Are Effects of Macroprudential Policies Frequency-Specific? A Design Limit Approach*

Junghwan Mok† Myungkyu Shim‡

August 12, 2016

Abstract

Is a macroprudential policy an effective tool for stabilization? The answer to this question depends on the frequency and the sector that we consider. Using a financial-sector augmented New Keynesian model, we find that a set of macroprudential policies which are commonly used both in theory and in practice have different frequency-effects on the economy. Loan volatility is reduced at every frequency while inflation rate volatility is amplified at every frequency when a countercyclical capital requirement policy is implemented in the economy. When the Taylor rule is extended to respond to loan, on contrary, loan is stabilized only at the relatively high frequency while output volatility increases at every frequency. Lastly, LTV policy is not effective at all in our model economy. More aggressive policies cannot resolve such problems. Hence, our findings unveil the design limit of current policies; they can be effective at the designated markets and frequencies while having significant perverse effects at other sectors and frequencies.

*First Draft: Sep, 2015. Myungkyu Shim gratefully acknowledges financial support from the Bank of Korea. The views in this paper are those of the authors, and do not necessarily reflect those of the Bank of Korea. We are grateful to Soyoung Kim, Jinill Kim, and Yongseung Jung for their helpful comments and suggestions. We would also like to thank seminar and conference participants at Bank of Korea, Korea University, Kyunghee University, and IFABS 2016 Barcelona.

†Financial Stability Department, The Bank of Korea. Email: jhmok@bok.or.kr

‡School of Economics, Shanghai University of Finance and Economics and School of Economics, Sogang University. Email: audrb917@gmail.com

1 Introduction

The worldwide recession from the Global Financial Crisis was different from other recessions in the history of the business cycle since the Great Depression, because it was initiated from the collapse of financial market. A number of researches have investigated the cause of the financial disruption. Among them, Borio (2012) and Drehmann, Borio, and Tsatsaronis (2012) argue that there exist financial cycles driving the movements of financial markets, independent of the traditional business cycle. Since financial cycles are shown to have much lower frequencies (8 to 32 years per cycle) than usual business cycles have (6 months to 8 years per cycle), the infrequent downturns of financial cycles, compared to business cycles, can initiate financial crises. In order to prevent such crises, they emphasize the need for different policy tools, so called macroprudential policies designed to strengthen the financial systems resilience to economic downturns and other adverse aggregate shocks, and actively limit the build-up of financial risks (BIS (2010)) in the financial cycle frequency. Thus, the goal of macroprudential policies is different from that of traditional monetary policies which are designed to stabilize output and inflation at the business cycle frequency. In this sense, asking the effectiveness of each policy can be translated into the question if a policy achieves its objective of lowering variance at the targeting frequency.

In order to answer to such question, we apply the idea of design limit approach, which refers to the trade-off that occurs when the decrease in variance at some frequencies induces the increase in variance at others (Brock, Durlauf, and Rondina (2013)), that is new to the literature on the macroprudential policy. We take this approach because existing papers mostly limited their focuses to the effects of macroprudential policies at the business cycle frequency although the main goal of such policies is to stabilize the aggregate economy at the low frequency. This paper particularly considers some questions such as "Are macroprudential policies effective in lowering the variance of output and loan at the relatively low frequency without increasing the volatility at the high frequency?." We will sometimes call this frequency-specific effects following Brock, Durlauf, and Rondina (2008). Importantly, knowledge on frequency-specific effects of different policy mixes will provide a guideline for the central bank and policy makers in implementing different policies.

In so doing, we consider the version of New Keynesian model with the financial sector, which extends the business cycle model introduced in Iacoviello (2014); the financial market consists of retail banks (receive deposits from patient households and lend to impatient households and investment banks) and investment banks (obtain funds from retail banks and lend to entrepreneurs), which is similarly designed to that of Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Hence, financial frictions are incorporated in the model as the borrowing (or capital adequacy regulation) constraints of different agents, which is the usual balance sheet channel that amplifies the propagation of shocks. Following Iacoviello (2014), we have the two classes of shocks; non-financial shocks (aggregate TFP shock, investment-specific technology shock, aggregate demand shock, and monetary policy shock) which are common shocks assumed in the business cycle literature and financial shocks including default shocks (transfers of wealth from savers to borrowers when the borrowers are default), loan-to-value shocks (changes in maximum loan-to-value ratios), and housing demand shocks (changes in the price of housing). The financial shocks are important since they approximately accounted for two-thirds of the output drops during the Great Recession, in which the financial sector played an important role for the propagation of the negative shock. Different from Iacoviello (2014), price stickiness in the goods market is further introduced so that the monetary policy has a role in our model economy.

We then consider the three versions of macroprudential policies that are very common both in the previous literature and in practice, and compare the effects of those policies to the benchmark economy with virtually no macroprudential policy. As the first policy, we extend the Taylor rule by explicitly taking loan into account so that the central bank responds to high loans as well. Second policy is to introduce the counter-cyclical capital requirement regulation, which is the core feature of BASEL III, so that the banks should accumulate more buffers in good times for the possible losses in bad times. The third policy is to tighten LTV ratio on impatient households and entrepreneurs so that they cannot borrow as they want for the given level of collateral value.

The steps for evaluating the performances of different policies are as follows. With the model introduced in Section 3, we first simulate the economies under different policy regimes for several times to obtain time series of key macroeconomic variables. Instead of following the conventional way to compute the welfare cost associated with each policy regime to compare policies, we focus on the extent to which each policy is effective to lower frequency-specific variances, which enables us to compare the frequency-specific effects of policies. In doing so, we compute the spectral density of each simulated series, following Otrok (2001a), since the spectral density provides us the variance of the series at each frequency so that we can examine if the effectiveness of policy is different at different frequency. After taking the average of the spectral densities at each frequency over the simulated series, we can finally obtain the expected frequency-specific variances under different policy regimes.

Our quantitative experiments provide several interesting findings. First, tightening LTV ratio is nearly ineffective in stabilizing key macro variables almost at every frequency. Second, the extended Taylor rule has several perverse effects on the aggregate economy mostly at the low frequency; output fluctuations are amplified almost at every frequency while the negative effect is the greatest at the low frequency; it amplifies loan fluctuations relatively at the low frequency so that it achieves the original goal of macroprudential policies only at the relatively high frequency (higher than 8 years/cycle) and amplifies the volatility of inflation rate almost at every frequency (lower than 8 years/cycle). In this sense, the extended Taylor rule is (relatively) successful at some frequency which is originally designated to stabilize at the cost of amplifying fluctuations at other frequencies. Third, the countercyclical capital requirement policy is very successful in stabilizing loan fluctuations at every frequency so that it satisfies the original goal described in BIS (2010), unlike the two policies above. However, it also incurs sizable costs; it amplifies both output and inflation fluctuations at every frequency. The adverse effects on these variables are more severe at the relatively high frequency, which is in contrast to the extended Taylor rule.

In summary, the design limit of the macroprudential policies considered in our paper is two folds. First, they can stabilize loan fluctuations at least at some frequencies while its effectiveness becomes lower at the other frequencies, sometimes even amplifying the financial market fluctuations. Second, even when the loan market is stabilized, the volatility of other key macro variables which are especially important for the real sector, becomes higher. In other words, if the output stabilization is the implicit dual-objective of macroprudential policies, current well-known policies are not successful. More importantly, these policies amplify the volatility of inflation at every frequency, which is exactly contrary to the main objective of the central bank. Lastly, we also find that more aggressive macroprudential policies amplify negative effects on output stabilization while their positive effects on loan stabilization become greater. In other words, our findings further raise the importance of carefully designed macroprudential policies in order to minimize such adverse effects.

Related Literature. Our paper is related to the literature that analyzes the role and the effectiveness of macroprudential policies. The conventional method to evaluate the performances of policies adopted by the previous literature is the welfare-cost approach; they compute the value of lifetime utility under different policy regimes and compare them using the compensational variation in terms of consumption.[1] One stream of literature focuses on measuring the welfare cost of policies. Van Den Heuvel (2008) measures the welfare cost of bank capital requirements and shows that the regulations produce 0.1% to 1% loss in consumption in the U.S economy. Nguyen (2014) applies a general equilibrium model to the dynamic banking sector to show that the increase in bank capital requirements to the optimal level can produce welfare gains greater than 1% of lifetime consumption. Another stream of the litera-

[1]See Schmitt-Grohé and Uribe (2006) for details of the approach.

ture tries to answer in which situations macroprudential policies are effective. Benes and Kumhof (2015) shows countercyclical bank capital requirements can create a precautionary motive to banks when the creditworthiness (or riskiness) of borrowers depreciates. Bailliu, Meh, and Zhang (2015) compare different sets of macroprudential regimes and find that welfare gains are largest when macroprudential policies react to financial shocks rather than productivity shocks. Lastly, a group of literature search for the optimal coordination between monetary and macroprudential policies. Quint and Rabanal (2014), and Suh (2012) find the optimal simple rule for monetary and macroprudential policies in the Euro Area and the U.S, respectively. Collard, Dellas, Diba, and Loisel (2014), and Angeloni and Faia (2013) support the view that implementing macroprudential policies along with monetary policies is important due to risk-taking behaviors by banks. On the other hand, Kiley and Sim (2014) find that an optimal monetary policy without macroprudential policies is sufficient to ensure efficiency even under the financial shock. Woodford (2012) suggests a modified inflation targeting framework to take account of financial stability concerns alongside traditional stabilization objectives. Our approach is unique since we focus on the frequency-specific effects of implementing macroprudential policies while the studies mentioned above use the welfare-cost approach to weigh policy benefits.

This paper is also related to the literature that applies the notion of design limit approach to macroeconomics. Brock, Durlauf, and Rondina (2008) and Brock, Durlauf, and Rondina (2013) show that unless the central bank implements a policy that is optimally designed to stabilize the economy at every frequency, the monetary policy can have unexpected negative effects at certain frequencies. The main difference between our paper and their series of papers is that we study the frequency-specific effects of macroprudential policies using a medium-scale New Keynesian model with financial frictions while they consider the small-scale New Keynesian model to derive the optimal monetary policy; to our best knowledge, our paper is the first to consider the possible design limit of macroprudential policies.

The remainder of this paper is organized as follows. We first introduce the notion of frequency specific effects of policy and design limit approach in Section 2. Our model is then introduced in Section 3 with parameterization and preliminary analysis in Section 4. Key findings from the model are presented in Section 5. In Section 6, we conclude the paper.

2 Frequency-specific effects and Design-Limit Approach

In this section, we introduce the primary concepts and steps taken in our main quantitative exercises.[2] Suppose that we have a covariance-stationary macro variable $\{Y_t\}_{t=-\infty}^{\infty}$, which is defined in the *time domain*. This variable oscillates over time so that it can be described as the weighted sum of periodic functions of the form cosine and sine functions. Then the spectral density function of the time series Y_t, $s_Y(\omega)$, can be described as follows.

$$s_Y(\omega) = \frac{1}{2\pi}\left[\sum_{k=-\infty}^{\infty} \lambda_k \exp(-i\omega k)\right] \tag{2.1}$$

where $\omega \in [0, \pi]$ is the frequency, λ_k is the k-th autocovariance of Y_t, and $i = \sqrt{-1}$. Then using De Moivere's theorem, symmetry of autocovariance, and properties of cosine and sine functions, we can obtain the spectral density of the following form:

$$s_Y(\omega) = \frac{1}{2\pi}\left[\lambda_0 + 2\sum_{k=1}^{\infty} \lambda_k \cos(\omega k)\right] \tag{2.2}$$

The spectral density function provides the information on the extent to which a specific frequency contributes to the variance of the series. To see this, we plot the spectral density of (1) a white noise process (Figure 2.1a) and (2) an AR (1) process (Figure 2.1b). In particular, we set the standard deviation of the innovation terms in each series as 0.038 and the persistence term ρ as 0.95 for the AR (1) process.[3] The horizontal axis denotes the frequency from low (0.13) to high (3.14) and the vertical axis denotes the spectral density corresponding to each frequency. Since the white noise process is i.i.d. across time, the contributions of variances at each frequency are equivalent in Figure 2.1a. However, for AR (1) process, the long-run frequency contributes more to the dynamics of the simulated series since it is generated to be very persistent over time.

One good property of the spectral density is that the sum of all spectral density is equal to the variance of the variable. Formally,

$$\mathbb{V}(Y_t|R_i) = \int_{-\pi}^{\pi} S_{Y_t|R_i}(\omega)d\omega \tag{2.3}$$

where R_i is the policy regime i under which the time series Y_t is simulated and $\mathbb{V}$ denotes variance. Hence, we can interpret spectral density at each frequency as the variance at each frequency.

[2] Some parts of this section are from Hamilton (1994).

[3] Simulated series are with $T = 1,000$ (total period) and $N = 1,000$ (numbers of simulation).

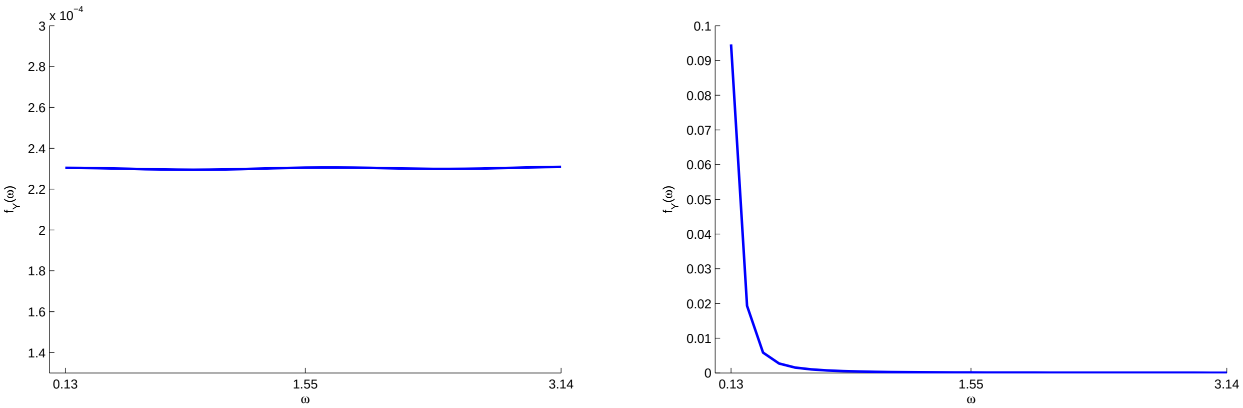

Figure 2.1a: White Noise   Figure 2.1b: AR(1) Process

Figure 2.1: Spectral Density Functions: Examples

We note that main objective of the (macroprudential) policies is to stabilize the economy. Put it differently, the spectral density can be used to evaluate the effectiveness of the policies, particularly on the heterogeneous effects on the variance at different frequency. This is important since a policy that is intended to stabilize the economy at some specific frequencies - a macroprudential policy is designed to stabilize the economy at the relative low frequency - may have an adverse effect at the different frequency. This property is known as the “design limit” of policy. For instance, Brock, Durlauf, and Rondina (2008) considers an example that shows a policy that is supposed to minimize the overall variance of variables can increase the variance of the series at the relatively short frequency. The finding that the welfare cost from different policies can vary across frequencies as the utility function is not time-separable (Otrok (2001b)) further raises the importance of our approach when it comes to studying the frequency-specific effects of macroprudential policies.

The steps to evaluate the effectiveness of different macroprudential policies can be described as follows.

1. Simulate the model economy exposed to all exogenous shocks in different policy regimes.

2. Compute the spectral density of the filtered series, take the average across simulations, and compare the density functions obtained from different policy regimes.

In particular, we will consider the effectiveness of macroprudential policies in comparison with the benchmark economy without any such policies. If the policy is effective at some frequency, the spectral density will become lower than that from the benchmark economy. If it has an adverse effect, the spectral density will be above that obtained from the benchmark economy. Therefore, if a policy is effective in stabilizing the economy as a whole, the spectral density of key macro variables, such as output, loan, consumption, and inflation rate, will be below the corresponding spectral density from the benchmark economy, which will be studied in details in Section 5.2.

3 The Model

The model introduced in this section is not particularly new by itself, but includes the key features of models with financial frictions. In particular, our model builds upon the model developed by Iacoviello (2014)[4]; we incorporate New-Keynesian features to the original setup by Iacoviello (2014), similar to Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015). Our strategy to keep the model consistent with the previous literature is in order to minimize the model-specific factors that can possibly affects equilibrium behaviors.

The economy consists of patient households, impatient households, entrepreneurs, retail banks, investment banks, retailers, and monetary authorities. Two financial intermediaries have different roles in the economy; retail banks lend funds to both impatient households and investment banks where they use deposits from patient households. Investment banks, however, obtain fund only from retail banks and lend to entrepreneurs. In order to obtain the hump-shape behavior of macro variables, habit formation and various adjustment costs are introduced.

3.1 Households

There is the measure of one patient household and another measure of one impatient household. As usually assumed in the literature, patient households have a higher discount factor than impatient households, namely $\beta_s > \beta_b > 0$. Hence, in equilibrium only patient households save while impatient households borrow.

3.1.1 Patient Households

The representative patient households (saver), denoted as s, solve the following expected lifetime utility maximization problem by choosing optimal consumption C_t^s, hours worked N_t^s, housing H_t^s, capital holding K_t^s and saving in the bank d_t^s, taking prices as given:

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \left[\varepsilon_t^c \ln\left(C_t^s - hC_{t-1}^s\right) + \varepsilon_t^c \varepsilon_t^h \nu_h^s \ln H_t^s - \nu_n^s \frac{(N_t^s)^{1+\phi}}{1+\phi} \right] \tag{3.1}$$

where $\beta_s \in (0,1)$ is a discount factor of patient households, $h \in [0,1]$ is a parameter that governs habit formation, $\phi > 0$ is the inverse Frisch elasticity, and $\nu_h^s > 0$ (resp. $\nu_n^s > 0$) is a relative utility parameter from housing (resp. working). ε_t^c is an exogenous shock to preference for consumption and

[4] A model in Iacoviello (2005) also shares similar features.

housing jointly,[5] and ε_t^k is an investment-specific technology shock. ε_t^h is an exogenous shock to housing preference, one of the financial shocks in our model economy.

Budget constraints for the patient households are as follows.

$$C_t^s + \frac{K_t^s}{\varepsilon_t^k} + p_t^H\left(H_t^s - H_{t-1}^s\right) + d_t + AC_{d^s,t} + AC_{K^s,t} = w_t^s N_t^s + r_t^d d_{t-1} + \left(r_t^K + \frac{1-\delta}{\varepsilon_t^k}\right) K_{t-1}^s \tag{3.2}$$

where the price of consumption goods is normalized to 1 ($P_t \equiv 1$), p_t^H is the real price of housing, r_t^d is a gross real interest rate from the deposit. Households rent capital to entrepreneurs at the rental rate r_t^K, and receive the real wage w_t^s for labor supply. We define $AC_{x,t}$, a convex real external adjustment cost for any variable x_t, as follows:

$$AC_{x,t} = \frac{\iota_x}{2}\frac{\left(x_t - x_{t-1}\right)^2}{x} \tag{3.3}$$

where $\iota_x \geq 0$ is an adjustment cost parameter and x is the steady state level for x_t.

3.1.2 Impatient Households

Similar to patient households, the representative impatient households (borrowers), denoted as b, also choose the optimal level of consumption, C_t^b, hours worked N_t^s, and housing stock H_t^b. As the discount factor of impatient households β_b is smaller than that of patient households, they prefer spending to saving and borrow from the banking sector to fund their spending. However, due to the financial friction, they cannot borrow as much as they want and lenders (retail banks) ask for collateral to secure loans. Since the only asset of impatient households is housing stock, the level of new bank loans depends on the discounted value of the house they own.

The problem of impatient households can be written as follows:

$$\max \;\; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t \left[\varepsilon_t^c \ln\left(C_t^b - hC_{t-1}^b\right) + \varepsilon_t^c \varepsilon_t^h \nu_h^b \ln H_t^b - \nu_n^b \frac{\left(N_t^b\right)^{1+\phi}}{1+\phi}\right] \tag{3.4}$$

subject to

$$C_t^b + p_t^H\left[H_t^b - H_{t-1}^b\right] + r_t^b l_{t-1}^b + AC_{l^b,t} = w_t^b N_t^b + l_t^b + \varepsilon_t^b \tag{3.5}$$

$$l_t^b \leq \rho_b l_{t-1}^b + (1-\rho_b)\left[\gamma_t^{Hb}\mathbb{E}_t \frac{p_{t+1}^H H_t^b}{r_{t+1}^b} - \varepsilon_t^b\right] \tag{3.6}$$

[5] Iacoviello (2014) interprets it as an aggregate spending shock

where l_t^b denotes bank loans, paying a gross interest rate r_t^b, and w_t^b is the real wage rate. $\varepsilon_t^b \geq 0$ is a default shock for impatient households, which is another financial shock in our model; this can be interpreted as a wealth redistribution shock between borrowers and lenders since this shock increases the net wealth of impatient household (borrower) while it lowers the net wealth of retail banks (lenders).[6] Contrary to Iacoviello (2014), we assume that the default shock also negatively affects the borrowing constraint of the impatient households in order to capture the idea that the default on existing loans can limit the level of new loans.

Equation (3.6) is the borrowing constraint of impatient households, where $\rho_b \in [0, 1]$ allows for the slow adjustment of bank loans over time.[7] Borrowers cannot borrow more than the fraction of γ_t^{Hb} of the expected value of their housing stock. Here we assume that this constraint is imposed by government policies, so called loan-to-value(LTV) ratio regulations.

The LTV ratio regulation γ_t^{Hb} is composed of two parts as follows:

$$\gamma_t^{Hb} = \gamma_0^{Hb}\varepsilon_t^{lb} - \gamma_1^{Hb}\left(\frac{p_t^H}{p^H} - 1\right) \tag{3.7}$$

where the first term is a constant LTV ratio regulation and the other term is a time-varying regulation. γ_0^{Hb} in the first term is the constant maximum LTV ratio cap, imposed by the policy, while ε_t^{lb} captures lenders' subjective perceptions of the riskiness of the housing stock. We call this shock a risk perception shock (or LTV shock). The time-varying LTV ratio regulation is one of the popular macroprudential tools to stabilize housing prices. If $\gamma_1^{Hb} > 0$, the LTV cap becomes tighter (lower) as housing prices increase. That is, it becomes more difficult for impatient households to borrow from banks with the collateral (housing) she/he holds.

3.2 Entrepreneurs

A continuum of entrepreneurs, denoted as e, produces intermediate goods X_t^e and sell at a price of p_t^X in a competitive market. They hire workers and combine them with housing stock H_{t-1}^e and capital (both produced by themselves, K_{t-1}^e, and rent from patient households, K_{t-1}^s). The Cobb-Douglass production technology can be written as:

$$X_t = \varepsilon_t^z\left(\left(K_{t-1}^e\right)^{\omega^k}\left(K_{t-1}^s\right)^{1-\omega^k}\right)^{\alpha}\left(H_{t-1}^e\right)^{\nu}\left(\left(N_t^s\right)^{\omega^n}\left(N_t^b\right)^{1-\omega^n}\right)^{(1-\alpha-\nu)} \tag{3.8}$$

[6] See Iacoviello (2014) for more discussions.

[7] For the parameters to govern slow adjustment of loans, see Canova, Coutinho, Mendicino, Pappa, Punzi, and Supera (2015).

where ε_t^z is a neutral productivity shock and $(1-\omega^k)$ and ω^n are shares of patient households' capital and labor, respectively.

Similarly to impatient households, entrepreneurs also face a borrowing constraint when making financing decisions. Their utility function is:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t \log\left(C_t^e - hC_{t-1}^e\right) \tag{3.9}$$

where $\beta_e < \beta_s$ is assumed and C_t^e is the consumption of the entrepreneur. They are subject to the following constraints:

$$\begin{aligned} C_t^e + \frac{K_t^e}{\varepsilon_t^k} + p_t^H[H_t^e - H_{t-1}^e] + w_t^s N_t^s + w_t^b N_t^b + r_t^K K_{t-1}^s + r_t^e l_{t-1}^e + AC_{K^e,t} + AC_{l^e,t} \\ = p_t^X X_t + \frac{1-\delta}{\varepsilon_t^k} K_{t-1}^e + l_t^e + \varepsilon_t^e \end{aligned} \tag{3.10}$$

$$l_t^e \leq \rho_e l_{t-1}^e + (1-\rho_e)\left(\gamma_t^{He}\mathbb{E}_t\frac{p_{t+1}^H H_t^e}{r_{t+1}^e} + \gamma_t^{Ke}K_t^e - \gamma_t^{Ne}(w_t^s N_t^s + w_t^b N_t^b) - \varepsilon_t^e\right) \tag{3.11}$$

Equation (3.10) is the budget constraint of the representative entrepreneur where r_t^e is a gross real interest rate on entrepreneur loans l_t^e. Similarly to equation (3.6), ε_t^e is a default shock to entrepreneurs, which captures losses on banks and gains from entrepreneurs. Equation (3.11) is the borrowing constraint for entrepreneurs. Contrary to impatient households, entrepreneurs can use both housing and capital stocks as collateral when borrowing from banks. γ_t^{He} and γ_t^{Ke} are the ratio of housing and capital they can pledge, respectively. γ_t^{He} shares the same implication with the LTV ratio regulation on impatient households' housing stock.

$$\gamma_t^{He} = \gamma_0^{He}\varepsilon_t^{le} - \gamma_1^{He}\left(\frac{p_t^H}{p^H} - 1\right) \tag{3.12}$$

However, the amount of loan capacity decreases due to the working capital assumption. Similarly to Iacoviello (2014), Aoki, Benigno, and Kiyotaki (2009), and Neumeyer and Perri (2005), entrepreneurs are assumed to pay for some portion of wage bills in advance, $i.e$ $\gamma_t^{Ne} \in (0,1]$. We assume $\gamma_t^{Ke} = \gamma_0^{Ke}\varepsilon_t^{le}$ and $\gamma_t^{Ne} = \gamma_0^{Ne}\varepsilon_t^{le}$, where ε_t^{le} is a risk perception shock which is applied to housing stock, capital and wage at the same time.

3.3 Retail Banks Retail banks, denoted as r, collect deposits from patient households and lend to impatient households l_t^b and investment banks l_t^i. As we assume $\beta_r < \beta_s$, retail banks prefer debt to equity.[8] To prevent banks from high leverage, regulators impose a cap on banks' capital ratio relative to the total asset. It is called as the minimum capital requirement.

The utility maximizing problem of retail banks is:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_r^t \log\left(C_t^r - hC_{t-1}^r\right) \tag{3.13}$$

subject to the following constraints:

$$C_t^r + l_t^b + l_t^i + r_t^d d_{t-1} + AC_{d^r,t} + AC_{l^{ir},t} + AC_{l^{br},t} = d_t + r_t^i l_{t-1}^i + r_t^b l_{t-1}^b - \varepsilon_t^b - \varepsilon_t^i \tag{3.14}$$

$$l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i \geq \rho_r(l_{t-1}^b + l_{t-1}^i - d_{t-1} - \varepsilon_{t-1}^b - \varepsilon_{t-1}^i) + (1-\rho_r)[\eta_t^b l_t^b + \eta_t^i l_t^i - \varepsilon_t^b - \varepsilon_t^i] \tag{3.15}$$

where r_t^i is a gross real interest rate on loans to investment banks. ε_t^b and ε_t^i in the budget constraint (3.14) are defaults shocks on loans to households and investment banks, which lower the level of bank equity. Equation (3.15) is the bank capital requirement regulation constraint. If we assume $\rho_r = 0$ and $\eta_t^b = \eta_t^i$ for simplicity, it can be rewritten as

$$\frac{(equity)}{(total\ assets)} = \frac{l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i}{l_t^b + l_t^i} \geq \eta_t^b$$

which means retail banks should retain a certain level of equity, proportional to assets.

Similarly to the LTV ratio regulation, the capital requirement regulation also consists of two parts as follows:

$$\eta_t^j = \eta_0^j + \eta_1^j \left(\frac{l_t/Y_t}{l/Y} - 1\right) \quad \text{where } j = \{b, i\} \tag{3.16}$$

where the first term is a constant capital requirement regulation and the next term is a time-varying regulation. η_0^j in the first term is the constant minimum capital requirement. The time-varying capital requirement regulation is called a counter-cyclical capital requirement regulation. It requires banks to hold more equity when loans expand much faster than output. That is, the policy is counter-cyclically

[8] The preference of debt over equity can also be introduced by tax treatment on debt, equity dilution cost, or liquidity premium on deposits.

tightened when the credit expands. Assuming $\eta_1^j > 0$, η_t^j is positively related to the deviation of loan to GDP ratio from the steady state value. In the extreme case when $\eta_t^j = 0$, the particular asset j is considered to be riskless.

3.4 Investment Banks Investment banks, denoted as i, obtain funds from the retail banks and lend to entrepreneurs. Investment banks are also subject to capital requirement regulation. The utility maximization problem of the representative investment bank is given by

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \log \left(C_t^i - hC_{t-1}^i \right) \tag{3.17}$$

subject to the following constraints:

$$C_t^i + l_t^e + r_t^i l_{t-1}^i + AC_{l^e,t} + AC_{l^i,t} = l_t^i + r_t^e l_{t-1}^e + \varepsilon_t^i - \varepsilon_t^e \tag{3.18}$$

$$l_t^i \leq \rho_i l_{t-1}^i + (1-\rho_i)[(1-\eta_t^e) l_t^e + \varepsilon_t^i - \varepsilon_t^e] \tag{3.19}$$

Budget constraint (3.18) and capital requirement constraint (3.19) (written in a form of borrowing constraint[9] are similar to other agents' constraints. We will skip the definition of η_t^e, which is exactly same with η_t^i and η_t^b.

3.5 Retailers Monopolistic competitive retailers purchase goods from the entrepreneurs in a competitive market and differentiate them into intermediate goods, as in the typical New Keynesian literature. The technology is linear: $Y_t(z) = X_t^e - F(z)$ where $F(z)$ are fixed costs to make the steady-state profit of the retailer zero. Then retailers sell intermediate goods, $Y_t(z)$, to the final goods-producing firm at a price of $P_t(z)$. Final output Y_t is given by

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.20}$$

where $\varepsilon > 1$.

The cost minimization problem of the final goods-producing firm yields the inverse demand function

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \tag{3.21}$$

[9] It is equivalent to $\frac{(equity)}{(total\ assets)} = \frac{l_t^e - l^i + \varepsilon_t^i - \varepsilon_t^e}{l_t^e} \geq l_t^e$, if $\rho^i = 0$

and the aggregate price index

$$P_t = \left(\int_0^1 P_t(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}} \tag{3.22}$$

Each retailer chooses optimal price $P_t(z)$; following Calvo (1983), the retailer can adjust the price with probability $1 - \theta$. If the retailer is not able to adjust its price, $P_t(z) = P_{t-1}(z)$. Each retailer maximizes its market value:

$$\max_{P_t(z)} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_t^s \left[P_t(z) Y_t(z) - P_t^X X_t^e \right] \tag{3.23}$$

subject to the equation (3.21). The optimal price level for firm z in period t is:

$$P_t^*(z) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X}{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j}} \tag{3.24}$$

We assume a symmetric equilibrium case where $P_t^* = P_t^*(z), \forall z$, thus the aggregate price level evolves according to

$$P_t = \left[(1-\theta)(P_t^*)^{1-\varepsilon} + \theta \left(P_{t-1} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{3.25}$$

3.6 Monetary Authority We assume that the monetary authority conducts a monetary policy following the extended Taylor rule, which incorporates the loan (or credit) as an additional determinant of the policy rate $R_{t+1}^i = r_t^i \mathbb{E}_t \pi_{t+1}$, the nominal inter-bank interest rate:

$$\frac{R_t^i}{R^i} = \left[\left(\frac{R_{t-1}^i}{R^i} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y} \right)^{\gamma_Y} \left(\frac{l_t}{l} \right)^{\gamma_L} \right]^{1-\rho_R} \right] \varepsilon_t^R \tag{3.26}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is a gross inflation rate, $l_t = l_t^b + l_t^e + l_t^i$ is a total loan in the economy. Variables without time subscript indicates steady-state levels. ρ_R is a smoothing parameter of policy rate and γ_π, γ_Y, and γ_L are feed-back parameters of corresponding variables. It becomes a standard Taylor rule if we set $\gamma_L = 0$.

3.7 Housing Market We assume that housing supply is exogenously given as $\bar{H}$. Then the housing market clearing condition is given by

$$\bar{H} = H_t^s + H_t^b + H_t^e \tag{3.27}$$

In what follows, we normalize $\bar{H}$ as one, without loss of generality.

3.8 Exogenous Shocks We have four non-financial shocks (Spending Shock (ε_t^c), Investment-specific technology shock(ε_t^k), TFP shock (ε_t^z) and monetary policy shock (ε_t^R)) and six financial shocks (housing demanding shock (ε_t^h), three default shocks (ε_t^b, ε_t^e, and ε_t^i), two risk perception shocks (ε_t^{lb} and ε_t^{le})) hence 10 exogenous shocks as total. For $x \in \{c, k, z, R, h, lb, le\}$, the exogenous shock process ε_t^x is assumed to follow an AR (1) process:

$$\log \varepsilon_t^x = \rho^x \log \varepsilon_{t-1}^x + u_t^x \tag{3.28}$$

where u_t^x is the i.i.d. shock that is normally distributed with mean 0 and variance σ_x. Default shocks $x \in \{b, e, i\}$, are defined as level instead of log level.

4 Calibration

4.1 Parameterization In calibrating parameters, we use the estimated values from Iacoviello (2014) as the parameters which are common between our and his model. If the parameters were not present in his paper, we might use parameter values that are generally used in the literature. For instance, we set the patient households discount factor at 0.9925 to target 3% annual risk-free interest rate. As in Iacoviello (2014), our value for capital depreciation is higher than the typical number in the literature, 0.025, because housing is the additional factor of production which does not depreciate. Following the standard NK-DSGE literature, the elasticity of substitution for intermediate varieties, ε, is calibrated as 11 to target the steady state mark-up at 10%. The coefficients in the Taylor rule are also usual numbers to ensure the unique equilibrium of the model. Parameters related to macroprudential policies will be described later. Table 4.1 shows our benchmark calibration for the parameters and Table 4.2 represents the parameterization for exogenous shocks used in our model.

4.2 Basic Results: Impulse Response Functions We will compare four model economies which only differ in the macroprudential policies implemented. The benchmark economy (Model 1) is set to

Table 4.1: Benchmark Calibration (Benchmark Economy)

Parameter	Value	Description
β_s	0.9925	Discount factor, patient household
β_b	0.94	Discount factor, impatient household
β_e	0.94	Discount factor, entrepreneur
β_r, β_i	0.945	Discount factor, banks
ν_s^h, ν_b^h	0.075	Housing preference parameter
ν_s^n, ν_b^n	2	Labor preference parameter
ϕ	1	Inverse Frisch elasticity
δ	0.035	Rate of capital depreciation
h	0.8	Habit formation
α	0.35	Total capital share in production
ω_n	0.67	Wage share of patient household
ω_k	0.64	Capital share of patient household
ν	0.04	Housing share in production
ε	11	Elasticity of substitution for intermediate varieties
θ	0.78	Calvo Parameter
ι_{K^s}	1.73	Capital adjustment cost, household
ι_{K^e}	0.59	Capital adjustment cost, entrepreneur
ι_{d^s}	0.10	Deposit adjustment cost, household
ι_{d^r}	0.14	Deposit adjustment cost, bank
ι_{l^b}	0.37	Household loan adjustment cost, household
$\iota_{l^{br}}$	0.47	Household loan adjustment cost, retail bank
ι_{l^e}	0.07	Entrepreneur loan adjustment cost, entrepreneur
$\iota_{l^{ei}}$	0.06	Entrepreneur loan adjustment cost, investment bank
$\iota_{l^{ir}}$	0.47	Interbank loan adjustment cost, retail bank
ι_{l^i}	0.05	Interbank loan adjustment cost, investment bank
ρ_b	0.70	Speed of deleveraging, impatient household
ρ_e	0.65	Speed of deleveraging, entrepreneur
ρ_r	0.24	Speed of loan adjustment, retail bank
ρ_i	0.70	Speed of loan adjustment, investment bank
γ_0^{Hb}, γ_0^{He}	0.7	LTV ratio on housing
γ_0^{Ke}	0.9	LTV ratio on entrepreneur capital
η_0^b	0.08	Minimum capital requirement, households loan
η_0^e	0.08	Minimum capital requirement, entrepreneur loan
η_0^i	0.08	Minimum capital requirement, interbank loan
ρ^R	0.75	Interest rate inertia, monetary policy
γ^π	1.5	Inflation targeting parameter, monetary policy
γ^Y	0.125	Output targeting parameter, monetary policy
γ^L	0	Financial targeting parameter, monetary policy

have 70% LTV ratio ($\gamma^{Hb} = 0.7$), 8% constant minimum capital requirement, and the monetary policy neutral to loan changes. Other economies have different policy measures as follows:

Table 4.2: Benchmark Calibration: Exogenous Shocks

Parameter	Value	Description
ρ^C	0.994	Autocorr. of spending shock
ρ^K	0.916	Autocorr. of investment-specific technology shock
ρ_z	0.839	Autocorr. of TFP shock
ρ_H	0.932	Autocorr. of housing demand shock
ρ_b	0.969	Autocorr. of default shock (impatient HH)
ρ_e	0.992	Autocorr. of default shock (entrepreneur)
ρ_i	0.916	Autocorr. of default shock (investment bank)
ρ_{lb}	0.839	Autocorr. of LTV shock (impatient HH)
ρ_{le}	0.873	Autocorr. of LTV shock (entrepreneur)
σ_C	0.025	s.d of spending shock
σ_K	0.025	s.d of investment-specific technology shock
σ_z	0.007	s.d. of TFP shock
σ_H	0.0348	s.d. of housing demand shock
σ_b	0.0013	s.d. of default shock (impatient HH)
σ_e	0.0011	s.d. of default shock (entrepreneur)
σ_i	0.0011	s.d. of default shock (investment bank)
σ_{lb}	0.0115	s.d. of Risk perception(LTV) shock (impatient HH)
σ_{le}	0.0204	s.d. of Risk perception(LTV) shock (entrepreneur)

- Model 1: No macroprudential policy (benchmark economy)
- Model 2: Extended Taylor rule ($\gamma^L = 0.0125$)
- Model 3: Counter-cyclical capital requirement ($\eta_j^1 = 0.25$ for $j = b, i, e$)
- Model 4: Time-varying LTV ratio regulation on housing ($\gamma_1^{Hb} = \gamma_1^{Eb} = 0.3$)

The parameter values are chosen in the following sense. In Model 2, the responsiveness to loans from the central bank is the same as to output. In Model 3, the capital requirement increases by 0.25% in response to 1% increase in loans. In Model 4, LTV regulation decreases by 3% in response to 10% increase in housing prices.

Before we present our main results, we first show the impulse response functions of our model economies to selected exogenous shocks (TFP shock, risk perception shock (on entrepreneur), default shock (on entrepreneur), housing preference shock, and monetary policy shock) to check if the models behave well consistently with the usual economic intuition and different policies result in the different impulse responses to the variables.

4.2.1 Comparing Effects of Different Policies with IRFs First of all, Figure 4.1 is the collection of impulse response functions to one-time-one-unit shock to the aggregate productivity. As usually argued in the literature, different sets of macroprudential policies do not have much impact on the response of macro variables when the real shock hits the economy. Shapes are consistent with the usual intuition; key variables all increase due to high productivity in this economy.

Figure 4.2 is the collection of impulse response functions to one-time-one-unit shock to risk perception (LTV) on entrepreneurs. Since positive shock to the risk perception of entrepreneurs means that they can borrow more with the same asset values, this shock stimulates the economy. Overall, different macroprudential policies are still not effective to lower the responsiveness of the economy to the shock. One noticeable observation is that inflation moves in opposite direction when the extended Taylor rule is implemented (Model 2): this comes from the fact that interest rates further change from the changes in loan size. Interest rates increase more in this case so that the incentive to consume decreases. However, the incentive of consumers for consumption smoothing, which is enhanced by the habit formation, requires less changes in interest rates. As a result, the inflation rate needs to decrease in equilibrium so that interest rates do not much change by the Taylor rule. The response of consumption is greater than that under different policies, which is directly following. Lastly the greater response of consumption and output in Model 3 versus Model 1 comes from the successfully controlled loan market; the circumstance where less loan in equilibrium implies less deposit is required by financial intermediary so that patient households consumption instead increases with more incomes. In what follows, explanations on the effects of the extended Taylor rule on the inflation rate and those of countercyclical capital requirement policies are omitted since intuitions are the same.

Figure 4.3 plots impulse response functions to the positive shock to the default of entrepreneurs; as this is a negative redistribution shock to banks, they will lower loans, which triggers economic downturns. While some macroprudential policies (countercyclical capital requirements in particular) seem to be effective in lowering loan fluctuations, most other policies are not effective. Figure 4.4 presents impulse response functions to the shock to the housing preference of households. Given fixed housing supply, the increase in housing demand means soaring housing prices, which grows the wealth of average agents in this economy. Hence, the economy experiences boom. Note again, macroprudential policies are not effective in terms of lowering the responsiveness of the economy to the exogenous shocks. Lastly, Figure 4.5 shows the impulse responses to the positive monetary policy shock. Higher policy rates usually dampen the economy; all variables exhibit patterns commonly observed in the recession, and different macroprudential policies do not show different patterns.

Therefore, the quick preview of the effectiveness of different policies with impulse response functions shows that in most cases macroprudential policies do not achieve their goals aiming to lower the effects of exogenous shocks. If any, it is mostly observed from the policy requiring banks to accumulate countercyclical capital buffers, or when the financial shock hits the economy.

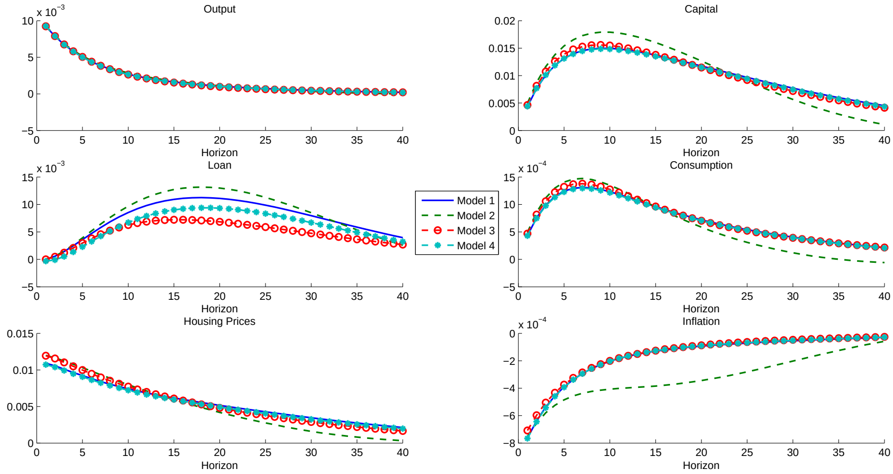

Figure 4.1: **Impulse Response Functions: Productivity Shock**
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

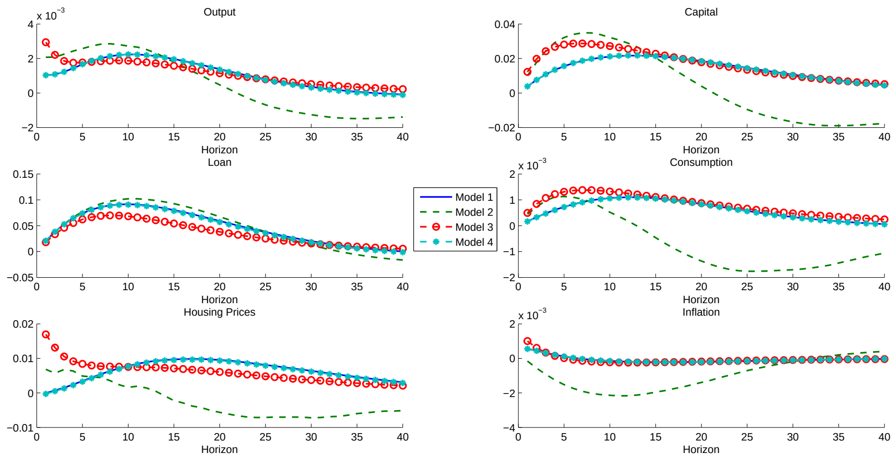

Figure 4.2: Impulse Response Functions: Risk Perception Shock to Entrepreneur
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

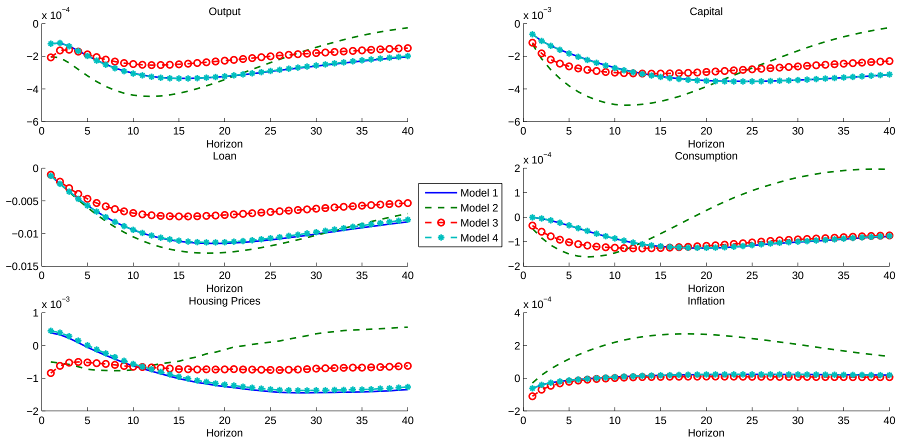

Figure 4.3: Impulse Response Functions: Default Shock to Entrepreneur

Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

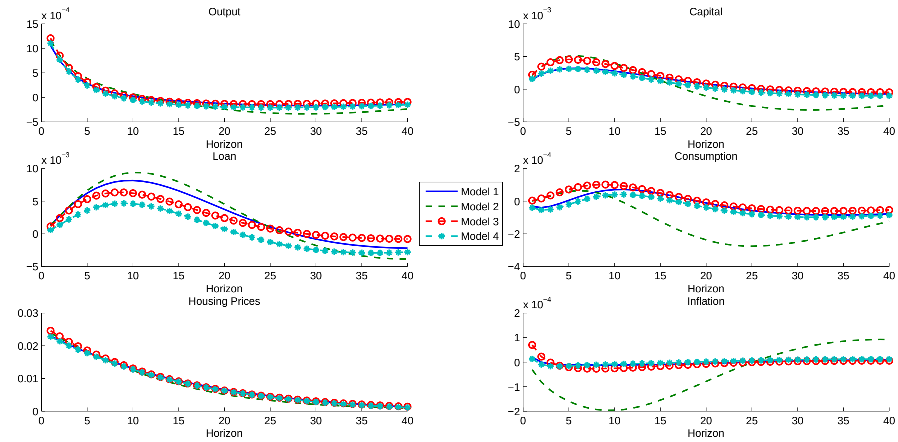

Figure 4.4: Impulse Response Functions: Housing Preference Shock

Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

4.2.2 Effectiveness of More Aggressive Policy

In this section, we evaluate the performance of more aggressive macroprudential policies. In particular, we consider the macroprudential policies on countercyclical capital requirements since it seems to be more effective than other policies in stabilizing loan fluctuations. Figure 4.6 to 4.9 show the impulse response functions to the productivity shock, default shock to entrepreneurs, housing preference shock, and monetary policy shock, respectively. The thick blue line represents the responses from the benchmark economy, the dotted green line represents the economy with the weak macroprudential policy ($\eta_j^1 = 0.25$ for $j = b, i, e$), and the circled red line represents the

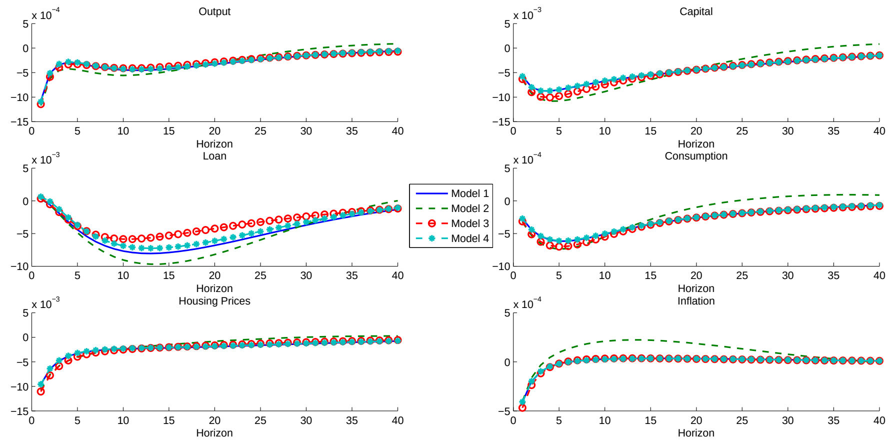

Figure 4.5: Impulse Response Functions: Monetary Policy Shock
Note: Model 1 is the benchmark economy, Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with time-varying LTV on impatient household.

economy with the aggressive macroprudential policy ($\eta_j^1 = 0.5$ for $j = b, i, e$ hence the coefficient is twice from the weak policy economy).

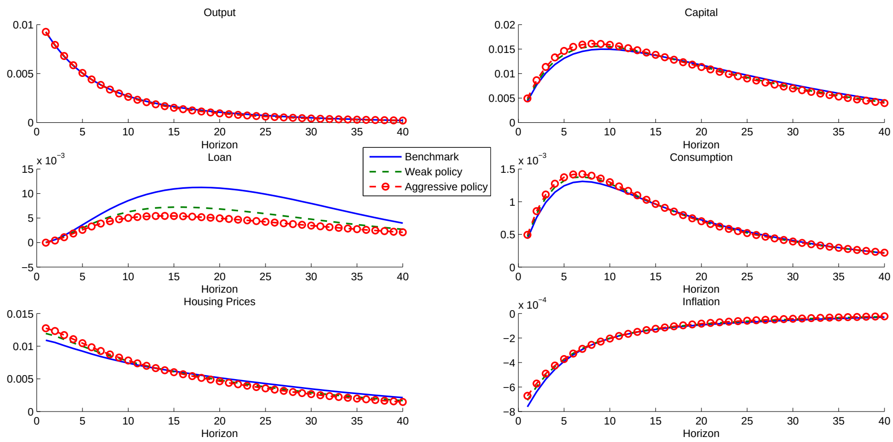

Figure 4.6: Impulse Response Functions: Productivity Shock
Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

We first note that in terms of lowering loan responses, the aggressive macroprudential policy is mostly effective. In other words, the aggressive policy achieves its goal to stabilize the financial market more effectively than the weak policy does. However, the negative impacts on the real sector are also amplified (see Figure 4.7 to 4.9); this comes from the fact that smaller response in loan results in smaller response

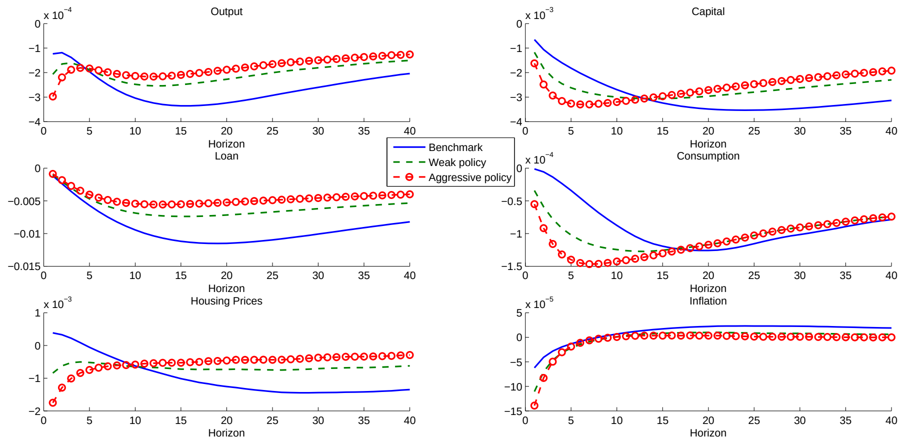

Figure 4.7: Impulse Response Functions: Default Shock to Entrepreneur

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Strong policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

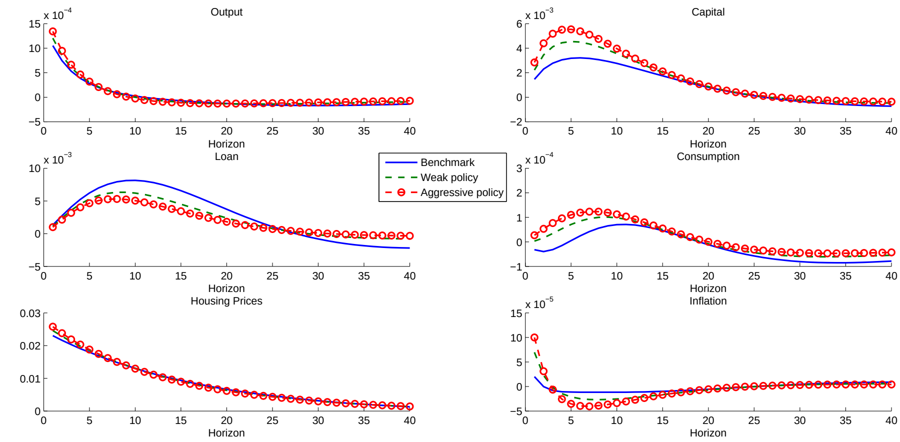

Figure 4.8: Impulse Response Functions: Housing Preference Shock

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Strong policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

in deposit, which thus increases the responsiveness of consumption and output as discussed above.

5 Frequency-Specific Effects from the Model Economy

5.1 Volatility at Business Cycle Frequency Before we proceed to analyze the frequency-specific effectiveness of macroprudential policies, we first report the variance of the key variables at the business

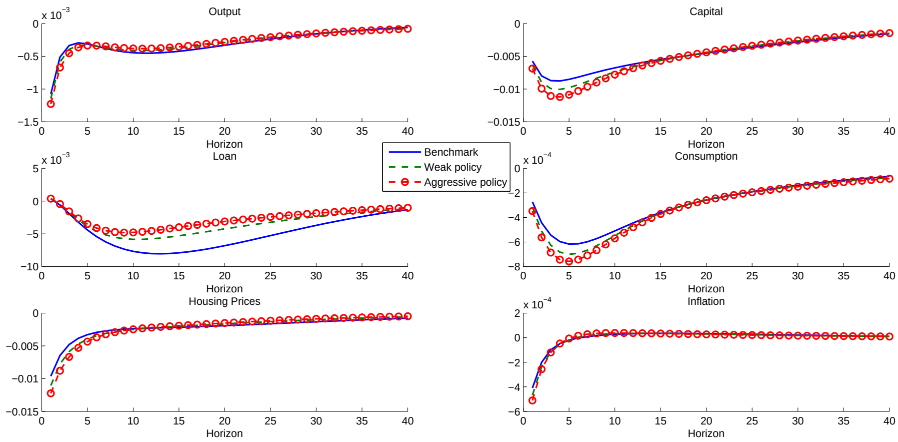

Figure 4.9: Impulse Response Functions: Monetary Policy Shock

Note: 'Benchmark' denotes the economy with $\eta_j^1 = 0$, 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.25$, and 'Strong policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$ for $j = b, i, e$.

cycle frequency. In particular, we simulate the model economy 250 times with each simulation setting the total period at $1,024$. We then filter each of the series with $\kappa = 200$ by applying the band-pass filter (Baxter and King (1999)) to obtain the series of frequency between 2 years/cycle and 8 years/cycle where κ is the number of leads/lags used in the approximation of the filtering[10]. We then compute the variance of each series[11], and take the average of variances from each simulation. Table 5.1 summarizes the results.

Table 5.1: Relative Variance of Key Variables

	Var(Y)	Var(C)	Var(π)	Var (L)	Var(L/Y)	Var(p^H)
Model 2	1.11	3.47	2.78	1.22	1.12	2.96
Model 3	1.01	1.12	1.28	0.76	0.86	1.15
Model 4	1.00	1.00	1.00	0.94	0.97	1.01
Model 5	1.04	1.16	1.60	0.58	0.77	1.47

Note: All values are relative to the benchmark economy. Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, Model 4 is the economy with tight LTV on housing. Model 5 is the economy with aggressive countercyclical capital requirement policy.

Several observations are noteworthy. First, the macroprudential policy that tightens the LTV ratio in response to housing prices (Model 4) is not essentially effective at all. Second, the extended Taylor rule (Model 2) much more amplifies the variance of consumption and output than other policies when

[10] See Otrok (2001b) for choosing $T = 1,024$ and $\kappa = 200$.

[11] Here we use the variance since the spectral density provides variance at each frequency. Information contained in the analysis is the same when we use the standard deviation.

compared to the benchmark economy, while it does not achieve its original goal of stabilizing the financial market. Even worse, the volatility of inflation rate increases dramatically. Hence, from the traditional perspective on the role of central bank, which emphasizes output and inflation stabilities, it is not recommended for the central bank to directly take loan into account. Third, stabilizing the financial market at the business cycle frequency is the most successful in the economies with countercyclical capital requirement policies (Model 3 (weak policy) and Model 5 (aggressive policy)), while the output fluctuations are somewhat similar to the benchmark economy. Finally, the aggressive macroprudential policy (Model 5) is more effective in the financial market stabilization than the weak policy (Model 3) at the cost of amplifying the fluctuation in the real sector.

5.2 Frequency-Specific Effects of Different Policies

We now turn to our main analysis, which analyzes the frequency-specific effects of macroprudential policies. In so doing, we take the steps described in Section 2. Again, the number of simulations is 250 and the total period of each simulation is set to be $1,024$. Since our main purpose here is to evaluate the performance of different policies versus the benchmark economy without any macroprudential policy, we compute the spectral density of each variable of the specific policy regime and compare the results to those of the benchmark model. Since our main purpose here is to evaluate the performance of different policies versus the benchmark economy without any macroprudential policy, we compute the spectral density of each variable of the specific policy regime and compare the results to those of the benchmark model. We particularly consider following macro variables in this exercise; output, loan, consumption, loan to output ratio, housing price, and inflation rate. Figure 5.1 to 5.6 show the results. In each figure, the vertical axis denotes the spectral density relative to model 1 at each frequency and the horizontal axis denotes frequencies from low to high frequency.

Firstly, when the extended Taylor rule is implemented in the economy, output fluctuations are amplified at every frequency while the negative effect is greater at the relatively low frequency. In contrast, the countercyclical capital requirement policy is also overall ineffective in stabilizing output compared to the benchmark economy, but the negative effect is much smaller than the economy with the extended Taylor rule. In addition, the output fluctuations are more amplified at the relatively high frequency. Therefore, this policy, even though it could be effective in stabilizing the financial market as shown later, clearly has a slight negative effect on the real sector, which is one of the design limit of the macroprudential policy. This finding is in line with our previous discussions on impulse response functions; lower loan volatility would also decrease deposit volatility, which would amplify the consumption fluctuation and then the output fluctuation. Consumption volatility exhibits similar patterns; it is dramatically exacerbated under the extended Taylor rule. Almost at every frequency, the relative magnitude of the consumption volatility is about three times higher than that of the benchmark economy. This is because interest rates directly respond to loan so that the Euler equation more affects consumption than economies under different policy regimes. In contrast, the LTV policy (Model 4) does not affect consumption and output fluctuations once again.

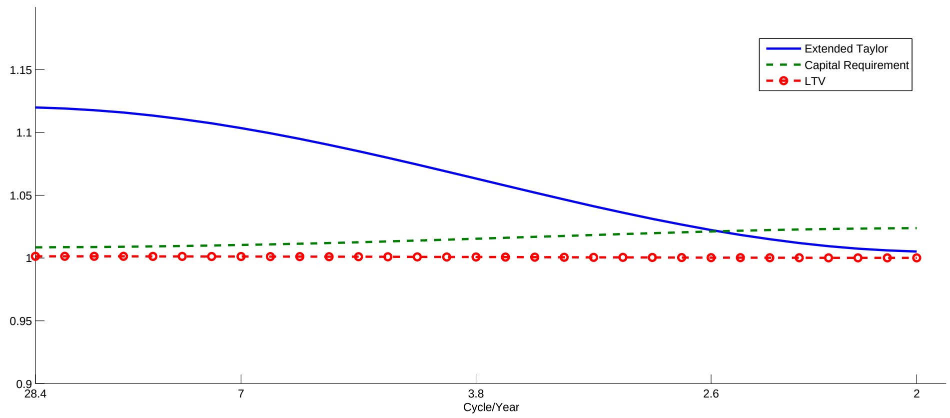

Figure 5.1: Frequency-Specific Effects: Output
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

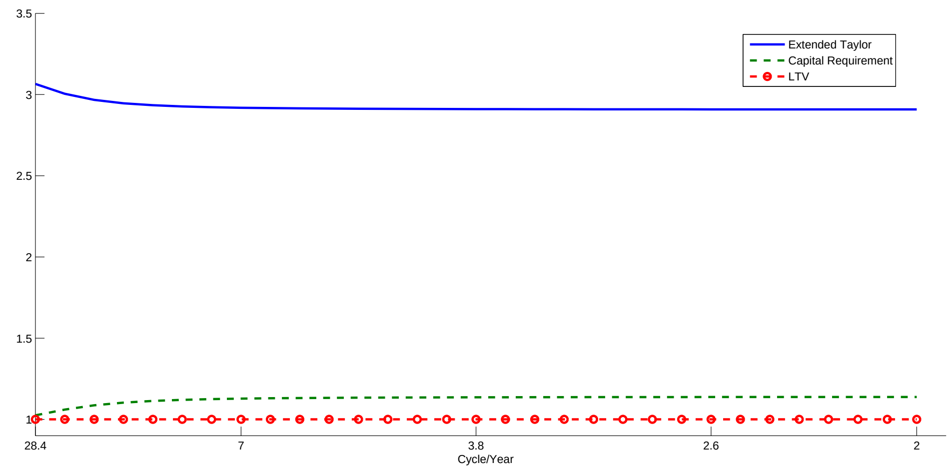

Figure 5.2: Frequency-Specific Effects: Consumption
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

How about loan fluctuations, which is the main objective of macroprudential policies? Figure 5.3 shows the result; importantly, the extended Taylor rule satisfies the main goal only at the relatively high frequency (higher than 8 years/cycle). It rather amplifies the financial market fluctuations at the relatively low frequency. In the sense that the financial cycle exhibits a much lower frequency (8 to 32 years/cycle), this implies that the extended Taylor rule does not achieve its goal to stabilize the financial cycles. Together with our observations from Figure 5.1, this further implies that the extended Taylor affects output and loan fluctuations in the opposite direction to the original objective of the macroprudential policy. On contrary, the countercyclical capital requirement policy is very effective in stabilizing loan fluctuations regardless of the frequency. However the output fluctuations are amplified at every frequency under this policy, which is similar to our observation for the extended Taylor rule, even though the size is small. This implies that the macroprudential policies that effectively stabilize the financial market pay the cost to destabilize the goods market. In the mean time, the policy to tighten LTV ratio on housing (Model 4) is not that effective in the financial market as well as in the goods market. This means that tightening LTV ratio is not overall effective in any market in our model economies. The observation from Figure 5.5 is consistent with the above discussions. Figure 5.4 and 5.5 provide the similar findings.

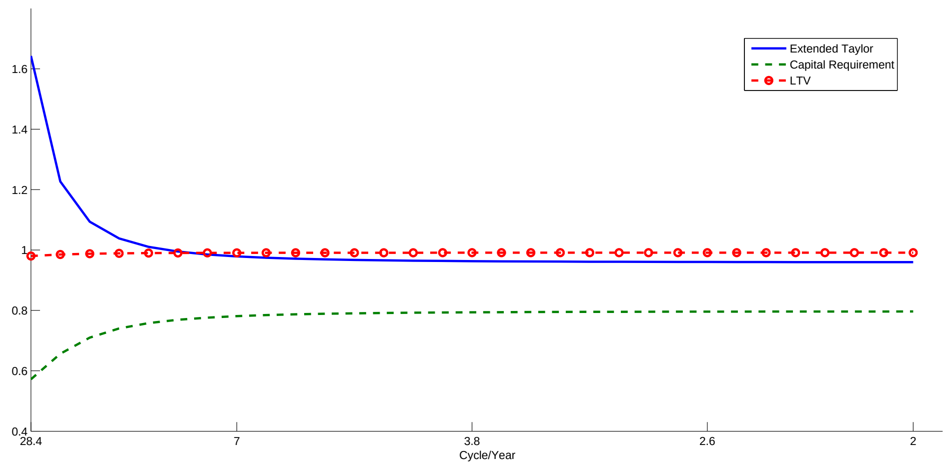

Figure 5.3: Frequency-Specific Effects: Loan
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

We first point out that tightening LTV ratio is not overall effective since it is not the policy that reacts to the changes in loan; other policies, in contrast, react to the changes in loan (potentially loan-to-output ratio). Rather, LTV policy responds to housing prices, but its propagation is relatively week compared to the effects of the overall loan on the decision of the banks. In contrast, the countercyclical capital requirement policy is effective since it really reacts to the fluctuations in loan to GDP ratio. It is effective in lowering loan fluctuations at every frequency since banks accumulates enough capital during boom times in preparation for possible losses during recessions, which results in less changes in the overall loan level.

Lastly, we also consider the effects of macroprudential policies on inflation rate (Figure 5.6). First, we can observe that the negative effect of the extended Taylor rule (Model 2) on inflation rate volatility is observed for the frequency lower than about 3 years/cycle. The high frequency which cannot be observed from the business cycle analysis can actually lower the volatility of inflation rate while the variance of inflation rate becomes much greater as we consider lower frequency. Therefore the negative effect on the inflation rate volatility is maximized at the lowest frequency under the extended Taylor rule. The intuition for this result is in line with our discussions on the impulse response functions in Section 4.2.1; as the interest rate directly responds to loan in this economy, the inflation rate should adjust in order for consumption smoothing. As a result, the inflation rate volatility becomes higher in this economy. The countercyclical capital requirement policy (Model 3) on the fluctuations of inflation rate is amplified at every frequency. Hence, our result provides an important lesson for the central bank; the currently well-known macroprudential policies can ruin the goal of the central bank which aims to stabilize the inflation rate.

In summary, our exercise implies that the currently well-known macroprudential policies can be effective in stabilizing the financial market, especially when the countercyclical capital requirement policy is implemented. However, there is a substantial cost to achieve its effectiveness in the financial sector; it can amplify the fluctuations in the real sector or in inflation rate. Especially the performance of the extended Taylor rule is worse than that of countercyclical capital requirement policy in every dimension. Contrary to these policies, tightening LTV ratio on housing does not much affect the economy compared to other policies. Hence, the design limit of macroprudential policies in our model is two folds. First, the financial sector stabilization is associated with the real sector (or inflation rate) destabilization. Second, the effectiveness of policy is different across frequencies; for instance, the inflation rate volatility increases as the frequency becomes lower when the extended Taylor rule is implemented, while the opposite is observed when the countercylcical capital requirement policy is implemented.

Next, we study if the effects of the aggressive macroprudential policy can be different from the less aggressive macroprudential policy. As in the previous section, we only consider the countercyclical capital

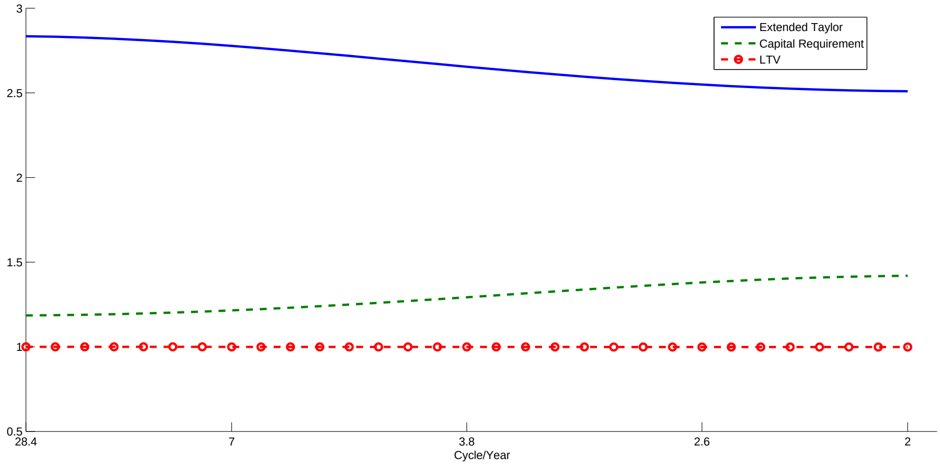

Figure 5.4: Frequency-Specific Effects: Housing Price
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

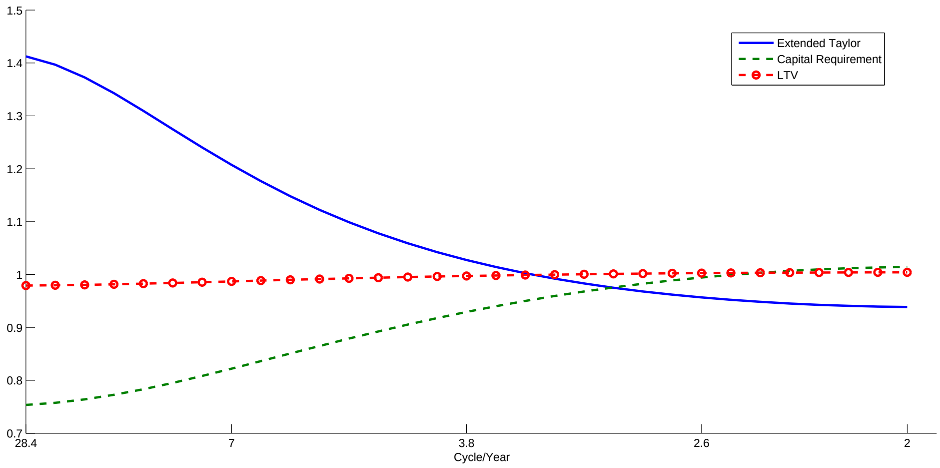

Figure 5.5: Frequency-Specific Effects: Loan to Output
Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

requirement policy since it is the most effective policy in terms of the loan market stabilization. We again compute the relative spectral densities of the economy with the weak macroprudential policy ($\eta_j^1 = 0.25$ for $j = b, i, e$, thick blue line) and with the aggressive macroprudential policy ($\eta_j^1 = 0.5$ for $j = b, i, e$, circled green line), and plot the spectral densities in Figure 5.7.

Two conclusions can be drawn from the figures. First, the aggressive policy is more effective in stabilizing the financial market than the weak one as expected. This achievement, however, leads to amplified

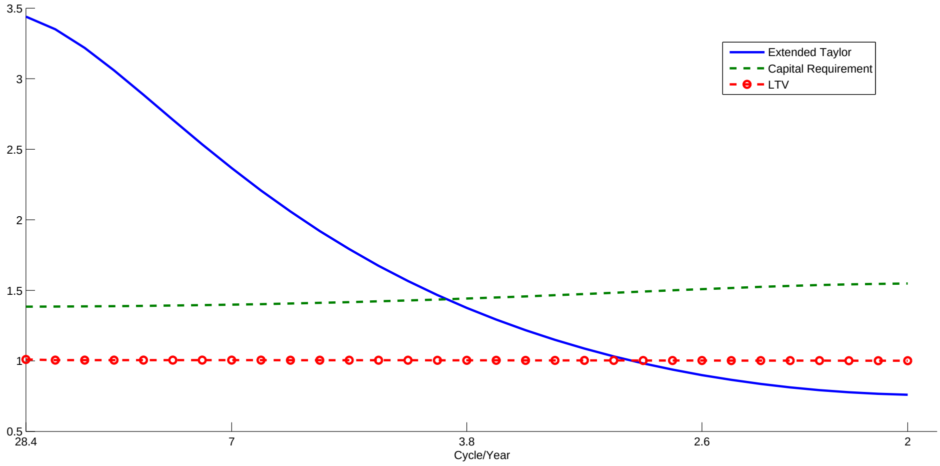

Figure 5.6: Frequency-Specific Effects: Inflation

Note: Relative volatility compared to model 1 at each frequency; Model 2 is the economy with extended Taylor rule, Model 3 is the economy with countercyclical capital requirement policy, and Model 4 is the economy with tight LTV on housing.

fluctuations in the real sector compared to the weak policy. Particularly, the output, consumption, and inflation rate volatility at each frequency are much more exacerbated under the aggressive policy. Even worse, the negative effects on the real sector increase as frequencies become higher. If monetary authorities care about the aggregate fluctuations at the business cycle frequency, which is usually defined as the fluctuations between about 2 years/cycle and 8 years/cycle, the aggressive macroprudential policy is not recommended; it increases output volatility compared to the benchmark economy approximately by 10% point, consumption volatility approximately by 30% point, and inflation rate volatility by more than 100% point at the business cycle frequency.

6 Conclusion

We evaluate the performances of various macroprudential policies in this paper within the financial sector augmented New Keynesian model. Our results from the conventional macroprudential policy are somewhat negative. A policy that makes LTV ratio respond to housing prices does not change the equilibrium properties of the model. Moreover, the extended Taylor rule is not recommended since its perverse effects on the real sector and inflation rate are observed at almost every frequency. On the other hand, the countercyclical capital requirement policy achieves its original goal of the stabilization of the financial market at the cost of destabilizing the inflation rate and consumption.

One possibility to understand our results is that the macroprudential policies considered in our analysis

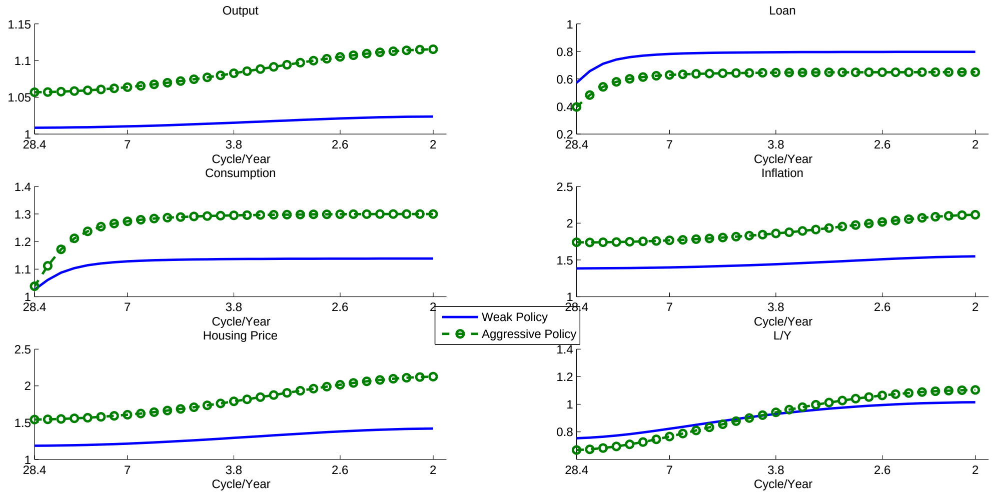

Figure 5.7: Frequency-Specific Effects: Weak vs. Aggressive Policy
Note: Relative volatility compared to model 1 at each frequency. 'Weak policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 0.5$, and 'Aggressive policy' is the economy with countercyclical capital requirement policy with $\eta_j^1 = 1$ for $j = b, i, e$.

are not optimally designed; they are originally intended to stabilize the financial market, hence there can exist an unexpected cost associated with such policies, as shown in our paper. Therefore, the future research needs to find the optimally designed macroprudential policy so that it can achieve its main goal while minimizing adverse effects. Other possibility is that the model we consider in this paper is not able to fully capture the positive effects of such policies on the real sector. In this regard, the studies with alternative models could be another direction of the future research.

References

Angeloni, I., and E. Faia (2013): "Capital Regulation and Monetary Policy with Fragile Banks," *Journal of Monetary Economics*, 60(3), 311–324.

Aoki, K., G. Benigno, and N. Kiyotaki (2009): "Capital Flows and Asset Prices," in *NBER International Seminar on Macroeconomics 2007*, ed. by R. Clarida, and F. Giavazzi, pp. 175–216. The University of Chicago Press.

Bailliu, J., C. Meh, and Y. Zhang (2015): "Macroprudential rules and monetary policy when financial frictions matter," *Economic Modelling*, 50, 148–161.

Baxter, M., and R. G. King (1999): "Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series," *Review of Economics and Statistics*, 81(4), 575–593.

Benes, J., and M. Kumhof (2015): "Risky bank lending and countercyclical capital buffers," *Journal of Economic Dynamics and Control*, 58, 58–80.

BIS (2010): "Macroprudential Instruments and Frameworks : A Stocktaking of Issues and Experiences," *Committee on the Global Financial System*.

Borio, C. (2012): "The financial cycle and macroeconomics: What have we learnt?," *BIS Working Papers No. 395*.

Brock, W. A., S. N. Durlauf, and G. Rondina (2008): "Frequency-Specific Effects of Monetary Policy Rules," *American Economic Review Papers and Proceedings*, 98(2), 241–245.

——— (2013): "Design Limits and Dynamic Policy Analysis," *Journal of Economic Dynamics and Control*, 37(12), 1421–1436.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Canova, F., L. Coutinho, C. Mendicino, E. Pappa, M. T. Punzi, and D. Supera (2015): "The Domestic and the International Effects of Financial Disturbances," *Working Paper*.

Collard, F., H. Dellas, B. Diba, and O. Loisel (2014): "Optimal Monetary and Prudential Policies," *Working Paper*.

Drehmann, M., C. Borio, and K. Tsatsaronis (2012): "Characterising the Financial Cycle : Don't Lose Sight of the Medium Term!," *BIS Working Papers*.

Hamilton, J. D. (1994): *Time Series Analysis*. Princeton University Press.

Iacoviello, M. (2005): "House Prices, Collateral Constraints and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739–764.

——— (2014): "Financial Business Cycles," *Review of Economic Dynamics*, Forthcoming.

Kiley, M. T., and J. W. Sim (2014): "Optimal Monetary and Macroprudnetial Policies: Gains and Pitfalls in a Model of Financial Intermediation," *Working Paper*.

Neumeyer, P. A., and F. Perri (2005): "Usiness Cycles in Emerging Economies: the Role of Interest Rates," *Journal of Monetary Economics*, 52(2), 345–380.

Nguyen, T. T. (2014): "Bank Capital Requirements : A Quantitative Analysis," *Working Paper*.

Otrok, C. (2001a): "On Measuring the Welfare Cost of Business Cycles," *Journal of Monetary Economics*, 47, 61–92.

——— (2001b): "Spectral Welfare Cost Functions," *International Economic Review*, 42(2), 345–367.

Quint, D., and P. Rabanal (2014): "Monetary and Macroprudential Policy in an Estimated DSGE Model of the Euro Area," *International Journal of Central Banking*.

Schmitt-Grohé, S., and M. Uribe (2006): "Optimal Simple and Implementable Monetary and Fiscal Rules: Expanded Version," *NBER Working Paper No. 12402*.

Suh, H. (2012): "Macroprudential Policy: Its Effects and Relationship to Monetary Policy," *Federal Reserve Bank of Philadelphia Working Paper No.12-28*.

Van Den Heuvel, S. J. (2008): "The welfare cost of bank capital requirements," *Journal of Monetary Economics*, 55(2), 298–320.

Woodford, M. (2012): "Inflation Targeting and Financial Stability," *Sveriges Riksbank Economic Review*, 1.

A Equilibrium Conditions

A.1 Patient Household λ_t^s the Lagrangian multiplier attached to the budget constraint.

$$[C_t^s] \qquad \lambda_t^s = \frac{\varepsilon_t^c}{C_t^s - hC_{t-1}^s} - h\beta_s \mathbb{E}_t \left[\frac{\varepsilon_{t+1}^c}{C_{t+1}^s - hC_t^s} \right] \tag{A.1}$$

$$[K_t^s] \qquad \frac{1}{\varepsilon_t^k} + \frac{\partial AC_{K^s,t}}{\partial K_t^s} = \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} \left(r_{t+1}^K + \frac{1-\delta}{\varepsilon_{t+1}^K} \right) \right] \tag{A.2}$$

$$[H_t^s] \qquad p_t^H = \frac{\varepsilon_t^c \varepsilon_t^h \nu_h^s}{H_t^s \lambda_t^s} + \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} p_{t+1}^H \right] \tag{A.3}$$

$$[N_t^s] \qquad w_t^s = \frac{\nu_n^s (N_t^s)^\phi}{\lambda_t^s} \tag{A.4}$$

$$[d_t^s] \qquad 1 + \frac{\partial AC_{d,t}}{\partial d_t} = \beta_s \mathbb{E}_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} r_{t+1}^d \right] \tag{A.5}$$

$$[\lambda_t^s] \quad C_t^s + \frac{K_t^s}{\varepsilon_t^k} + p_t^H \left[H_t^s - H_{t-1}^s \right] + d_t + AC_{d^s,t} + AC_{K^s,t} = w_t^s N_t^s + r_t^d d_{t-1} + \left(r_t^K + \frac{1-\delta}{\varepsilon_t^k} \right) K_{t-1}^s \tag{A.6}$$

A.2 Impatient Household λ_t^b (resp. μ_t^b) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^b] \qquad \lambda_t^b = \frac{\varepsilon_t^c}{C_t^b - hC_{t-1}^b} - h\beta_b \mathbb{E}_t \left[\frac{\varepsilon_{t+1}^c}{C_{t+1}^b - hC_t^b} \right] \tag{A.7}$$

$$[H_t^b] \qquad p_t^H = \frac{\varepsilon_t^c \varepsilon_t^h \nu_h^b}{H_t^b \lambda_t^b} + \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1}^b}{\lambda_t^b} p_{t+1}^H \right] + \mu_t^b (1-\rho_b) \gamma_t^b \mathbb{E}_t \left[\frac{p_{t+1}^H}{r_{t+1}^b} \right] \tag{A.8}$$

$$[N_t^b] \qquad w_t^b = \frac{\nu_n^b (N_t^b)^\phi}{\lambda_t^b} \tag{A.9}$$

$$[l_t^b] \qquad 1 - \frac{\partial AC_{l^b,t}}{\partial l_t^b} = \mu_t^b + \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1}^b}{\lambda_t^b} (r_{t+1}^b - \rho_b \mu_{t+1}^b) \right] \tag{A.10}$$

$$[\lambda_t^s] \qquad C_t^b + p_t^H \left[H_t^b - H_{t-1}^b \right] + r_t^b l_{t-1}^b + AC_{l^b,t} = w_t^b N_t^b + l_t^b + \varepsilon_t^b \tag{A.11}$$

$$[\mu_t^b] \qquad l_t^b = \rho_b l_{t-1}^b + (1-\rho_b) \left[\gamma_t^{Hb} \mathbb{E}_t \frac{p_{t+1}^H H_t^b}{r_t^b} - \varepsilon_t^b \right] \tag{A.12}$$

A.3 Entrepreneur λ_t^e (resp. μ_t^e) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^e] \qquad \lambda_t^e = \frac{1}{C_t^e - hC_{t-1}^e} - \beta_e \mathbb{E}_t \frac{h}{C_{t+1}^e - hC_t^e} \tag{A.13}$$

$$[H_t^e] \qquad p_t^H = \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} p_{t+1}^H (1 + r_{t+1}^H) \right] + \mu_t^e (1-\rho_e) \gamma_t^{He} \frac{p_{t+1}^H}{r_{t+1}^e} \tag{A.14}$$

$$[K_t^e] \qquad \frac{1}{\varepsilon_t^k} + \frac{\partial AC_{K^e,t}}{\partial K_t^e} = \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} (1 + r_{t+1}^K - \delta) \right] + \mu_t^e (1-\rho_e) \gamma_t^{Ke} \tag{A.15}$$

$$[N_t^s] \qquad (1 + (1-\rho_e)\gamma_t^{Ne}) w_t^s N_t^s = (1-\alpha-\nu)\omega^n p_t^X X_t \tag{A.16}$$

$$[N_t^b] \qquad (1 + (1-\rho_e)\gamma_t^{Ne}) w_t^b N_t^b = (1-\alpha-\nu)(1-\omega^n) p_t^X X_t \tag{A.17}$$

$$[r_t^K] \qquad r_t^K = \alpha(1-\omega^k) p_t^X X_t / K_{t-1}^s \tag{A.18}$$

$$[l_t^e] \qquad 1 - \frac{\partial AC_{l^e,t}}{\partial l_t^e} = \mu_t^e + \beta_e \mathbb{E}_t \left[\frac{\lambda_{t+1}^e}{\lambda_t^e} (r_{t+1}^e - \rho_e \mu_{t+1}^e) \right] \tag{A.19}$$

$$[\lambda_t^e] \quad C_t^e + \frac{K_t^e}{\varepsilon_t^k} + p_t^H [H_t^e - H_{t-1}^e] + w_t^s N_t^s + w_t^b N_t^b + r_t^K K_{t-1}^s + r_t^e l_{t-1}^e + AC_{K^e,t} + AC_{l^e,t}$$
$$= p_t^X X_t + \frac{1-\delta}{\varepsilon_t^k} K_{t-1}^e + l_t^e + \varepsilon_t^e \tag{A.20}$$

$$[\mu_t^e] \qquad l_t^e = \rho_e l_{t-1}^e + (1-\rho_e) \left(\gamma_t^{He} \mathbb{E}_t \frac{p_{t+1}^H H_t^e}{r_{t+1}^e} + \gamma_t^{Ke} K_t^e - \gamma_t^{Ne} (w_t^s N_t^s + w_t^b N_t^b) - \varepsilon_t^e \right) \tag{A.21}$$

where $X_t = \varepsilon_t^z \left(K_{t-1}^e\right)^{\alpha\omega^k} \left(K_{t-1}^s\right)^{\alpha(1-\omega^k)} \left(H_{t-1}^e\right)^{\nu} \left(N_t^s\right)^{(1-\alpha-\nu)\omega^n} \left(N_t^b\right)^{(1-\alpha-\nu)(1-\omega^n)}$. From the firm's production function, $p_t^H r_t^H = \nu p_t^X X_t / H_{t-1}^e$

A.4 Retail Banks λ_t^r (resp. μ_t^r) the Lagrangian multiplier attached to the budget constraint (resp. capital requirement constraint).

$$[C_t^r] \qquad \lambda_t^r = \frac{1}{C_t^r - hC_{t-1}^r} - \beta_r \mathbb{E}_t \frac{h}{C_{t+1}^r - hC_t^r} \tag{A.22}$$

$$[d_t] \qquad 1 - \frac{\partial AC_{d^r,t}}{\partial d_t} = \mu_t^r + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} (r_{t+1}^d - \rho_r \mu_{t+1}^r) \right] \tag{A.23}$$

$$[l_t^b] \qquad 1 + \frac{\partial AC_{l^{br},t}}{\partial l_t^b} = \mu_t^r (1 - (1-\rho_r)\eta_t^b) + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} (r_t^b - \rho_r \mu_{t+1}^r) \right] \tag{A.24}$$

$$[l_t^i] \qquad 1 + \frac{\partial AC_{l^{ir},t}}{\partial l_t^i} = \mu_t^r (1 - (1-\rho_r)\eta_t^i) + \beta_r \mathbb{E}_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} (r_t^i - \rho_r \mu_{t+1}^r) \right] \tag{A.25}$$

$$[\lambda_t^r] \qquad C_t^r + l_t^b + l_t^i + r_t^d d_{t-1} + AC_{d,t} + AC_{l^i,t} + AC_{l^b,t} = d_t + r_t^i l_{t-1}^i + r_t^b l_{t-1}^b - \varepsilon_t^b - \varepsilon_t^i \tag{A.26}$$

$$[\mu_t^r] \quad l_t^b + l_t^i - d_t - \varepsilon_t^b - \varepsilon_t^i = \rho_r (l_{t-1}^b + l_{t-1}^i - d_{t-1} - \varepsilon_{t-1}^b - \varepsilon_{t-1}^i) + (1-\rho_r)[\eta_t^b l_t^b + \eta_t^i l_t^i - \varepsilon_t^b - \varepsilon_t^i] \tag{A.27}$$

$$\tag{A.28}$$

A.5 Investment Banks λ_t^i (resp. μ_t^i) the Lagrangian multiplier attached to the budget constraint (resp. borrowing constraint).

$$[C_t^i] \qquad \lambda_t^i = \frac{1}{C_t^i - hC_{t-1}^i} - \beta_i \mathbb{E}_t \frac{h}{C_{t+1}^i - hC_t^i} \tag{A.29}$$

$$[l_t^e] \qquad 1 + \frac{\partial AC_{l^{ei},t}}{\partial l_t^e} = \mu_t^i (1-\rho_i)(1-\eta_t^e) + \beta_i \mathbb{E}_t \left[\frac{\lambda_{t+1}^i}{\lambda_t^i} r_{t+1}^e \right] \tag{A.30}$$

$$[l_t^i] \qquad 1 + \frac{\partial AC_{l^i,t}}{\partial l_t^i} = \mu_t^i + \beta_i \mathbb{E}_t \left[\frac{\lambda_{t+1}^i}{\lambda_t^i} (r_t^i - \rho_i \mu_{t+1}^i) \right] \tag{A.31}$$

$$[\lambda_t^i] \qquad C_t^i + l_t^e + r_t^i l_{t-1}^i + AC_{l^e,t} + AC_{l^i,t} = l_t^i + r_t^e l_{t-1}^e + \varepsilon_t^i - \varepsilon_t^e \tag{A.32}$$

$$[\mu_t^i] \qquad l_t^i = \rho_i l_{t-1}^i + (1-\rho_i)[(1-\eta_t^e) l_t^e + \varepsilon_t^i - \varepsilon_t^e] \tag{A.33}$$

A.6 Retailers

$$\max_{P_t^*(z)} \quad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta \beta_s)^j \lambda_{t+j}^s \left[P_t^*(z) - P_{t+j}^X \right] Y_{t+j}(z)$$

subject to

$$Y_{t+j}(z) = \left(\frac{P_t^*(z)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}$$

We can rewrite the problem as:

$$\max_{P_t^*(z)} \quad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} \left[(P_t^*(z))^{1-\varepsilon} - P_{t+j}^e \left(P_t^*(z)\right)^{-\varepsilon} \right]$$

$$[P_t^*(z)] \qquad \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} \left[(1-\varepsilon) P_t^*(z) + \varepsilon P_{t+j}^X \right] = 0$$

$$P_t^*(z) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X}{\mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j}} \equiv \frac{\varepsilon}{\varepsilon - 1} \frac{F_{1,t}}{F_{2,t}} \tag{A.34}$$

$$\begin{aligned} F_{1,t} &= \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} P_{t+j}^X \\ &= \lambda_t^s P_t^{\varepsilon} Y_t P_t^X + \theta\beta_s \mathbb{E}_t F_{1,t+1} \qquad \text{(A.35)} \\ F_{2,t} &= \mathbb{E}_t \sum_{j=0}^{\infty} (\theta\beta_s)^j \lambda_{t+j}^s P_{t+j}^{\varepsilon} Y_{t+j} \\ &= \lambda_t^s P_t^{\varepsilon} Y_t + \theta\beta_s \mathbb{E}_t F_{2,t+1} \qquad \text{(A.36)} \end{aligned}$$

or equivalently, with $P_t^* = P_t^*(z)$,

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{f_{1,t}}{f_{2,t}} P_t \tag{A.37}$$

$$f_{1,t} = \lambda_t^s Y_t p_t^e + \theta\beta_s \mathbb{E}_t f_{1,t+1} \pi_{t+1}^{\varepsilon+1} \tag{A.38}$$

$$f_{2,t} = \lambda_t^s Y_t + \theta\beta_s \mathbb{E}_t f_{2,t+1} \pi_{t+1}^{\varepsilon} \tag{A.39}$$

where $f_{1,t} = F_{1,t}/P_t^{\varepsilon+1}$ and $f_{2,t} = F_{2,t}/P_t^{\varepsilon}$.

Therefore, substituting equation (A.37) to the following equation yields

$$\pi_t = \left[(1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\varepsilon} + \theta\right]^{\frac{1}{1-\varepsilon}}$$
$$= \left[(1-\theta)(\frac{\varepsilon}{\varepsilon - 1}\frac{f_{1,t}}{f_{2,t}}\pi_t)^{1-\varepsilon} + \theta\right]^{\frac{1}{1-\varepsilon}} \quad \text{(A.40)}$$

hence the equations (A.38), (A.39), and (A.40) implicitly determines $f_{1,t}$, $f_{2,t}$, and π_t.

A.7 Market Clearing Conditions

We have

$$1 \equiv \bar{H} = H_t^s + H_t^b + H_t^e \quad \text{(A.41)}$$
$$Y_t = C_t + K_t + (1-\delta)K_{t-1} \quad \text{(A.42)}$$
$$l_t = l_t^e + l_t^i + l_t^b \quad \text{(A.43)}$$

where $C_t = C_t^s + C_t^b + C_t^e + C_t^e + C_t^r + C_t^i$, $K_t = K_t^e + K_t^s$, and $Y_t = X_t - \frac{Y}{1-\varepsilon}$. Finally, we have a monetary policy rule (3.26).