial AC_{l^{ei},t}}{\partial l_t^e} = \mu_t^i(1-\rho_i)(1-\eta_t^e) + \beta_i \mathbb{E}_t \left[\frac{\lambda_{t+1}^i}{\lambda_t^i} r_{t+1}^e\right] \tag{A.30}$$

$$[l_t^i] \qquad 1 + \frac{\partial AC_{l^i,t}}{\partial l_t^i} = \mu_t^i + \beta_i \mathbb{E}_t \left[\frac{\lambda_{t+1}^i}{\lambda_t^i}(r_t^i - \rho_i \mu_{t+1}^i)\right] \tag{A.31}$$

$$[\lambda_t^i] \qquad C_t^i + l_t^e + r_t^i l_{t-1}^i + AC_{l^e,t} + AC_{l^i,t} = l_t^i + r_t^e l_{t-1}^e + \varepsilon_t^i - \varepsilon_t^e \tag{A.32}$$

$$[\mu_t^i] \qquad l_t^i = \rho_i l_{t-1}^i + (1-\rho_i)[(1-\eta_t^e) l_t^e + \varepsilon_t^i - \varepsilon_t^e] \tag{A.33}$$

A.6 Retailers

$$\max_{P_t^*(z)} \quad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s \left[P_t^*(z) - P_{t+j}^X\right] Y_{t+j}(z)$$

subject to

$$Y_{t+j}(z) = \left(\frac{P_t^*(z)}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}$$

We can rewrite the problem as:

$$\max_{P_t^*(z)} \quad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} \left[(P_t^*(z))^{1-\varepsilon} - P_{t+j}^e \left(P_t^*(z)\right)^{-\varepsilon} \right]$$

$$[P_t^*(z)] \qquad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} \left[(1-\varepsilon) P_t^*(z) + \varepsilon P_{t+j}^X \right] = 0$$

$$P_t^*(z) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X}{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j}} \equiv \frac{\varepsilon}{\varepsilon - 1} \frac{F_{1,t}}{F_{2,t}} \tag{A.34}$$

$$F_{1,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X$$

$$= \lambda_t^s P_t^{\varepsilon} Y_t P_t^X + \theta\beta_s \mathbb{E}_t F_{1,t+1} \tag{A.35}$$

$$F_{2,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j}$$

$$= \lambda_t^s P_t^{\varepsilon} Y_t + \theta\beta_s \mathbb{E}_t F_{2,t+1} \tag{A.36}$$

or equivalently, with $P_t^* = P_t^*(z)$,

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{f_{1,t}}{f_{2,t}} P_t \tag{A.37}$$

$$f_{1,t} = \lambda_t^s Y_t p_t^e + \theta\beta_s \mathbb{E}_t f_{1,t+1} \pi_{t+1}^{\varepsilon+1} \tag{A.38}$$

$$f_{2,t} = \lambda_t^s Y_t + \theta\beta_s \mathbb{E}_t f_{2,t+1} \pi_{t+1}^{\varepsilon} \tag{A.39}$$

where $f_{1,t} = F_{1,t}/P_t^{\varepsilon+1}$ and $f_{2,t} = F_{2,t}/P_t^{\varepsilon}$.

Therefore, substituting equation (A.37) to the following equation yields

$$\pi_t = \left[(1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\varepsilon} + \theta\right]^{\frac{1}{1-\varepsilon}}$$
$$= \left[(1-\theta)(\frac{\varepsilon}{\varepsilon-1}\frac{f_{1,t}}{f_{2,t}}\pi_t)^{1-\varepsilon} + \theta\right]^{\frac{1}{1-\varepsilon}} \quad \text{(A.40)}$$

hence the equations (A.38), (A.39), and (A.40) implicitly determines $f_{1,t}$, $f_{2,t}$, and π_t.

A.7 Market Clearing Conditions We have

$$1 \equiv \bar{H} = H_t^s + H_t^b + H_t^e \quad \text{(A.41)}$$
$$Y_t = C_t + K_t + (1-\delta)K_{t-1} \quad \text{(A.42)}$$
$$l_t = l_t^e + l_t^i + l_t^b \quad \text{(A.43)}$$

where $C_t = C_t^s + C_t^b + C_t^e + C_t^e + C_t^r + C_t^i$, $K_t = K_t^e + K_t^s$, and $Y_t = X_t - \frac{Y}{1-\varepsilon}$. Finally, we have a monetary policy rule (3.26).